

DS 2500 Mar 20

From last time (included for completeness today):

- bayes net motivation, definition

New:

- compute conditional probabilities with multiple random variables:
 - $P(ABC|XYZ) = P(ABCXYZ) / P(XYZ)$
- computing conditional probabilities
 - via spreadsheet ("computer" method)

Admin:

- project data & analysis plan due monday

WHAT ARE BAYES NETS
GOOD FOR?

source: <https://sites.pitt.edu/~druzdzel/psfiles/cbmi99a.pdf>

Bayesian Network (Bayes Net)



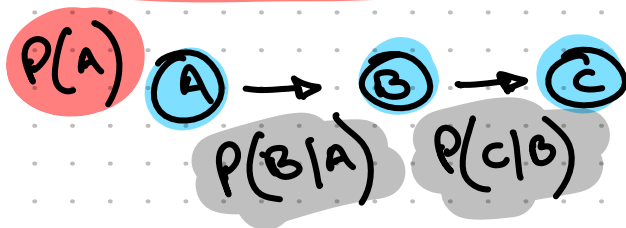
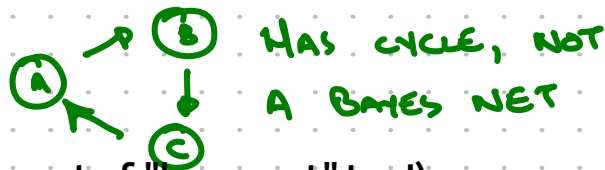
(formally):

A directed, **acyclic** graph which represents conditional distributions / independences between a set of random variables.

each node represents a random variable

directed edges represent conditional distributions

any node without inward edges has prob specified (its part of "Bayes net" too!)



(informally):

a network which describes how random variables influence each other. can be used to compute conditional probabilities of interest

ANATOMY OF BAYES NET

Prob Cloudy = True is 50%

	P(C=T)	P(C=F)
	0,5	0,5

$P(S=F|C=T) = .9$
Prob sprinkler is
off given it's cloudy
out is 90%

C			P(S=T)	P(S=F)
T	F	T	0,1	0,9
			0,5	0,5

Cloudy

Rain

C			P(R=T)	P(R=F)
T	F	T	0,8	0,2
			0,2	0,8

$P(W=T|S=T, R=T) = .99$
Prob that grass is wet
given sprinkler is on and
its raining is 99%

WetGrass

		S		R		P(W=T)	P(W=F)
T	T	F	T	F	T	0,99	0,01
						0,9	0,1
	F	T	F	T	F	0,9	0,1
						0,0	1,0

BAYES NET NOTATION (OUR CONVENTION)

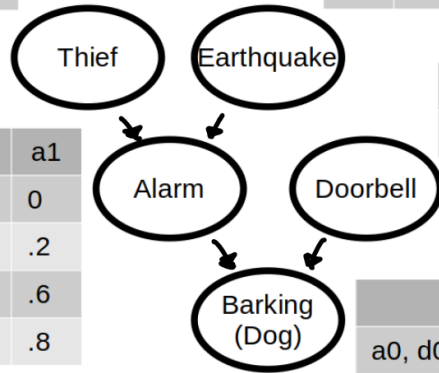
Each random variable is denoted with a capital letter (T for Thief). Each outcome in sample space has its own lowercase letter :

t0 = no thief
t1 = thief

t0	t1
.99	.01

e0	e1
.95	.05

	a0	a1
t0, e0	1	0
t0, e1	.8	.2
t1, e0	.4	.6
t1, e1	.2	.8



d0	d1
.8	.2

	b0	b1
a0, d0	1	0
a0, d1	.2	.8
a1, d0	.5	.5
a1, d1	.01	.99

(quick) ICA X:

what's prob of earthquake?

given a thief in house, but no earthquake, what's prob alarm goes off?

interpretation question:

- is alarm better at detecting thieves or earthquakes?

- which sound bothers the dog more, the alarm or doorbell?

In Class Assignment (last time):

Estimate / intuit the four probabilities below. Is it greater / lesser / equal to other prob immediately above?

What is the prob of thief? $P(T=1) = 1\%$

Given that alarm is going off, what is prob of thief?

$$P(T=1|A=1) > P(T=1)$$

THIEF IS MORE COMMON
WHEN ALARM GOES OFF THAN
NOT

Given that alarm is going off & dog is barking, what is prob of thief?

$$P(T=1|A=1, B=1) = P(T=1|A=1)$$

GIVEN ALARM THIEF AND BARK
ARE INDEPENDENT

Given that alarm is going off, dog is barking & earthquake, what is prob of thief?

$$P(T=1|A=1, B=1, E=1) < P(T=1|A=1, B=1)$$

EARTHQUAKE EXPLAINS AWAY
ALARM

How do we compute conditional probabilities from a Bayes Net?

