CHAPTER 3

The Value of Information

Often there is an option in a decision to collect additional information, and this chapter presents procedures for determining when it is worth collecting additional information.

3.1 Calculating the Value of Perfect Information

We begin by determining the value of **perfect** information. Perfect information removes all uncertainty about the outcomes for the decision alternatives. While there is rarely an option in real-world business decisions that would actually remove all uncertainty, the value of perfect information provides an easily calculated benchmark about the worth of collecting additional information. If all the available options for collecting information cost more than the value of perfect information, then these options do not need to be analyzed in further detail. This is because **imperfect** information cannot be worth more than **perfect** information.

Box 3.1: The Value of Perfect Information

No source of information can be worth more than the **value of perfect information**.

The following example illustrates how to compute the value of perfect information.

Example 3.1

Xanadu Traders. This is a continuation of the Xanadu Traders decision that was discussed in Example 1.8. (Figure 1.6 shows this decision.) Suppose a source of perfect information existed that would let Xanadu know if the import license would be issued.

> **Question 3.1:** How much money would it be worth to obtain perfect information about issuance of the import license?

Figure 3.1 shows a decision tree with this (hypothetical) source of perfect information. The topmost three branches of the root node for this decision tree are the same as the corresponding branches in Figure 1.6. The lowest branch of the root node is the perfect information alternative. At a quick glance, the perfect information may appear to be similar to the wait alternative, since for both of these alternatives George Xanadu learns whether the license will be issued before he purchases the molyzirconium. However, with the perfect information alternative, information is available *immediately* about whether the license will be issued. Therefore, with the perfect information alternative, Xanadu does not run the risk that a competitor will purchase the molyzirconium before he learns whether the license will be issued.

Since the probability is 0.5 that the license will be issued, this is the probability that the perfect information source will report that the license will be issued. After learning this perfect information, Xanadu then can decide whether or not to purchase the molyzirconium. Of course, if Xanadu learns that the license will be issued, then he purchases the molyzirconium, and if Xanadu learns that the license will not be issued, then he does not purchase the molyzirconium.

By the standard calculation procedure, it is determined that the perfect information alternative has an expected value of \$1.5 million, and this is shown on the Figure 3.1 decision tree. Since the best alternative without perfect information (purchase) has an expected value of \$1 million, the value of perfect information is \$1 5 − \$1 0 = \$0 5 million. Therefore, this places an upper limit on how much it is worth paying for *any* information about whether the license will be issued. It cannot be worth paying more than \$0.5 million for such information, since \$0.5 million if the value of perfect information. ■

3.2 The Value of Imperfect Information

The calculation procedure is more complicated for determining the value of imperfect information. This procedure is illustrated by the following example.

