

Log-Linear Models with Structured Outputs

Natural Language Processing
CS 4120/6120—Spring 2015
Northeastern University

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(some slides from Andrew McCallum)

Overview

- Sequence labeling task (cf. POS tagging)
- Independent classifiers
- HMMs
- (Conditional) Maximum Entropy Markov Models
- Conditional Random Fields
- Beyond Sequence Labeling

Sequence Labeling

- Inputs: $x = (x_1, \dots, x_n)$
- Labels: $y = (y_1, \dots, y_n)$
- Typical goal: Given x , predict y
- Example sequence labeling tasks
 - Part-of-speech tagging
 - Named-entity-recognition (NER)
 - Label people, places, organizations

NER Example:

Red Sox and Their Fans Let Loose



Elise Amendola/Associated Press

Fans of the slugger David Ortiz in Boston's Copley Square.

By [PETE THAMEL](#)

Published: October 31, 2007

BOSTON, Oct. 30 — **Jonathan Papelbon** turned Boston's World Series victory parade into a full-scale dance party Tuesday as the [Red Sox](#) put an exclamation point on the 2007 season.

 E-MAIL

 PRINT

 REPRINTS

 SAVE

First Solution:

Maximum Entropy Classifier

- Conditional model $p(y|x)$.
 - Do not waste effort modeling $p(x)$, since x is given at test time anyway.
 - Allows more complicated input features, since we do not need to model dependencies between them.
- Feature functions $f(x,y)$:
 - $f_1(x,y) = \{ \text{word is Boston} \ \& \ y=\text{Location} \}$
 - $f_2(x,y) = \{ \text{first letter capitalized} \ \& \ y=\text{Name} \}$
 - $f_3(x,y) = \{ x \text{ is an HTML link} \ \& \ y=\text{Location} \}$

First Solution: MaxEnt Classifier

- How should we choose a classifier?
- Principle of maximum entropy
 - We want a classifier that:
 - Matches feature constraints from training data.
 - Predictions maximize entropy.
- There is a unique, exponential family distribution that meets these criteria.

First Solution: MaxEnt Classifier

- Problem with using a maximum entropy classifier for sequence labeling:
- It makes decisions at each position independently!

Second Solution: HMM

$$P(\mathbf{y}, \mathbf{x}) = \prod_t P(y_t | y_{t-1}) P(x | y_t)$$

- Defines a generative process.
- Can be viewed as a weighted finite state machine.

Second Solution: HMM

- How can represent we multiple features in an HMM?
 - Treat them as conditionally independent given the class label?
 - The example features we talked about are not independent.
 - Try to model a more complex generative process of the input features?
 - We may lose tractability (i.e. lose a dynamic programming for exact inference).

Second Solution: HMM

- Let's use a conditional model instead.

Third Solution: MEMM

- Use a series of maximum entropy classifiers that know the previous label.
- Define a Viterbi algorithm for inference.

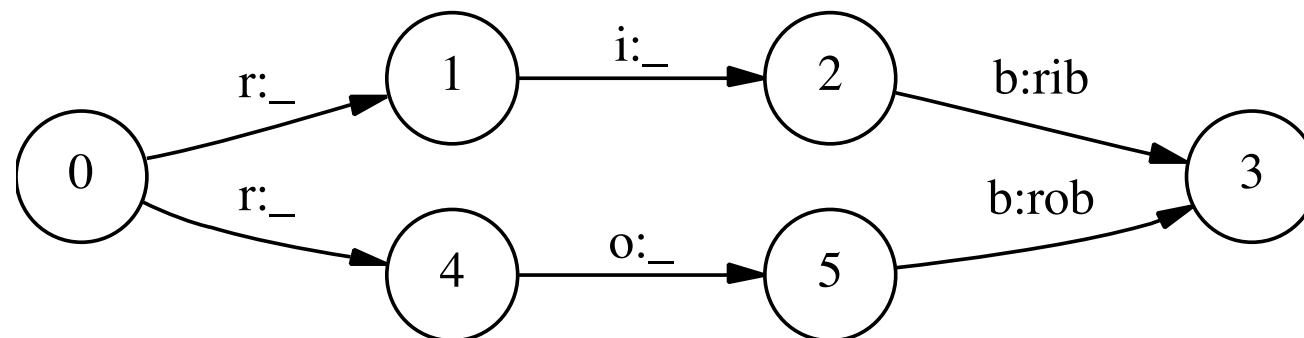
$$P(\mathbf{y} \mid \mathbf{x}) = \prod_t P_{y_{t-1}}(y_t \mid \mathbf{x})$$

Third Solution: MEMM

- Combines the advantages of maximum entropy and HMM!
- But there is a problem...

Problem with MEMMs: Label Bias

- In some state space configurations, MEMMs essentially completely ignore the inputs.



- This is not a problem for HMMs, because the input sequence is generated by the model.

Fourth Solution: Conditional Random Field

- Conditionally-trained, undirected graphical model.
- For a standard linear-chain structure:

$$P(\mathbf{y} \mid \mathbf{x}) = \prod_t \Psi_k(y_t, y_{t-1}, \mathbf{x})$$

$$\Psi_k(y_t, y_{t-1}, \mathbf{x}) = \exp\left(\sum_k \lambda_k f(y_t, y_{t-1}, \mathbf{x})\right)$$

Fourth Solution: CRF

- Have the advantages of MEMMs, but avoid the label bias problem.
- CRFs are globally normalized, whereas MEMMs are locally normalized.
- Widely used and applied. CRFs give state-the-art results in many domains.

Fourth Solution: CRF

- Have the advantages of MEMMs, but avoid the label bias problem.
- CRFs are globally normalized, whereas MEMMs are locally normalized.
- Widely used and applied to achieve state-the-art results in n

Remember, Z is the normalization constant. How do we compute it?

CRF Applications

- Part-of-speech tagging
- Named entity recognition
- Document layout (e.g. table) classification
- Gene prediction
- Chinese word segmentation
- Morphological disambiguation
- Citation parsing
- Etc., etc.

