Machine Learning 2

DS 4420 - Spring 2020

Structured prediction, I

Byron C Wallace

Today

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- We'll switch gears a bit today and consider *structured* spaces where an instance is associated with multiple labels

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- We'll switch gears a bit today and consider *structured* spaces where an instance is associated with multiple labels
- Material today based (mostly) on

Daumé, CIML reading

Structured output spaces

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$$(x_i, y_i)$$

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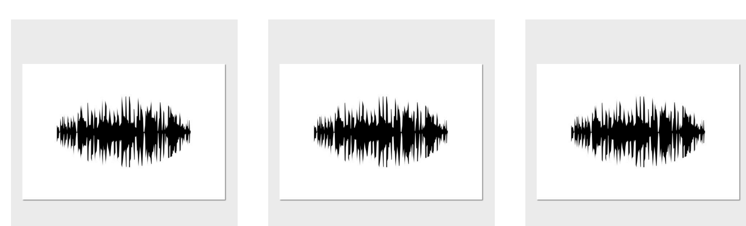
Consider *speech transcription*

$$y_1 \quad y_2 \quad y_3$$



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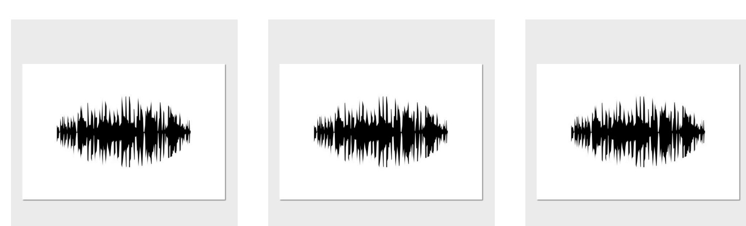

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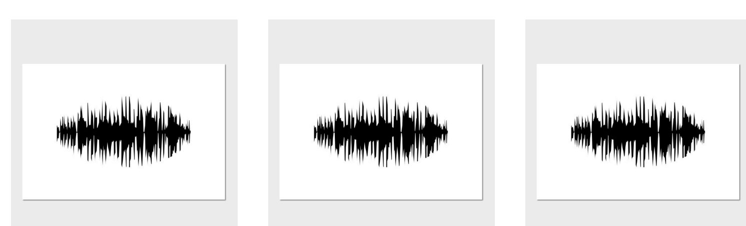

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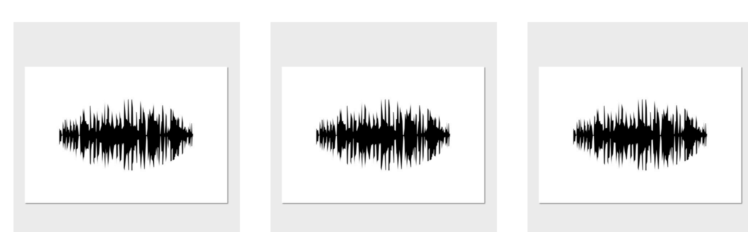

$x_1 \quad x_2 \quad x_3$

Structured output spaces

Source: http://cocodataset.org/

Structured output spaces

John	lives	in	New	York	and	works	for	the	European	Union
B-PER	O	O	B-LOC	I-LOC	O	O	O	O	B-ORG	I-ORG

Source: https://guillaumegenthial.github.io/sequence-tagging-with-tensorflow.html

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y is now a vector (or *tensor)*

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$$(x, \hat{y})$$

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Often called **sequence labeling**

Space of problems

Given	**Predict**	**Type?**
An image	Contains a cat?	

Space of problems

Given	Predict	Type?
An image	Contains a cat?	**Classification**

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Models

- Structured perceptron
- Hidden Markov Models (HMMs)
- Conditional Random Fields (CRFs)