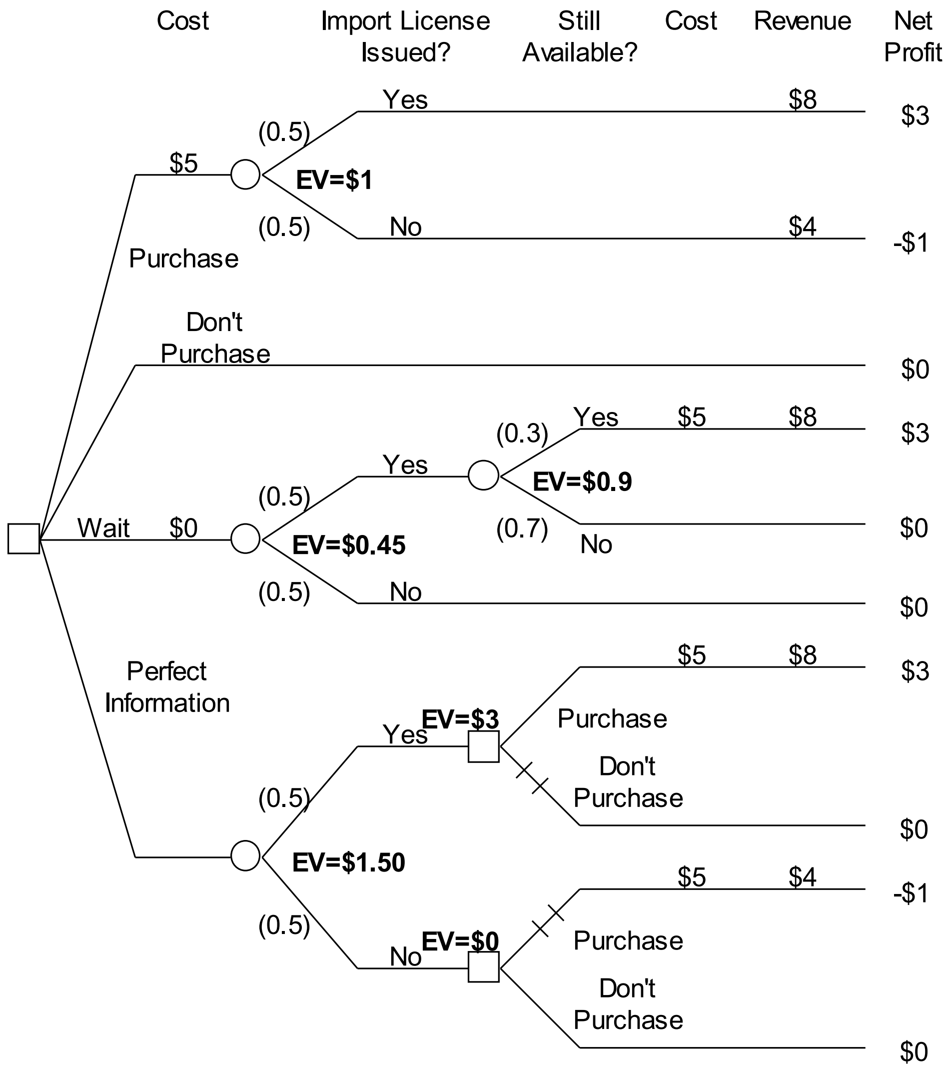

Figure 3.1 *Xanadu Traders decision tree, with perfect information alternative*

Example 3.2

Xanadu Traders. Now consider a potential source of imperfect information in the Xanadu Traders case last discussed in Example 3.1. We continue with the discussion between Daniel Analyst and George Xanadu.

Analyst: Is there any way of obtaining additional information about the chances of obtaining a license other than waiting and seeing what happens? Perhaps there is something that doesn t take as long as waiting for the import approval.

Xanadu: Well, there s always John S. Lofton. He is a Washington-based business consultant with good connections in the import licensing bureaucracy. For a fee, he will consult his contacts and see if they think the license will be granted. Of course, his assessment that the license will come through is no guarantee. If somebody in Congress starts screaming, they might shut down imports from Zeldavia. They are really upset about this in the Industrial Belt, and Congress is starting to take some heat. On the other hand, even if Lofton thinks the license won t come through, he might be wrong. He has a pretty good record on calling these things, but not perfect. And he charges a lot for making a few telephone calls.

Analyst: How good has he been?

Xanadu: He s done some assessments for me, as well as other people I know. I d say in cases where the import license was ultimately granted, he called it right 90% of the time. However, he hasn t been so good on the license requests that were turned down. In those cases, he only called it right 60% of the time.

Analyst: You commented earlier that he was expensive. How much would he charge?

Xanadu: This is a pretty standard job for him. His fee for this type of service is $10,000.

> **Question 3.2:** Should Xanadu hire Lofton, and if so, what is the maximum amount that he should pay Lofton for his services?

We know from our earlier analysis of the value of perfect information in Example 3.1 that the maximum amount that it could possibly be worth to purchases Lofton s services is $0.5 million. Since he would only charge $10,000 it is possible that it would be worth purchasing his services. However, it is clear from the discussion above that Lofton often makes mistakes, and perhaps Xanadu would not learn enough to warrant paying Lofton the $10,000.