With a computer:

Step 1: Rewrite conditional probability without conditional

Step 2: In a spreadsheet, compute prob of every possible combination of outputs for all vars

Step 3: Compute the needed probabilities from step 1 via marginalization

Step 1: write conditional probabilities as ratio of (not conditional) probabilities

$$P(\underset{\substack{\uparrow \\ \text{TARGET} \\ \text{VARS}}}{ABC} \mid \underset{\substack{\uparrow \\ \text{EVIDENCE} \\ \text{VARS}}}{XYZ}) = \frac{P(ABC \text{ } XYZ)}{P(XYZ)}$$

PROB OF TARGET AND EVIDENCE TOGETHER

PROB OF EVIDENCE

(ex: Given that alarm is going off & dog is barking, what is prob of thief?)

$$P(t_i \mid a_i, b_i) =$$

Step 2: In a spreadsheet, compute prob of every possible combination of outputs for all vars

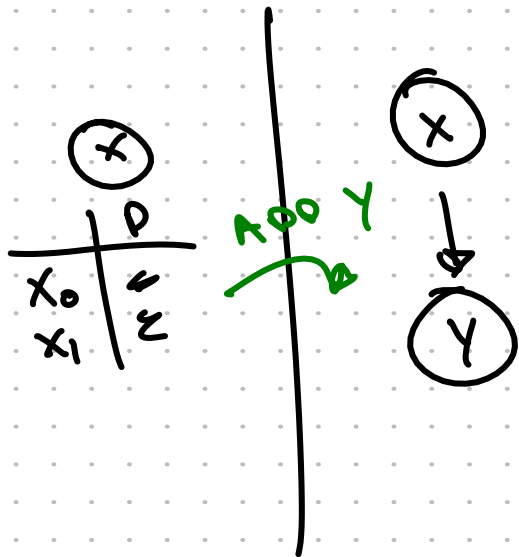
("JOINT DISTRIBUTION TABLE")

B: Barking	D: Doorbell	A: Alarm	T: Thief	E: Earthquake	P(BDATE)
b0	d0	a0	t0	e0	0.7524
b0	d0	a0	t0	e1	0.03168
b0	d0	a0	t1	e0	0.00304
b0	d0	a0	t1	e1	8E-05
b0	d0	a1	t0	e0	0
b0	d0	a1	t0	e1	0.00396
b0	d0	a1	t1	e0	0.00228
b0	d0	a1	t1	e1	0.00016
b0	d1	a0	t0	e0	0.03762
b0	d1	a0	t0	e1	0.001584
b0	d1	a0	t1	e0	0.000152
b0	d1	a0	t1	e1	4E-06
b0	d1	a1	t0	e0	0
b0	d1	a1	t0	e1	1.98E-05
b0	d1	a1	t1	e0	1.14E-05
b0	d1	a1	t1	e1	8E-07
b1	d0	a0	t0	e0	0
b1	d0	a0	t0	e1	0

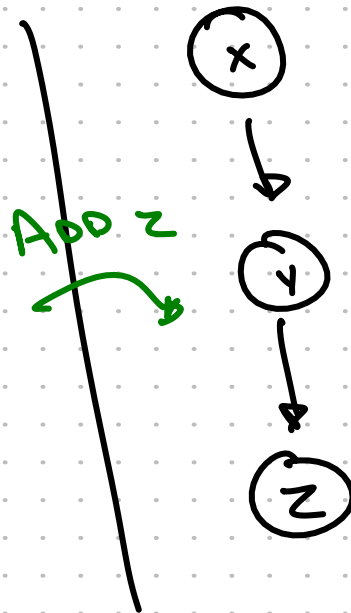
EVERY
COMBINATION
VARS

PROB

PRODUCING A JOINT TABLE ITERATIVELY



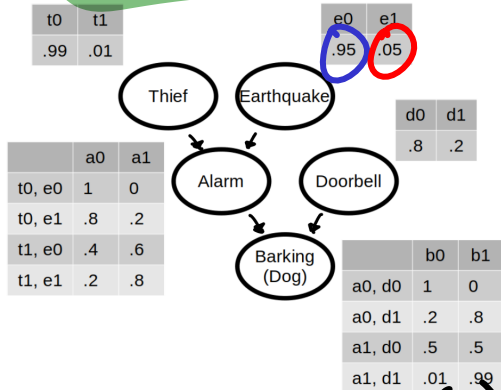
X	Y	P
x_0	y_0	p
x_0	y_1	p
x_1	y_0	p
x_1	y_1	p



