The method of induction can be stated as follows. Let \( S(1), S(2), \ldots \) be a sequence of statements such that a statement \( S(i) \) is either true or false. The Axiom of Induction says:

If \( S(1) \) is true and if \( S(k) \Rightarrow S(k + 1) \) for all \( k > 0 \), then \( S(n) \) is true for all \( n \geq 1 \).

In the above statement, \( S(1) \) is the “base case”, \( S(k) \) is the “induction hypothesis”, \( S(k) \Rightarrow S(k + 1) \) is the “induction statement”, and \( S(n) \) is true for all \( n \geq 1 \) is the “conclusion of the induction”.

Recall that “\&” means “and”. An alternative form is:

If \( S(1) \) is true and if \( S(1) \land S(2) \land \cdots \land S(k) \Rightarrow S(k + 1) \) for all \( k > 0 \), then \( S(n) \) is true for all \( n \geq 1 \).

Here is an example. Recall that \( \lg 7 \) means \( \log_2 7 \).

Given: \( T(n) = 7T(n/2) \)
Induction: \( T(n) \leq n^{\lg 7} \)

Base case: \( T(1) \leq 1^{\lg 7} \) is true.

Show: \( T(k) \leq k^{\lg 7} \) for all \( k \Rightarrow T(r) \leq r^{\lg 7} \)
Proof: \( T(r) = 7T(r/2) \leq 7(r/2)^{\lg 7} = r^{\lg 7} \)

(Note that the inequality “\( \leq \)” follows because \( r/2 < r \) and so we can invoke the induction hypothesis for \( K < r \), where \( k = r/2 \).)

We have satisfied the prerequisites for induction. So \( T(r) \leq r^{\lg 7} \).

Hence, \( T(n) \leq n^{\lg 7} \) for all \( n \)
So, \( T(n) = O(n^{\lg 7}) \).

If we had not been able to guess \( T(n) \leq n^{\lg 7} \) at the beginning, we would have done induction on \( T(n) \leq c_1 n^{c_2} \) for some constant \( c_1 \) and \( c_2 \) that will be decided later. Then we would have written:

\[ T(r) = 7T(r/2) \leq 7c_1(r/2)^{c_2} \]

We would then have noted that in order to make \( 7c_1(r/2)^{c_2} \leq r^{\lg 7} \), we will now choose \( c_1 = 1 \) and \( c_2 = \lg 7 \).