A partial decision tree for the Hire Lofton alternative is shown in Figure 3.2. To simplify this tree, the possibility of hiring Lofton and then still waiting to see if the import license is issued has been eliminated from the tree. In this tree, each of the two subtrees starting from the decision nodes after the outcome of predict import license issued? has the same structure. Each of these subtrees also has the same structure as the top two branches of the decision tree in Figure 1.6. However, as we will see below, the probabilities on the import license issued? branches differ in Figure 3.2 from those in the Figure 1.6 tree. ■

3.3 Flipping a Probability Tree

In order to complete the analysis of the alternative in Example 3.2, we need the probabilities for the two branches labeled predict import license issued? in Figure 3.2. Additionally, we need the probabilities for the two sets of branches under the label import license issued? Unfortunately, as often happens in real problems, the information presented about Lofton s accuracy in his predictions is

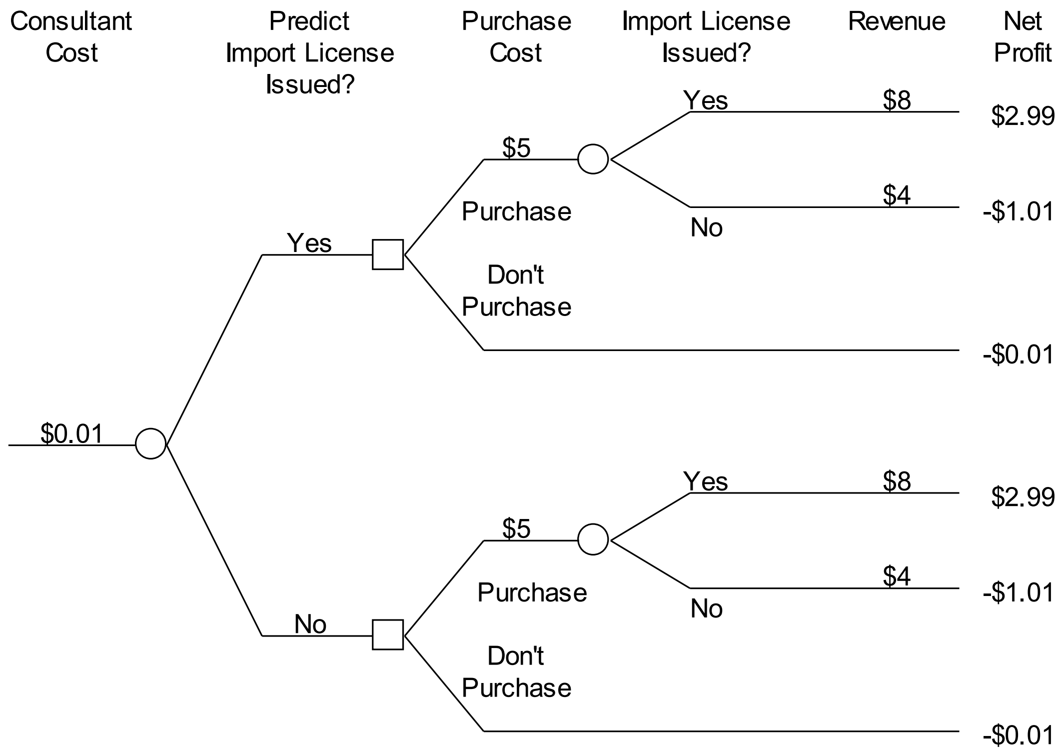

Figure 3.2 *Hire consultant alternative*

not in a form that directly provides the required probabilities. Figure 3.3 shows in probability tree form the information that is given above about the accuracy of Lofton. The root node on the left side of the tree shows the probabilities for import license issued? specified in earlier discussions of this decision problem. The two chance nodes on the right side of the tree show the probabilities that Lofton will call the licensing decision right, based on the conversation presented in Example 3.2 between Daniel Analyst and George Xanadu.