X, Y, Z	P
x_0, y_0, z_0	p
$\vdots$	p
x_1, y_1, z_1	p

PRODUCING JOINT TABLE (ADDING INDEPENDENT NODES)

SINCE T, E ARE INDEPENDENT
 $P(TE) = P(T)P(E)$



T	P(T)
t ₀	.99
t ₁	.01

T	E	P(TE)
t ₀	e ₀	.99 .95
t ₀	e ₁	.99 .05
t ₁	e ₀	.01 .95
t ₁	e ₁	.01 .05

PRODUCING JOINT TABLE (DEPENDENT NODES)

X



Y

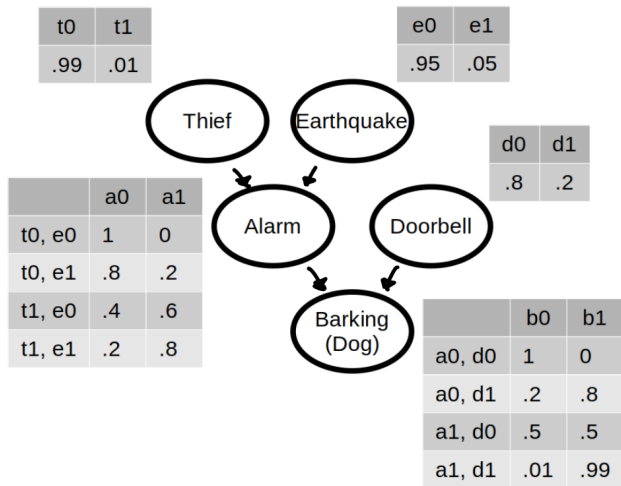
	Y ₀	Y ₁
X ₀	1/3	2/3
X ₁	1/4	3/4

THESE ARE $P(Y|X)$ VALUES

SINCE Y DEPENDS ON X
 $P(X, Y) = P(Y|X) P(X)$

X	P_{X00}
X ₀	1/7
X ₁	6/7

X	Y	$P(X, Y)$
X ₀	Y ₀	1/7 · 1/3
X ₀	Y ₁	1/7 · 2/3
X ₁	Y ₀	6/7 · 1/4
X ₁	Y ₁	6/7 · 3/4



In Class Exercise (don't submit):

Build the joint distribution table for the bayes net on the left.

(You needn't submit for credit. You can check your work with the given final answer csv on website)

MARGINALIZING IN JOINT TABLE (step 3)

COMPUTE $P(X_0 Z_0)$

X	Y	Z	PROB
x_0	y_0	z_0	$1/4$
x_0	y_0	z_1	0
x_0	y_1	z_0	0
x_0	y_1	z_1	$1/8$
x_1	y_0	z_0	$3/8$
x_1	y_0	z_1	0
x_1	y_1	z_0	0
x_1	y_1	z_1	$1/4$

MARGINALIZING IN JOINT TABLE (step 3)

X	Y	Z	PROB
x_0	y_0	z_0	$1/4$
x_0	y_0	z_1	0
x_0	y_1	z_0	0
x_0	y_1	z_1	$1/8$
x_1	y_0	z_0	$3/8$
x_1	y_0	z_1	0
x_1	y_1	z_0	0
x_1	y_1	z_1	$1/4$

QUICK PRACTICE

COMPUTE $P(y_1, x_1)$

COMPUTE $P(x_0)$

Putting it all together:

Step 1: Rewrite conditional probability without conditional

Step 2: In a spreadsheet, compute prob of every possible combination of outputs for all vars

Step 3: Compute the needed probabilities from step 1 via marginalization

Example:

Given alarm is going off and dog is barking, what is the probability of a thief?

$$p(t_i | a, b) =$$

In Class Exercise 1:

Explicitly compute each of the following

1. What is the prob of thief?
2. Given that alarm is going off, what is prob of thief?
3. Given that alarm is going off & dog is barking, what is prob of thief?
4. Given that alarm is going off, dog is barking & earthquake, what is prob of thief?

Answer each question below with one sentence (please avoid algebraic motivations and appeal to our intuition):

- Why is the prob of 2 greater than the prob of 1?
- Why is the prob of 3 equal to the prob of 2?
- Why is the prob of 4 less than the prob of 2?

That spreadsheet work sure was cumbersome ... if only we could make the computer do all that busy work!

In Class Assignment 2:

Design the interface of a Bayes Net "library" which allows the user to

1. specify a bayes net on discrete random variables (as shown here)

as well as querying the bayes net for arbitrary:

2. conditional distributions

3. marginal distributions (i.e. not conditional distributions)

This design is intentionally open-ended. You're welcome to write out notes on paper, though you can write function (or method) docstrings as well if you're ready.