Homework 7: Undecidablility & Complexity

CS 390 — Spring 2009

Assigned: April 10
Due: April 17 (Friday) 3:25 pm

Problem 1

Prove that the following language is undecidable:

$$L = \{\langle M \rangle \mid M \text{ is a Turing machine, and } M \text{ accepts the string "000"}\}$$

Hint: Reduce from A_{TM}. Your reduction will be similar to the reduction we used to show that $REGULAR_{TM}$ was undecidable (pg. 191 Sipser): M' will filter its inputs and take one action if the input string is "000" and another if the input string is not.

Problem 2

Prove that the following language is undecidable:

$$L = \{\langle M \rangle \mid M \text{ is a Turing machine accepting the regular language } 0^*1^*\}$$

Hint: Reduce from A_{TM}. Your reduction will be similar to the reduction we used to show that $REGULAR_{TM}$ was undecidable: M' will filter its inputs and take one action if the input string is of the form 0^*1^* and another if the input string is not. You may need to "flip" the output bit....

Problem 3

Prove that the following language is undecidable:

$L = \{\langle M_1, M_2, k\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines, and } |L(M_1) \cap L(M_2)| \geq k\}$

Hint: You can reduce either from A_{TM} or from $EINT_{TM}$. If you reduce from $EINT_{TM}$, think about how you should set k and whether you should flip the output bit or not.

Note: The following language is also known to be undecidable:

$EINT_{TM} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines, and } |L(M_1) \cap L(M_2)| = \emptyset\}$

Problem 4

Show that P and NP are closed under concatenation.

Note: Use determinism to prove that P is closed under concatenation, and non-determinism to prove that NP is closed under concatenation. Don't forget to prove that your Turing Machines run in polynomial time in each case.

Problem 5

Problem 7.20 from Sipser

Problem 6

Problem 7.21 from Sipser

Hint: Reduce from SAT or 3SAT. You may do so by adding one extra clause to the given formula.

Problem 7 [Difficult]

Problem 7.24 from Sipser