Comparing Figure 3.2 with Figure 3.3, shows that the probabilities in Figure 3.3 are backwards from what is needed to assign probabilities to the branches of the chance nodes in Figure 3.2. That is, the probability of license approval is known, as well as the probability of Lofton s different predictions, *given the actual situation* regarding license approval. However, the decision tree in Figure 3.2 requires the probability of Lofton s different predictions and the probability of license approval *given Lofton s predictions.* This is shown in Figure 3.4, where the probabilities marked A, B, C, D, E, and F are required. If these probabilities were known, then they could be inserted into the Figure 3.2 decision tree, and the expected value could be determined for the alternative of hiring Lofton.

This may seem like an odd way to present the information about Lofton s accuracy, but information about the accuracy of an information source is often available in the form of Figure 3.3 when there is a historical record about the accuracy of the source. As an example, suppose that a new test instrument has

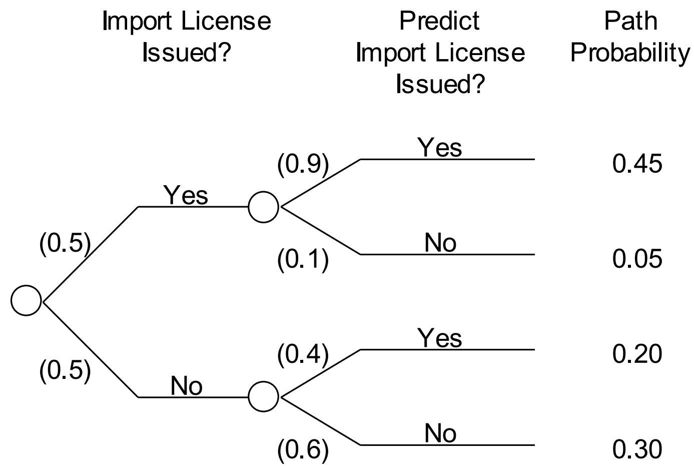

Figure 3.3 *Accuracy of consultant*

been developed to use in testing for defects in the parts that are manufactured on a production line. How would the accuracy of the test instrument be determined? Probably by using the instrument on a series of parts that have previously been tested by other methods. Thus, it would be known whether the parts that are being tested are good or bad, and hence it would be possible to determine what fraction of good parts the test instrument correctly identifies as good, and what fraction of bad parts the test instrument correctly identifies as bad. This is analogous to the way that the information is presented for Lofton in Figure 3.3.

In a similar manner, the accuracy of a proposed medical diagnostic procedure for some medical condition is often determined by applying the diagnostic procedure to patients who are known to either have the condition or not have the condition. Information from such tests would be in the form of Figure 3.3. Thus, the form of the information shown in Figure 3.3 is common, and we need to know how to use such information when analyzing the value of a potential information source.

To proceed with the analysis of the alternative of hiring Lofton, we need to flip the probabilities from the tree in Figure 3.3 to determine the probabilities needed in Figure 3.4.

Definition 3.1: Tree flipping

> **Tree flipping** is the process of calculating the probabilities for a probability tree with the order of the chance nodes reversed, as illustrated by Figures 3.3 and 3.4.