NER as Sequence Tagging

The Phoenicians came from the Red Sea

NER as Sequence Tagging

○

B-E

○

○

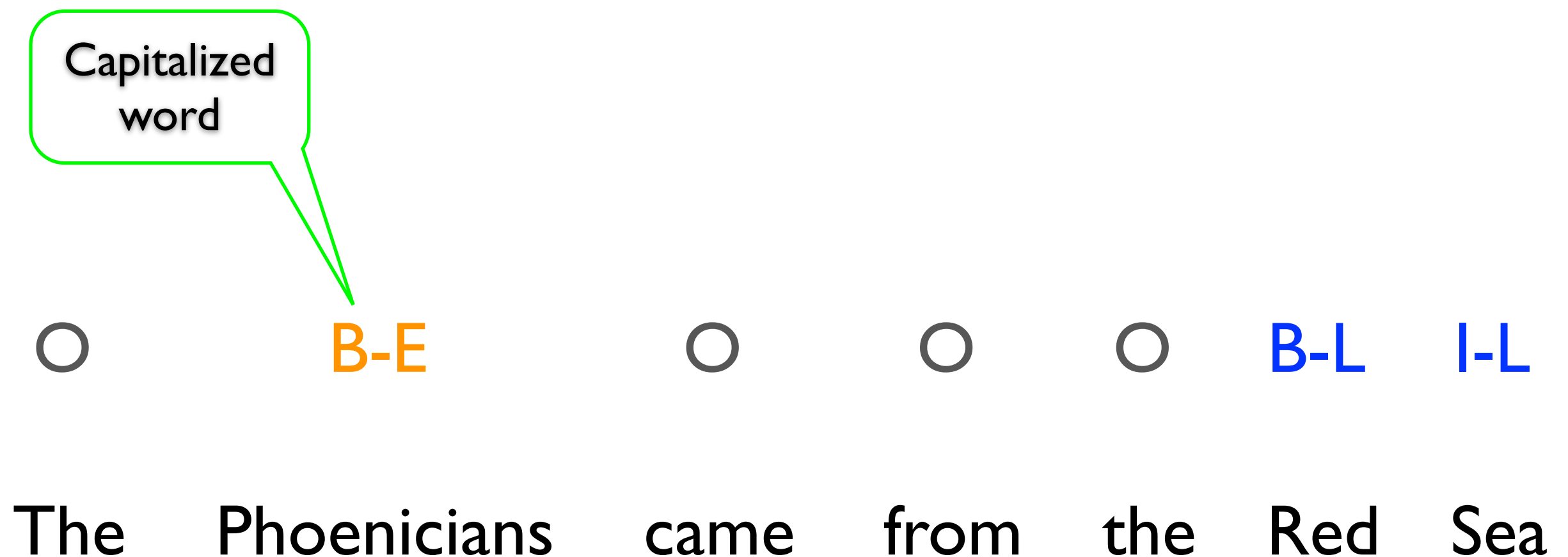
○

B-L

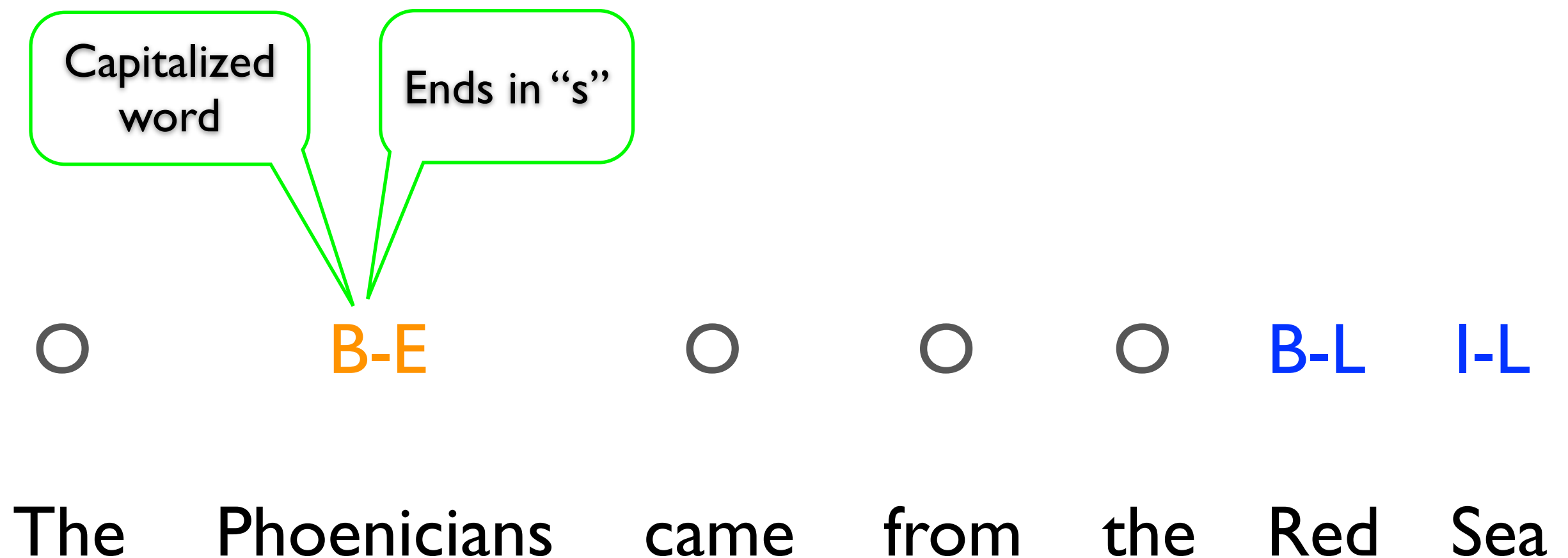
I-L

The Phoenicians came from the Red Sea

