Challenge Problem – 3

In order to get credit for this challenge, you must do two out of the three problems. You may do three problems, but only two are required.

1. Compact sorted linked lists

Problem 10-3 in the book discusses compact sorted linked lists. In class, we discussed part h: Why is the algorithm no longer $O(\sqrt{n})$ if keys may be repeated? (Essentially, if there is a key $\ell$, with $\ell < k$, and if $\ell$ is repeated often, then the condition key[$i$] < key[$j$] is almost never satisfied, since key[$i$] = key[$j$] = $\ell$ often.)

However, if we apply Random() near the beginning, we will still make progress with high probability, since $i$ is still near the beginning, and $j$ is likely to be later. Modify the logic around key[$i$] < key[$j$] so that it provides the best possible algorithm, given that keys can repeat. The “best possible algorithm” means the exact timing, not the asymptotic timing. Also state a $Theta(\cdot)$ estimate for the expected time of your algorithm.

2. There are $n$ men and $n$ women. Every man-woman pair is characterized by a number that captures the intensity of their mutual dislike. Give an algorithm to find the minimum intensity number such that all the men and women can be paired, with the intensity of no pair exceeding this number. What is the complexity of your algorithm? (It should be asymptotically optimal.)

3. There are $n$ men and $n$ women. Each man likes $d$ women and each woman likes $d$ men (“like” is mutual; if Joe likes Jane then Jane likes Joe). A round of dance requires a pairing of all the men and women such that the persons in every pair like each other. Prove that it is possible to conduct $d$ rounds of dances such that no pair is repeated.