The *structured* perceptron

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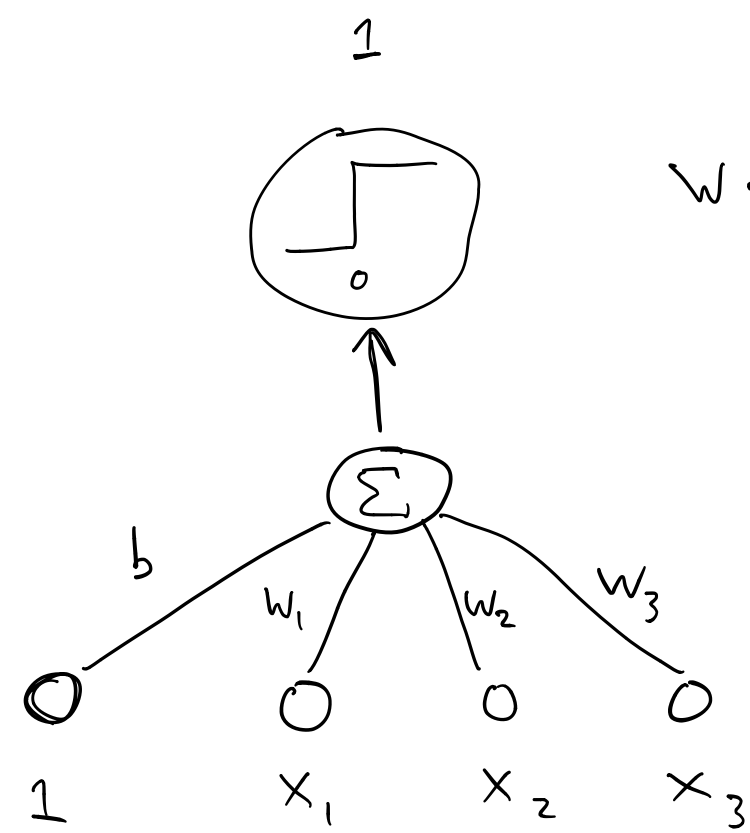
Will build up to this by first introducing the *multi-class* perceptron.

The Perceptron

Perceptron

$$\hat{y} = \begin{cases} 1 & \text{if } w \cdot x > 0 \\ -1 & \text{otherwise} \end{cases}$$



$$W \cdot X = 1 + w_1 x_1 + w_2 x_2 + w_3 x_3$$

Multiclass perceptron

Assume $y \in \{1, 2, ..., K\}$

Joint feature vector $\phi(x, y)$

We're after a *scoring function*

$$s(x, y) = w \cdot \phi(x, y)$$

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should be high for correct y

Multiclass perceptron

How should we design $\phi(x, y)$?

One option:

$$\phi(\boldsymbol{x}, k) = \Big\langle \underbrace{0, 0, \ldots, 0}_{D(k-1)\text{ zeros}}, \underbrace{\boldsymbol{x}}_{\in \mathbb{R}^D}, \underbrace{0, 0, \ldots, 0}_{D(K-k)\text{ zeros}} \Big\rangle \in \mathbb{R}^{DK}$$

Learning

Consider making a prediction, given $\boldsymbol{w}$

$$\hat{y} = \operatorname*{argmax}_{\hat{y} \in [1, K]} s(\boldsymbol{x}, \hat{y})$$

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Learning

If $\hat{y} = y$ we do nothing

If we are wrong ($\hat{y} \neq y$) then update

$$\boldsymbol{w} \leftarrow \boldsymbol{w} + \phi(\boldsymbol{x}, y) - \phi(\boldsymbol{x}, \hat{y})$$

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features for true label

features for wrong label

Learning

Is this doing what we want?

$$\boldsymbol{w}^{\text{(new)}} \leftarrow \boldsymbol{w} + \phi(\boldsymbol{x}, y) - \phi(\boldsymbol{x}, \hat{y})$$

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$$\boldsymbol{w}^{(\text{new})} \leftarrow \boldsymbol{w} + \phi(\boldsymbol{x}, y) - \phi(\boldsymbol{x}, \hat{y})$$

Consider updated prediction for true label

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$$\begin{aligned}
&\boldsymbol{w}^{\text{(new)}} \cdot \phi(\boldsymbol{x}, y) \\
&= \left(\boldsymbol{w}^{\text{(old)}} + \phi(\boldsymbol{x}, y) - \phi(\boldsymbol{x}, \hat{y})\right) \cdot \phi(\boldsymbol{x}, y)
\end{aligned}$$

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&= \left(\boldsymbol{w}^{(\text{old})} + \phi(\boldsymbol{x}, y) - \phi(\boldsymbol{x}, \hat{y})\right) \cdot \phi(\boldsymbol{x}, y) \\
&= \underbrace{\boldsymbol{w}^{(\text{old})} \cdot \phi(\boldsymbol{x}, y)}_{\text{old prediction}} + \underbrace{\phi(\boldsymbol{x}, y) \cdot \phi(\boldsymbol{x}, y)}_{\geq 0} - \underbrace{\phi(\boldsymbol{x}, \hat{y}) \cdot \phi(\boldsymbol{x}, y)}_{=0}
\end{aligned}$$

(by construction)

Sharing features

Suppose there are three classes: *music*, *movies,* and *oncology*.

We think the first two are probably more similar.

We can **encode** this in the feature space.