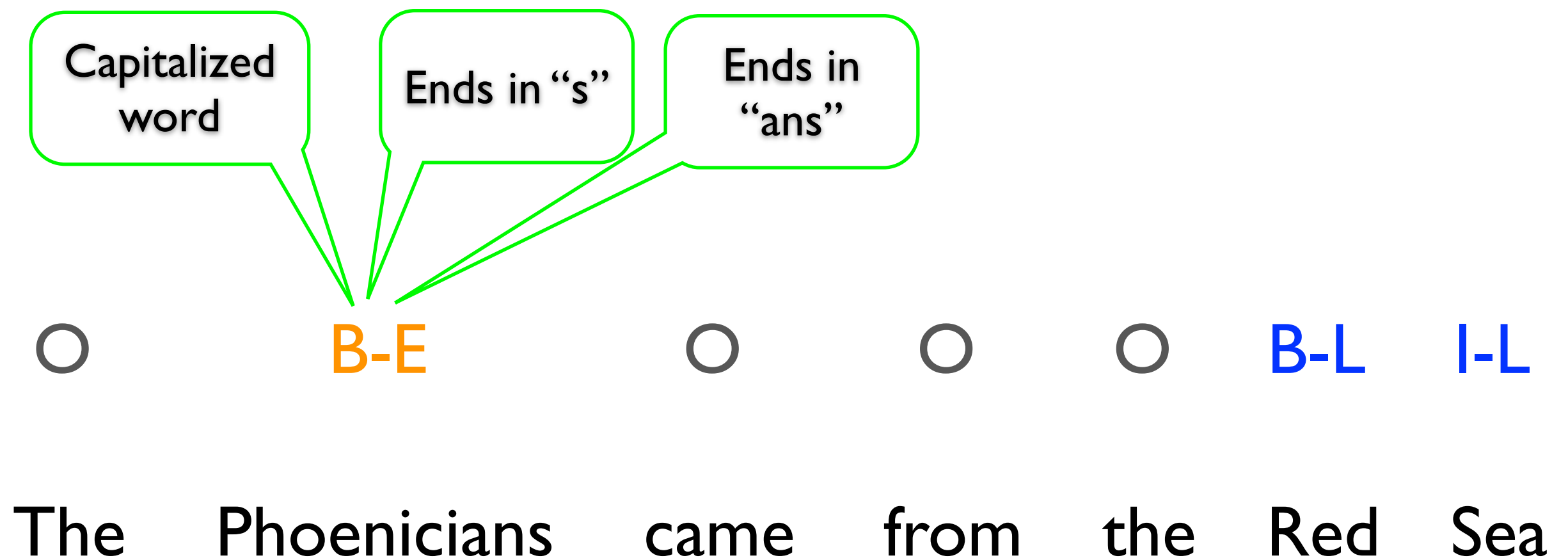
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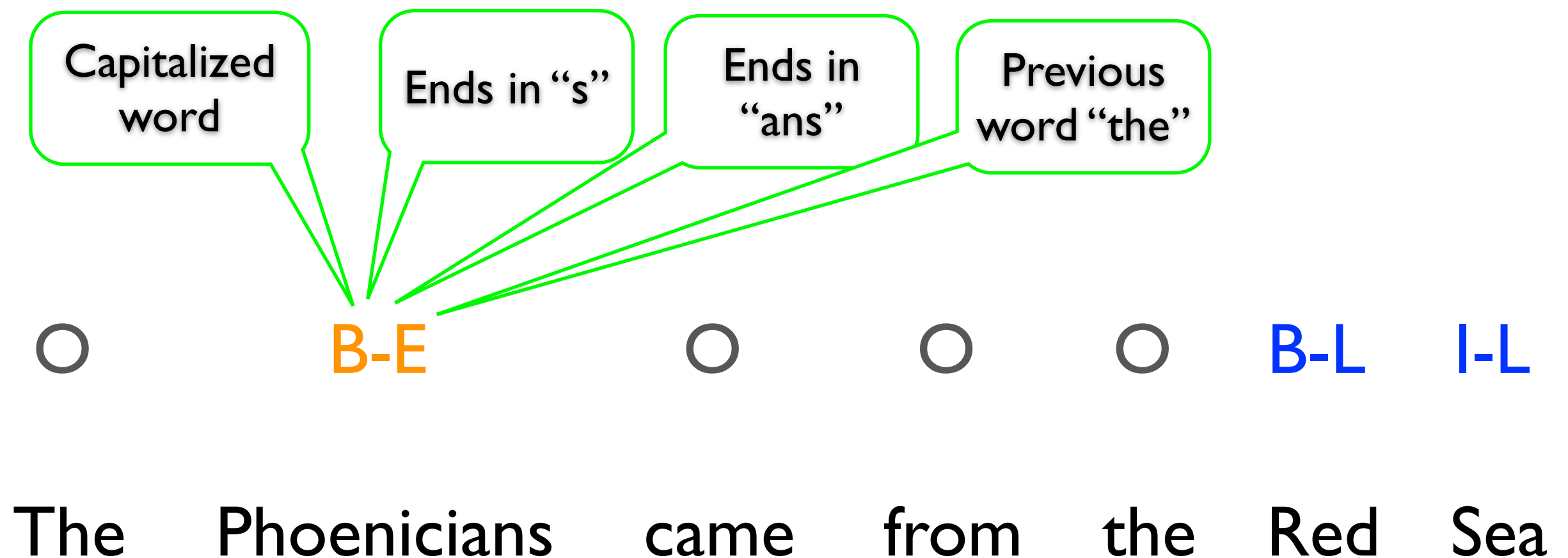
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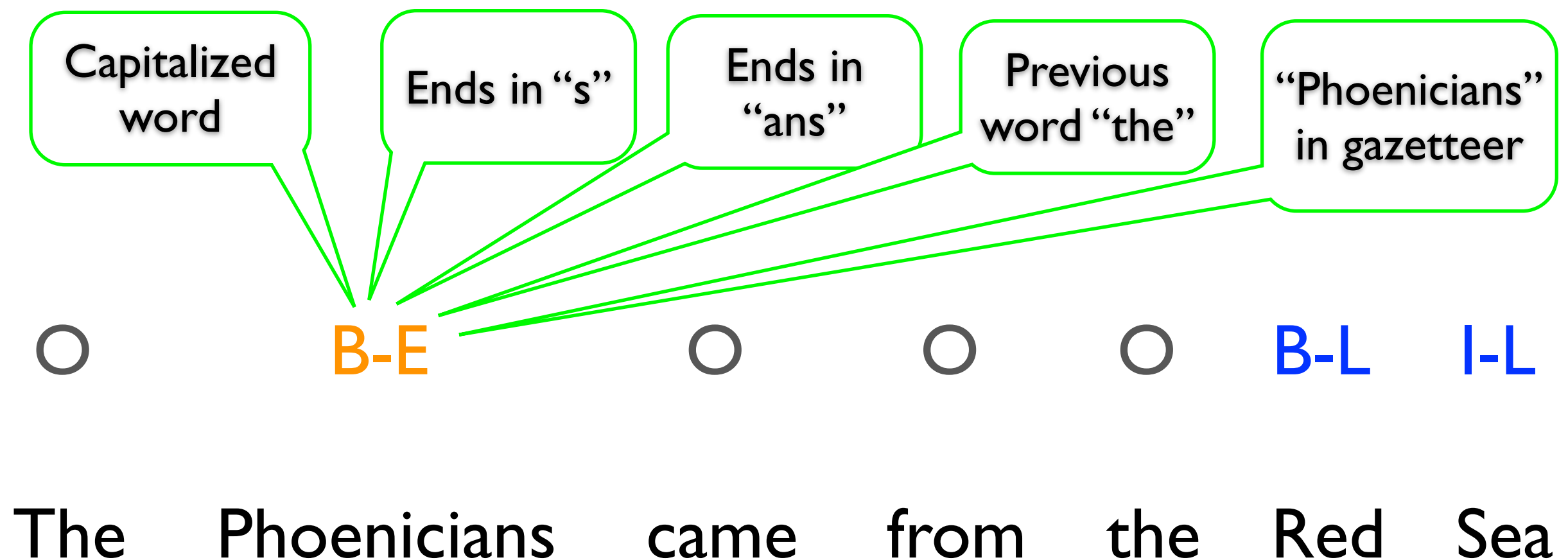
