College of Computer & Information Science
Northeastern University
CSG250: Wireless Networks

Fall 2008
17 September 2008
Handout 1

Problem Set 1 (due Wednesday, September 24)

1. Applying low-pass and bandpass filters to a digital signal

A square periodic signal is represented as the following sum of sinusoids:

$$s(t) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} cos((2k+1)\pi t).$$

(Note that this is just a rewriting of the formula we discussed in class.)

(a) Suppose that the signal is applied to an ideal low-pass filter with bandwidth 15 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 5 Hz; for 3 Hz. What happens as the bandwidth increases. [5]

(b) Suppose that the signal is applied to a bandpass filter that passes the frequencies from 5 to 9 Hz. Plot the output from the filter and compare to the original signal. [5]

For your plots, use an appropriate plotting tool. One such tool is gnuplot, available in Unix.

2. Fourier Transforms

(a) Show that convolution in the time domain is multiplication in the frequency domain. In other words show that if

$$y(t) = \int_{-\infty}^{\infty} x(\tau) n(t-\tau) d\tau$$

then

$$Y(f) = X(f)N(f)$$

[3]

(b) Consider the signal $x(t) = e^{-at}, a > 0$. Plot the phase and magnitude of the Fourier transform as a function of f. [4]

(c) Explain why the Fourier basis of complex exponentials is a good basis for representing (wireless) signals. In particular, why are complex exponentials superior to sinusoidals? [3]

3. Sampling

(a) State and prove the Nyquist sampling theorem [5]

(b) Consider the signal $x(t) = \cos(\frac{f_s}{2}t)$. If the sampling frequency is f_s then what does the sampled signal look like? What does the reconstructed signal look like? Plot both. Why are we unable to reconstruct the original signal? [5]

4. Shannon's theorem

(a) A digital signaling system is required to operate at 38.4 Kbps. If a signal element encodes a 8-bit word, what is the minimum required bandwidth of the channel. What signal-to-noise ratio is required to achieve the desired capacity on the bandwidth that you have computed? [4]

(b) Derive the "spectral efficiency" form of the Shannon theorem from the capacity version. Plot the spectral efficiency (bits per Hertz) in terms of the bit-energy-to-noise-density. Indicate the attainable region. Calculate the largest value (in dB) of bit-energy-to-noise-density below which there can be no error-free communication. [6]

5. Antenna

(a) Show that doubling the transmission frequency is equivalent to doubling the distance between the sending and receiving antennae. Calculate, in dB, attenuation of power in either case. [4]

(b) Suppose we used the thumbrule that the optical line of sight distance from an antenna h meters high to the horizon, in kilometers, is $4\sqrt{h}$. What is the radius of the earth in kilometers that would justify this approximation? Assume the earth to be a perfect sphere. [3]

(c) Assume that two antennas are half-wave dipoles, each with a directive gain of 10 dB. If the transmitted power is 0.5W and the two antennae are separated by a distance of 5km, what is the received power? Assume perfect alignment of the antennae and a frequency of 300MHz. [3]