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$$\phi(\boldsymbol{x}, \text{music}) = \langle \boldsymbol{x}, \mathbf{0}, \mathbf{0}, \boldsymbol{x} \rangle$$

$$\phi(\boldsymbol{x}, \text{movies}) = \langle \mathbf{0}, \boldsymbol{x}, \mathbf{0}, \boldsymbol{x} \rangle$$

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"extra" copy; allows ***w*** to capture shared aspects of movies/music

Structured perceptron

Let's come back to our motivation of *structured* outputs

To be concrete, we will focus on *sequence labeling*

$$\boldsymbol{x} = \text{“ monsters eat tasty bunnies “}$$

$$\boldsymbol{y} = \quad \text{noun} \;\; \text{verb adj} \quad \text{noun}$$

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So *y* is now a sequence. But goal is the same; Score true *y* (whole sequence) higher than other potential *y*'s.

How big is our output space?

Assume x has length L, and that there are K possible labels at each position

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$$\mathcal{Y} = K^L$$

Designing features

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$\boldsymbol{y} =$ noun verb adj noun

Want to design $\phi(x, y)$

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Some possibilities

- # of times *w* gets label *l* (for all *w*, *l*)
- # of times *l* is adjacent to *l'* (for all *l* and *l'*)

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- # of times *w* gets label *l* (for all *w*, *l*) **Unary**
- # of times *l* is adjacent to *l'* (for all *l* and *l'*) **Markov**

Algorithm 40 StructuredPerceptronTrain($\mathbf{D}$, *MaxIter*)

```
w ← 0                                                    // initialize weights
for iter = 1 ... MaxIter do
    for all (x,y) ∈ D do
        ŷ ← argmax_{ŷ∈𝒴(x)} w · φ(x, ŷ)                  // compute prediction
        if ŷ ≠ y then
            w ← w + φ(x, y) − φ(x, ŷ)                    // update weights
        end if
    end for
end for
return w                                                 // return learned weights
```

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$w \leftarrow \mathbf{0}$ // initialize weights
for *iter* = 1 … *MaxIter* **do**
  for all $(x,y) \in \mathbf{D}$ **do**
    $\boxed{\hat{y} \leftarrow \operatorname{argmax}_{\hat{y} \in \mathcal{Y}(x)} w \cdot \phi(x, \hat{y})}$ // compute prediction
    if $\hat{y} \neq y$ **then**
      $w \leftarrow w + \phi(x, y) - \phi(x, \hat{y})$ // update weights
    end if
  end for
end for
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argmax problem

$$\hat{\boldsymbol{y}} \leftarrow \operatorname*{argmax}_{\hat{\boldsymbol{y}} \in \mathcal{Y}(\boldsymbol{x})} \boldsymbol{w} \cdot \phi(\boldsymbol{x}, \hat{\boldsymbol{y}})$$

Why is this hard?

Decomposing structure

Problem We want to compute an argmax over ridiculously large set of elements

Key idea If we restrict ourselves to "local" features, we can decompose ϕ over the input

Decomposing structure

$$\boldsymbol{w} \cdot \phi(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{w} \cdot \sum_{l=1}^{L} \phi_l(\boldsymbol{x}, \boldsymbol{y}) \qquad \text{decomposition of structure}$$

$$= \sum_{l=1}^{L} \boldsymbol{w} \cdot \phi_l(\boldsymbol{x}, \boldsymbol{y}) \qquad \text{associative law}$$

$\phi_l(x, y)$ encodes only features about position *l*

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Markov

The Viterbi algorithm
(on board)

Face extraction

- Suppose you are given a dataset of 28 x 28 images containing within them faces

- In the train data, these have been labeled at the *pixel* level

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[[0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
 [0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 0.]
 [0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 0.]
 [0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 0.]
 [0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 0.]
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```

X → Y

Face extraction

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 [0. 0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 0. 0.]
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```

X $\longrightarrow$ Y

How would you design $\phi_l(x, y)$?

How would you construct your lattice?

A probabilistic view on structured learning

Some content that follows derived from
Michael Collins' materials

A probabilistic view on structured learning

- **Perceptrons** lack any probabilistic semantics
- We can introduce these for structured problems of course!