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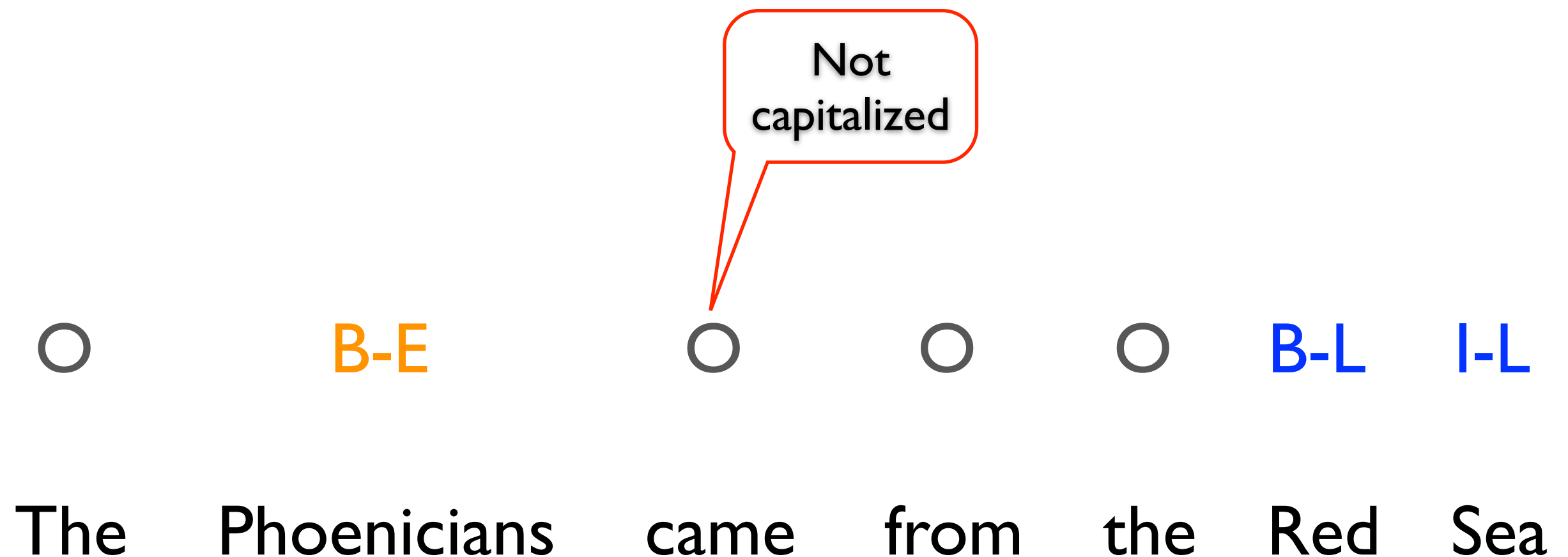
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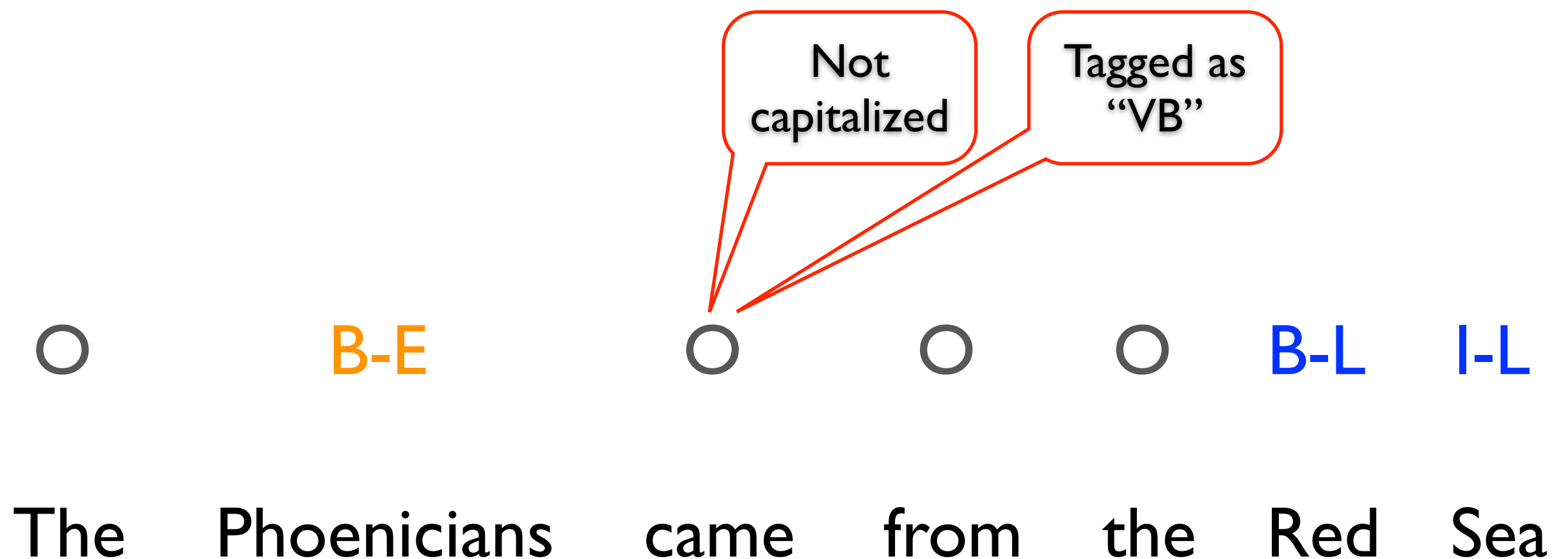
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