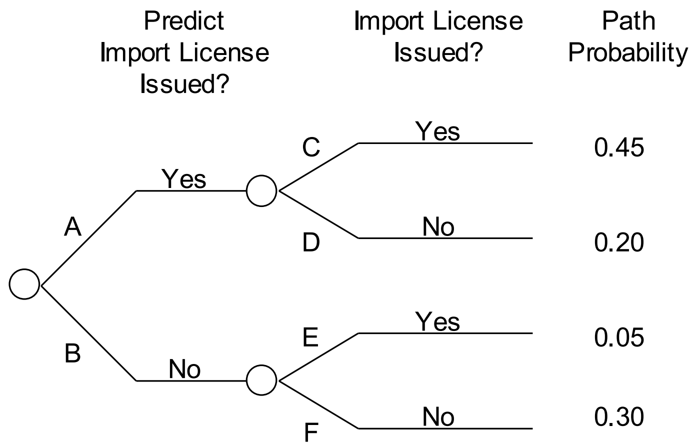

Figure 3.4 *Probabilities needed for the decision tree*

3.4 Calculating Flipped Probabilities

It is straightforward to determine the probabilities in Figure 3.4. The key to doing this is to recognize that the *paths* from the root node to the endpoints are the same in the Figure 3.3 and Figure 3.4 trees, but they are arranged in a different order. The probabilities for these paths can be determined in Figure 3.3 by following the multiplication rule for probabilities. Namely, the probabilities on the branches along a path are multiplied to determine the probability of following that path. For example, the probability of following the topmost path in Figure 3.3 is determined as $0\ 5 \times 0\ 9 = 0\ 45$.

Definition 3.2: Path probability

> A **path probability** is the probability of a particular sequence of branches from the root node to a specified endpoint in a probability tree. A path probability is determined by multiplying the probabilities on the branches included in the path.

Once the probabilities are determined for each path in Figure 3.3, they can be transferred to Figure 3.4, as shown at the right side of Figure 3.4. (The topmost and bottommost probabilities are transferred directly from the Figure 3.3 tree to the Figure 3.4 tree, and the other two path probabilities need to be reversed when they are transferred.)

Once the path probabilities are known, probabilities A and B can be determined. Probability A is the probability of a yes prediction regarding license approval, and this occurs only on the two topmost paths in the Figure 3.4 tree. Therefore, probability A is equal to the sum of the probabilities for the two topmost paths. That is, $\mathrm{A} = 0\ 45 + 0\ 20 = 0\ 65$. Similarly, probability B is

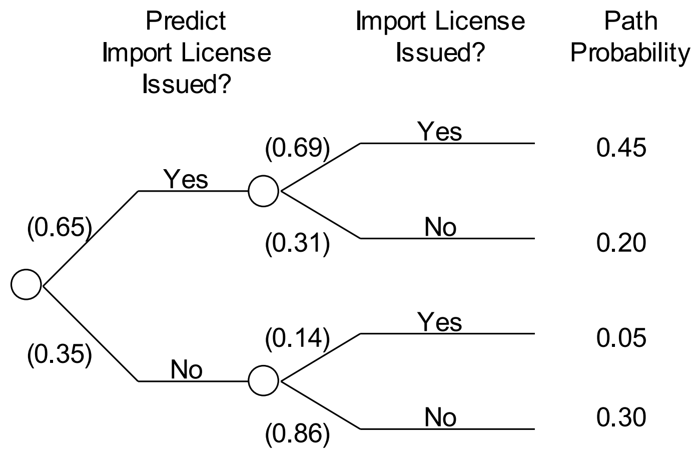

Figure 3.5 *Decision tree probabilities*

equal to the sum of the probabilities for the two bottommost paths. That is, B $= 0\ 05 + 0\ 30 = 0\ 35$.

Once A and B are known, then C, D, E, and F can be determined using the multiplication rule. Thus, $A \times$ C $= 0\ 45$, or C $= 0\ 45$ A $= 0\ 45\ 0\ 65 = 0\ 69$ (rounded). Similarly, D $= 0\ 20$ A $= 0\ 20\ 0\ 65 = 0\ 31$ (rounded), E $= 0\ 05$ B $=$ $0\ 05\ 0\ 35 = 0\ 14$ (rounded), and F $= 0\ 30$ B $= 0\ 30\ 0\ 35 = 0\ 86$ (rounded). Figure 3.5 shows the probabilities filled in for the Figure 3.4 probability tree.