Hidden Markov Models (HMMs)

- A fully generative model that explicitly bakes in Markov assumption

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Modeling Sequences

Want:

$$P(X_1 = x_1 \dots X_n = x_n, Y_1 = y_1 \dots Y_n = y_n)$$

Modeling Sequences

$$P(X_1 = x_1 \ldots X_n = x_n, Y_1 = y_1 \ldots Y_n = y_n)$$
$$= \prod_{i=1}^{n+1} P(y_i | y_{i-1}) \prod_{i=1}^{n} P(x_i | y_i)$$

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Transition probability

Modeling Sequences

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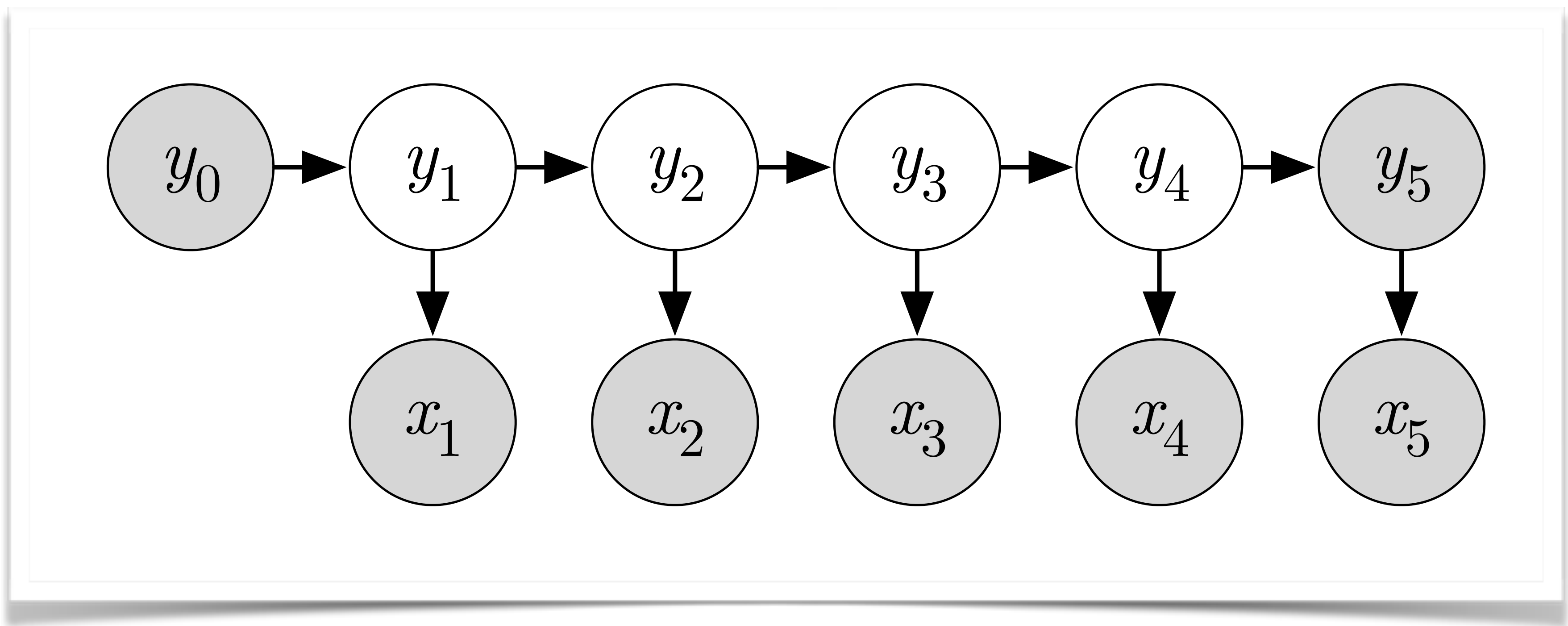
Transition probability

Emission probability

Graphical Model (HMMs)



Consider a “Tri-gram” variant

$$P(Y_i = y_i | Y_{i-2} = y_{i-2}, Y_{i-1} = y_{i-1}) = q(y_i | y_{i-2}, y_{i-1})$$

Transition probability

$$P(X_i = x_i | Y_i = y_i) = e(x_i | y_i)$$

Emission probability

How should we estimate our parameters?

Maximum Likelihood Estimates

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$$q(s|u,v) = \frac{c(u,v,s)}{c(u,v)}$$

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$$e(x|s) = \frac{c(s \rightsquigarrow x)}{c(s)}$$

$$\boxed{\text{i.e., counts!}}$$

What about *decoding*?

$$\arg\max_{y_1 \ldots y_{n+1}} p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

What about *decoding*?

$$\arg\max_{y_1 \cdots y_{n+1}} p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

Viterbi!

Consider: How might we do *unsupervised* or *semi-supervised* learning in HMMs?

Summary

Structured problems are those in which *y*'s are correlated

- Image segmentation
- Language modeling
- Credit fraud detection
- Any time we have *sequences*

Learning to recognize a structured problem when you see them is important!