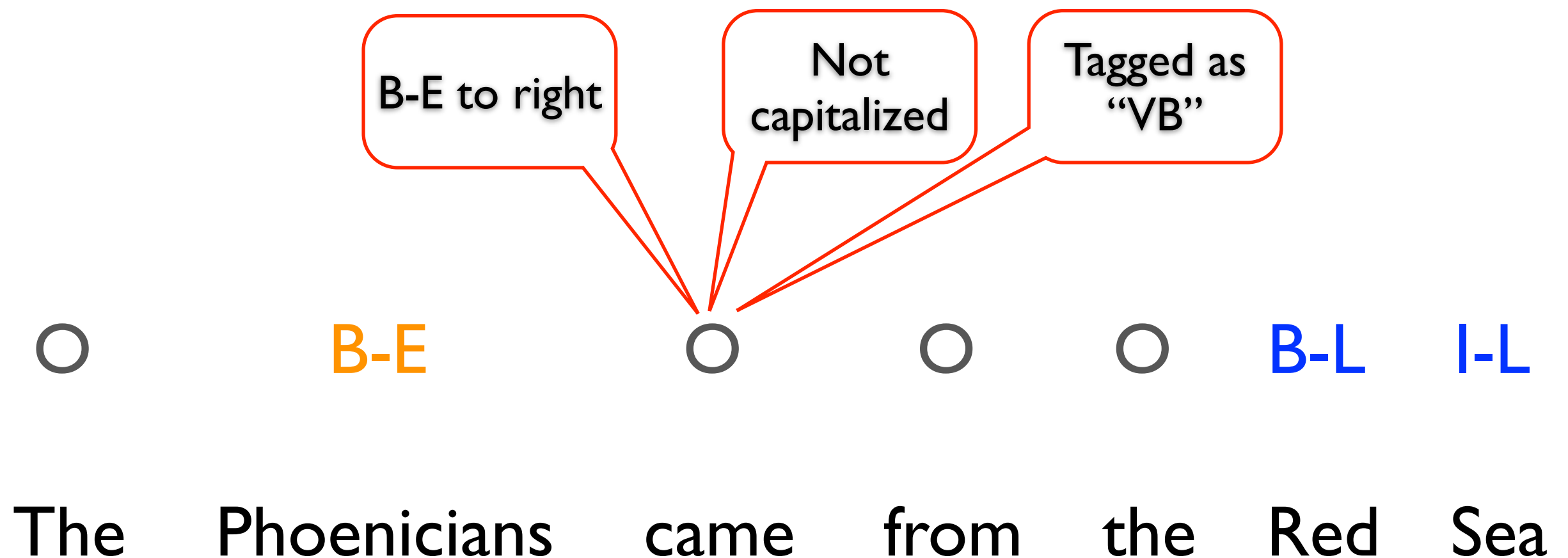
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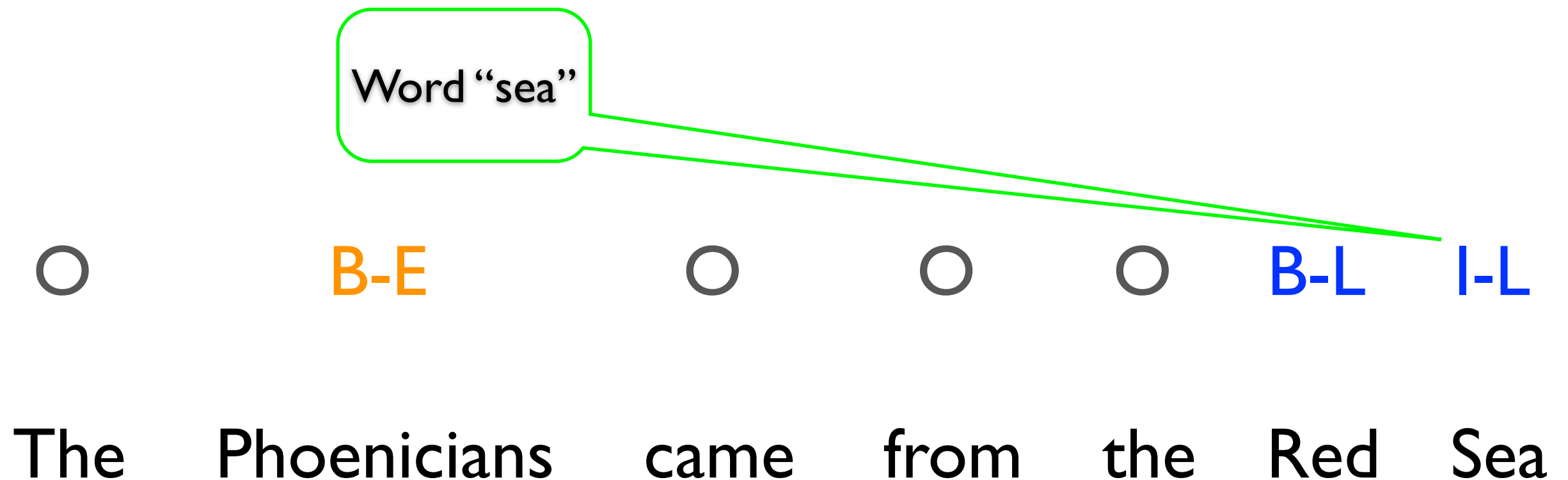
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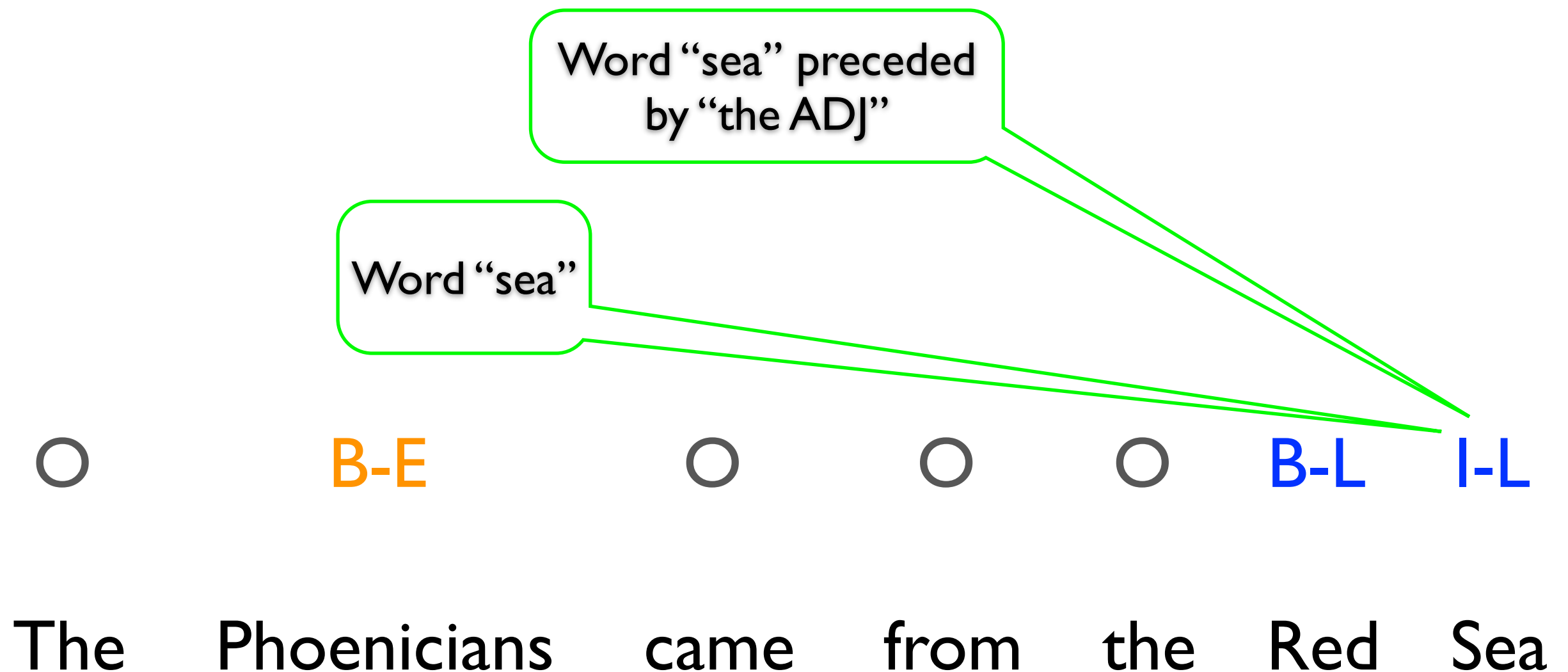