3.5 Finding the Expected Value of Imperfect Information

The probabilities in Figure 3.5 can now be transferred to the tree diagram in Figure 3.2, and the expected value can be calculated for the alternative of hiring Lofton by using the same process as in earlier decision trees. The result is shown in Figure 3.6, where the expected value for this alternative is $1.13 million. Figure 1.6 shows that the best alternative without hiring Lofton only has an expected value of $1 million, and so it is worth hiring Lofton. In fact, it is worth considerably more than $10,000 to hire Lofton, since the alternative with hiring him for $10,000 is worth $1.13 million. In fact, it is worth it to hire Lofton as long as he costs less than $\$130\ 000 + \$10\ 000 = \$140\ 000$.

3.6 Exercises

3.1 This is a continuation of Exercise 1.4. Assume that all the information presented in that exercise still holds. Determine the expected value of perfect information about whether Zyz will exercise its option.

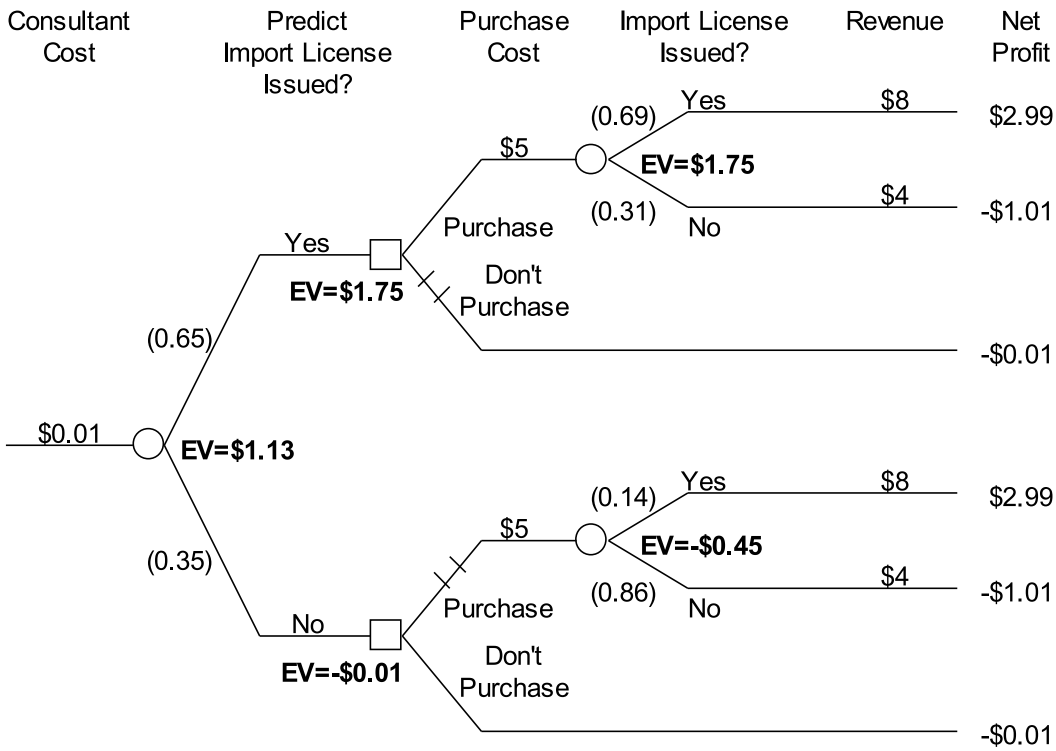

Figure 3.6 *Hire consultant alternative, with expected value calculation*

3.2 For the decision in the preceding exercise, Aba Manufacturing has created a new option: It can conduct some research and development in an attempt to lower the fixed setup cost associated with manufacturing a batch of the PC boards. This research and development would not be completed in time to influence the setup cost for the initial batch that Zyz has ordered, but would be completed before the second batch would have to be manufactured. The research and development will cost \$25,000, and there is a 0.4 probability that it will be successful. If it is successful, then the fixed setup cost per batch will be reduced by \$200,000 to \$50,000. If the research and development is not successful, then there will be no reduction in the setup cost. There will be no other benefits from the research and development besides the potential reduction in setup cost for the Zyz reorder.

