College of Computer & Information Science — Fall 2008
Northeastern University — 6 October 2008
CSG250: Wireless Networks — Handout 1

Problem Set 2 (due Wednesday, October 15)

1. Codes

(a) Hamming codes are *perfect* linear codes of distance 3. *Perfect* means that every word is at Hamming distance at most 1 from a unique codeword. Prove that it must be the case for any perfect $(n, k, 3)$-code that $n = 2^t - 1$ and $k = 2^t - t - 1$. [3]

(b) Consider the $2^t - 1xt$ matrix whose ith row is the t-bit binary representation of $i, 1 \leq i \leq 2^t - 1$. Prove that such a matrix is the parity check matrix of a Hamming code. [3]

(c) Consider the set of codes generated by the $tx2^t$ matrix whose ith column is the t-bit binary representation of $i, 0 \leq i \leq 2^t - 1$. Prove that this linear code has a distance of at least $2^(t - 1)$. (Such codes are known as Hadamard codes.) [4]

2. Hat problem Consider the hat problem presented in class where the professor tosses a coin and picks each student's hat. Students can see all hats except their own. If a nonzero subset speak up and each person (who speaks up) correctly reports the color of their own hat then the professor gives everybody an A grade. If no students speak up or if some student speaks up and says the wrong color of their hat then all the students fail.

(a) Prove that in the case of 3 students it is possible for them to succeed with probability at least $\frac{3}{4}$. [3]

(b) Prove that in the case of $2^t - 1$ students it is possible for them to succeed with probability at least $1 - \frac{1}{2^t}$. [3]

(c) Prove that it is not possible for n students to fail with probability less than $\frac{1}{n+1}$. [4]

3. Spread Spectrum

(a) What is spread spectrum? Describe the two main flavors of spread spectrum. [4]

(b) What are the advantages and disadvantages of spread spectrum? [2]

(c) Consider an MFSK scheme with carrier frequency $f_c = 250$ kHz, difference frequency $f_d = 25kHz$, and number of different signal elements $M = 8$. Give a frequency assignment for each of the possible 3-bit data combinations. Now apply slow FHSS to this MFSK scheme with a pseudo-noise of 2 bits or 4 different channels and show the set of all frequency assignments. [4]

4. Shannon's theorem

(a) What is the noise model for the discrete version of Shannon's theorem? What is the expected number of errors given block length of n and bit flip probability of p? [2]

(b) How many $0-1$ strings of length n have exactly pn 1's? Prove that this number is asymptotically equal to $2^(nH(p))$ where $H(p) = -plg(p) - (1-p)\lg(1-p)$. [4]

(c) Prove the converse of Shannon's theorem in the discrete case, i.e. all encoding/decoding schemes that achieve a rate in excess of the channel capacity do so with a success probability that is exponentially small. [4]

5. Fading and Modulation

(a) What is fading? Briefly explain the different causes of fading. What are three techniques for dealing with fading? [3]

(b) Explain how Amplitude Modulation works. How is the signal encoded and what filters are used in the demodulation process to extract the signal at the receiver. [4]

(c) Describe how QAM works with an example. How is the signal encoded and how is it decoded by filtering at the receiver's end? [3]