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○	B-E	○	○	○	B-L	I-L
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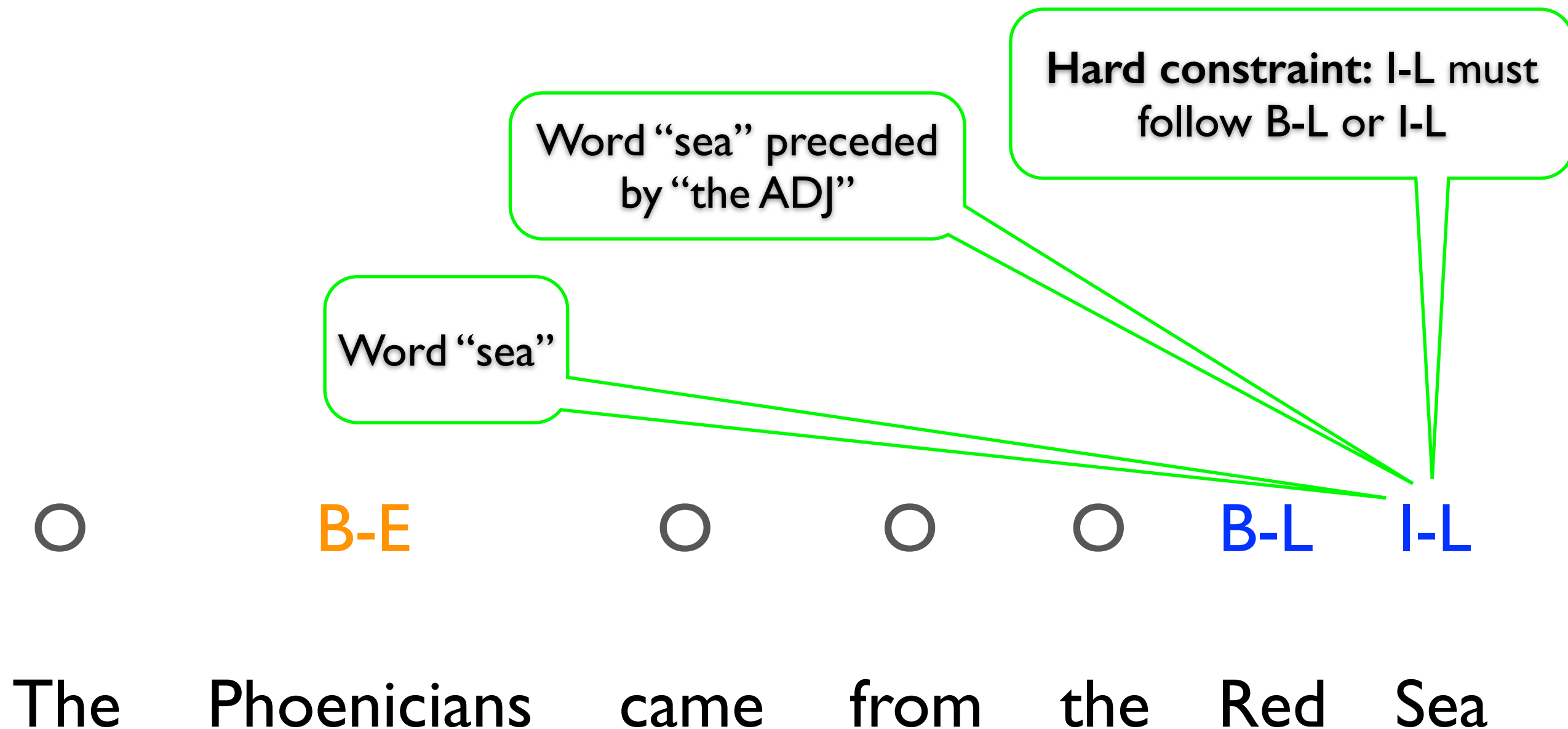
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NER as Sequence Tagging



