Problem 1

Prove that the following language is undecidable:

\[ L = \{ \langle M \rangle \mid M \text{ is a Turing machine, and } M \text{ accepts the string “000”} \} \]

*Hint: Reduce from \( A_{TM} \). Your reduction will be similar to the reduction we used to show that REGULAR\(_{TM}\) was undecidable (pg. 191 Sipser): \( M' \) will filter its inputs and take one action if the input string is “000” and another if the input string is not.*

Problem 2

Prove that the following language is undecidable:

\[ L = \{ \langle M \rangle \mid M \text{ is a Turing machine accepting the regular language } 0^*1^* \} \]

*Hint: Reduce from \( A_{TM} \). Your reduction will be similar to the reduction we used to show that REGULAR\(_{TM}\) was undecidable: \( M' \) will filter its inputs and take one action if the input string is of the form \( 0^*1^* \) and another if the input string is not. You may need to “flip” the output bit.*
Problem 3

Prove that the following language is undecidable:

\[ L = \{ \langle M_1, M_2, k \rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines, and } |L(M_1) \cap L(M_2)| \geq k \} \]

Hint: You can reduce either from \( A_{TM} \) or from \( EINT_{TM} \). If you reduce from \( EINT_{TM} \), think about how you should set \( k \) and whether you should flip the output bit or not.

Note: The following language is also known to be undecidable:

\[ EINT_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines, and } |L(M_1) \cap L(M_2)| = \emptyset \} \]

Problem 4

Show that \( P \) and \( NP \) are closed under concatenation.

Note: Use determinism to prove that \( P \) is closed under concatenation, and non-determinism to prove that \( NP \) is closed under concatenation. Don’t forget to prove that your Turing Machines run in polynomial time in each case.

Problem 5

Problem 7.20 from Sipser

Problem 6

Problem 7.21 from Sipser

Hint: Reduce from SAT or 3SAT. You may do so by adding one extra clause to the given formula.

Problem 7 [Difficult]

Problem 7.24 from Sipser