Challenge Problem 1 – Part 1

A function that is neither polynomial nor polylogarithmic

Give a function $f(n)$ such that: (i) $f(n) = o(n^\varepsilon)$ for any constant $\varepsilon > 0$, and (ii) $f(n) = \omega((\log n)^c)$ for any constant $c > 0$. Briefly justify your answer.

Challenge Problem 1 – Part 2

A fault-tolerant OR-gate

Assume we are given an infinite supply of two-input, one-output gates, most of which are OR gates and some of which are AND gates. Unfortunately the OR and AND gates have been mixed together and we can’t tell them apart. For a given integer $k \geq 0$, we would like to construct a two-input, one-output combinational “$k$-OR” circuit from our supply of two-input, one output gates such that the following property holds: If at most $k$ of the gates are AND gates then the circuit correctly implements OR. Assume for simplicity that $k$ is a power of two.

For a given integer $k \geq 0$, we would like to design a $k$-OR circuit that uses the smallest number of gates. Design the best possible circuit you can and derive a $\Theta$-bound (in terms of the parameter $k$) for the number of gates in your $k$-OR circuit.

Also, given an explicit function, $g(k)$ for the smallest number of gates needed to design the above $k$-OR circuit. As before, assume for simplicity that $k$ is a power of two. Briefly justify why your function $g(k)$ represents the minimum number.