NER as Sequence Tagging



Overview

- What computations do we need?
- Smoothing log-linear models
- MEMMs vs. CRFs again
 - Action-based parsing and dependency parsing

Recipe for Conditional Training of $p(y | x)$

1. Gather constraints/features from training data

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero

3. Classify training data with current parameters; calculate expectations

$$E_{\Theta}[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_{\Theta}(y' | x_j) f_{iy}(x_j, y')$$

4. Gradient is $\tilde{E}[f_{iy}] - E_{\Theta}[f_{iy}]$

5. Take a step in the direction of the gradient

6. Repeat from 3 until convergence

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EM!

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Where have we seen expected counts before?

Gradient-Based Training

- $\lambda := \lambda + \text{rate} * \text{Gradient}(F)$
- After all training examples? (batch)
- After every example? (on-line)
- Use second derivative for faster learning?
- A big field: numerical optimization

Overfitting

- If we have too many features, we can choose weights to model the training data perfectly
- If we have a feature that only appears in spam training, not ham training, it will get weight ∞ to maximize $p(\text{spam} \mid \text{feature})$ at 1.
- These behaviors
 - Overfit the training data
 - Will probably do poorly on test data

Solutions to Overfitting

- Throw out rare features.
 - Require every feature to occur > 4 times, and > 0 times with ling, and > 0 times with spam.
- Only keep, e.g., 1000 features.
 - Add one at a time, always greedily picking the one that most improves performance on held-out data.
- Smooth the observed feature counts.
- Smooth the weights by using a prior.
 - $\max p(\lambda | \text{data}) = \max p(\lambda, \text{data}) = p(\lambda)p(\text{data} | \lambda)$
 - decree $p(\lambda)$ to be high when most weights close to 0

Smoothing with Priors

- What if we had a prior expectation that parameter values wouldn't be very large?
- We could then balance evidence suggesting large (or infinite) parameters against our prior expectation.
- The evidence would never totally defeat the prior, and parameters would be smoothed (and kept finite)
- We can do this explicitly by changing the optimization objective to maximum posterior likelihood:

$$\log P(y, \lambda \mid x) = \log P(\lambda) + \log P(y \mid x, \lambda)$$

Posterior

Prior

Likelihood

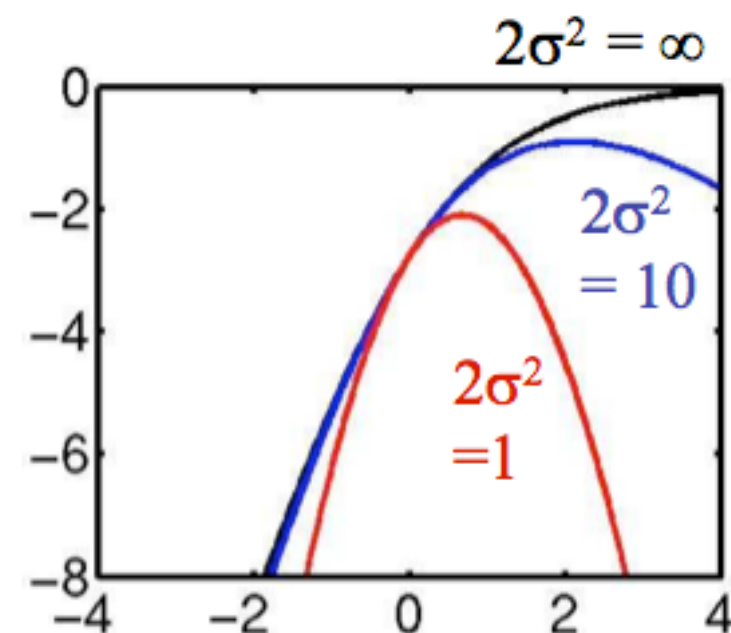


Smoothing: Priors

- Gaussian, or quadratic, priors:
 - Intuition: parameters shouldn't be large.
 - Formalization: prior expectation that each parameter will be distributed according to a gaussian with mean μ and variance σ^2 .

$$P(\lambda_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(\lambda_i - \mu_i)^2}{2\sigma_i^2}\right)$$

- Penalizes parameters for drifting to far from their mean prior value (usually $\mu=0$).
- $2\sigma^2=1$ works surprisingly well.



They don't even
capitalize my
name anymore!



Parsing as Structured Prediction

Shift-reduce parsing

Stack	Input remaining	Action
()	Book that flight	shift
(Book)	that flight	reduce, Verb $\rightarrow$ book, (Choice #1 of 2)
(Verb)	that flight	shift
(Verb that)	flight	reduce, Det $\rightarrow$ that
(Verb Det)	flight	shift
(Verb Det flight)		reduce, Noun $\rightarrow$ flight
(Verb Det Noun)		reduce, NOM $\rightarrow$ Noun
(Verb Det NOM)		reduce, NP $\rightarrow$ Det NOM
(Verb NP)		reduce, VP $\rightarrow$ Verb NP
(Verb)		reduce, S $\rightarrow$ V
(S)		SUCCESS!

Ambiguity may lead to the need for backtracking.

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Train log-linear model of
 $p(\text{action} \mid \text{context})$

Compare to an MEMM

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Structured Log-Linear Models

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- Linear model for scoring structures

$$\text{score}(\textit{out}, \textit{in}) = \theta \cdot \text{features}(\textit{out}, \textit{in})$$

Structured Log-Linear Models

- Linear model for scoring structures
- Get a probability distribution by normalizing

$$\text{score}(out, in) = \theta \cdot \text{features}(out, in)$$

$$p(out \mid in) = \frac{1}{Z} e^{\text{score}(out, in)} \quad Z = \sum_{out' \in GEN(in)} e^{\text{score}(out', in)}$$

Structured Log-Linear Models

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 - ✦ Viz. logistic regression, Markov random fields, undirected graphical models

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- Inference: sampling, variational methods, dynamic programming, local search, ...

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- Linear model for scoring structures
- Get a probability distribution by normalizing
 - ❖ Viz. logistic regression, Markov random fields, undirected graphical models
- Inference: sampling, variational methods, dynamic programming, local search, ...
- Training: maximum likelihood, minimum risk, etc.