(i) Using expected profit as the decision criterion, determine whether Aba should undertake the research and development.

(ii) Using expected profit as the decision criteria, determine the value of learning for certain whether the research and development will be successful before a decision has to be made about whether to initially manufacture 100,000 or 200,000 PC boards.

3.3 This is a continuation of Exercise 1.5. Assume that all the information presented in that exercise still holds. Using expected value as the decision criterion, determine the maximum amount that Kezo should pay for information about whether the antidumping tax will be imposed if this information can be obtained prior to making the ordering decision.

3.4 A college athletic department is considering a mandatory drug testing policy for all its athletes. Suppose that the test to be used will give either a positive or a negative indication. From previous testing it is known that if the tested person is a drug user there is a 0.92 probability that the test will be positive. In cases where the tested person is not a drug user, there is a 0.96 probability that the test will be negative. Assume that 10% of the athletes to be tested are drug users.

(i) Determine the probability that a randomly selected athlete will test positive for drug use.
(ii) Assuming that a randomly selected athlete tests positive, determine the probability that he or she is actually a drug user.
(iii) Assuming that a randomly selected athlete tests negative, determine the probability that he or she is actually a drug user.
(iv) In light of the results above, discuss the potential advantages and disadvantages of introducing a mandatory drug testing program using this test.

3.5 **Intermodular Semiconductor Systems, Part 2 The Value of Information.** This is a continuation of the case in Exercise 1.6. Assume that all information presented in that exercise still holds.

Analyst: Would it be possible to get a better handle on production costs before making the bid?

Iron: As I said earlier, the main issue is what it will cost to reinforce the electrotransponders to take the pressure. We could make up some material samples and borrow the high pressure chamber over in the Submersible Systems Division to do some tests. We d get some information out of that, but there would still be a lot of uncertainty. Also, it would be expensive I would have to put people on overtime to meet the bid schedule.

The main problem is that we don t have time to do very extensive testing before the bid is due. We could make up a rack of samples from materials we have in stock and take some measurements under pressure, but these materials aren t exactly the same as what we would use in the actual electrotransponders. Because of this, we would still not know for sure what we will have to do to make the electrotransponders work.

[This option was discussed at some length. Following this discussion Analyst summarizes as follows.]

Analyst: As I understand it, the result of doing material tests would be an indication that the production will either be expensive or inexpensive. If molyaluminum is going to work, it is more likely that you will get an inexpensive result while if you have to use molyzirconium you are more likely to get an expensive result.

Iron: Yes. In previous cases when we have done tests like this and molyaluminum ultimately worked, then 80% of the time we had gotten an inexpensive indication. On the other hand, when it has worked out that we needed molyzirconium, then 90% of the time we had gotten an expensive indication.

Analyst: What about if a mixture worked?

Iron: We haven t gotten very much useful information in those cases. In cases where a mixture has worked, 60% of the time we had gotten an inexpensive indication and 40% of the time it came out expensive.

Analyst: Based on our earlier discussion, I understand that if molyaluminum works the production costs will be \$2,000 per unit, if molyzirconium is needed the costs will be \$6,000 per unit, and if a mixture works the costs will be \$4,000.

Iron: That s correct for the 100-unit quantity we are discussing here.

Reynolds: How much would the material tests cost?

Iron: There will be a lot of hand labor. I ll go talk with my people and get a figure back to you in a couple of hours.

[Iron leaves the meeting and later reports that it would cost \$7,000 to conduct the material tests.]

(i) Determine the expected value of perfect information about what material must be used.

(ii) Determine whether it is worth doing the experiment that is outlined above.