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Structured Log-Linear Models

With latent variables

- Several layers of linguistic structure
- Unknown correspondences
- Naturally handled by probabilistic framework
- Several inference setups, for example:

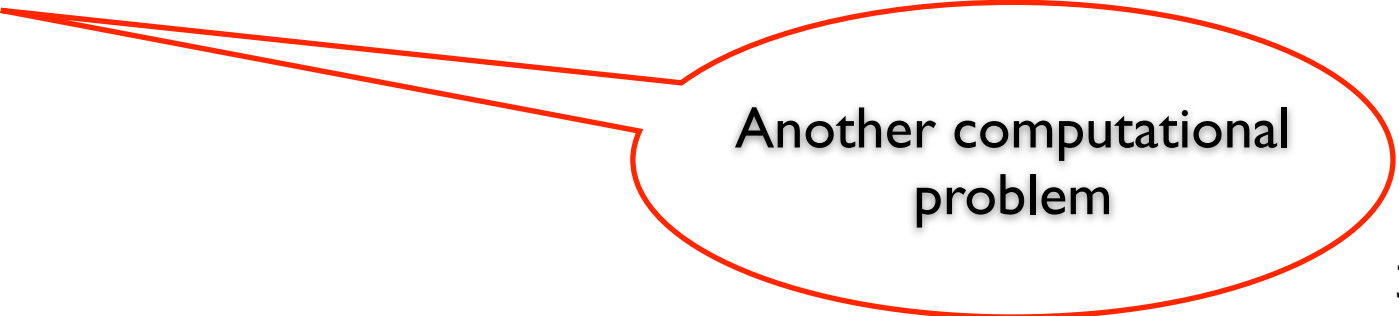
$$p(out_1 \mid in) = \sum_{out_2, alignment} p(out_1, out_2, alignment \mid in)$$

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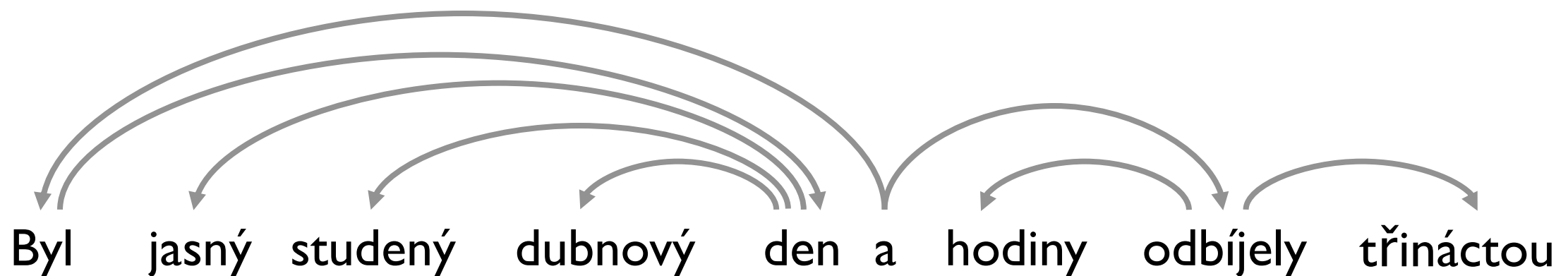
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Another computational problem

Edge-Factored Parsers

- No global features of a parse (*McDonald et al. 2005*)
- Each feature is attached to some edge
- *MST or CKY-like DP for fast $O(n^2)$ or $O(n^3)$ parsing*

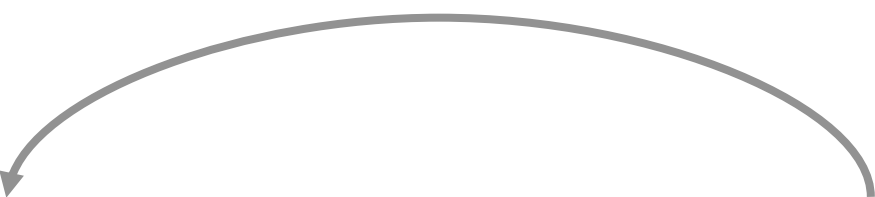


“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

- Is this a good edge?

Byl jasný studený dubnový den a hodiny odbíjely třináctou

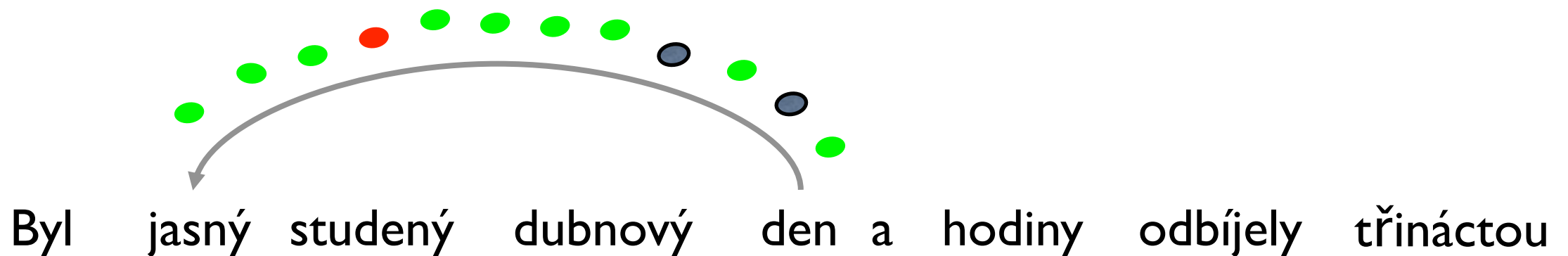
A curved arrow originates from the word 'den' and points back to the word 'jasný', illustrating a long-distance dependency between the subject and the object of the sentence.

“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

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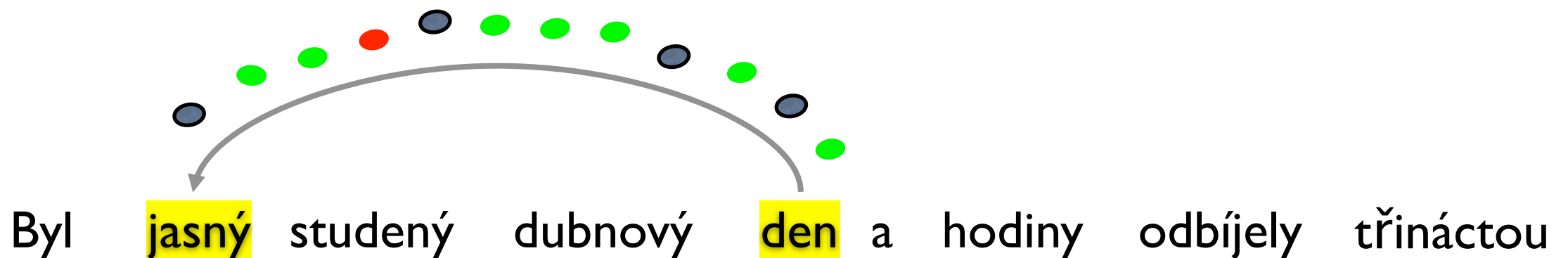
yes, lots of positive features ...



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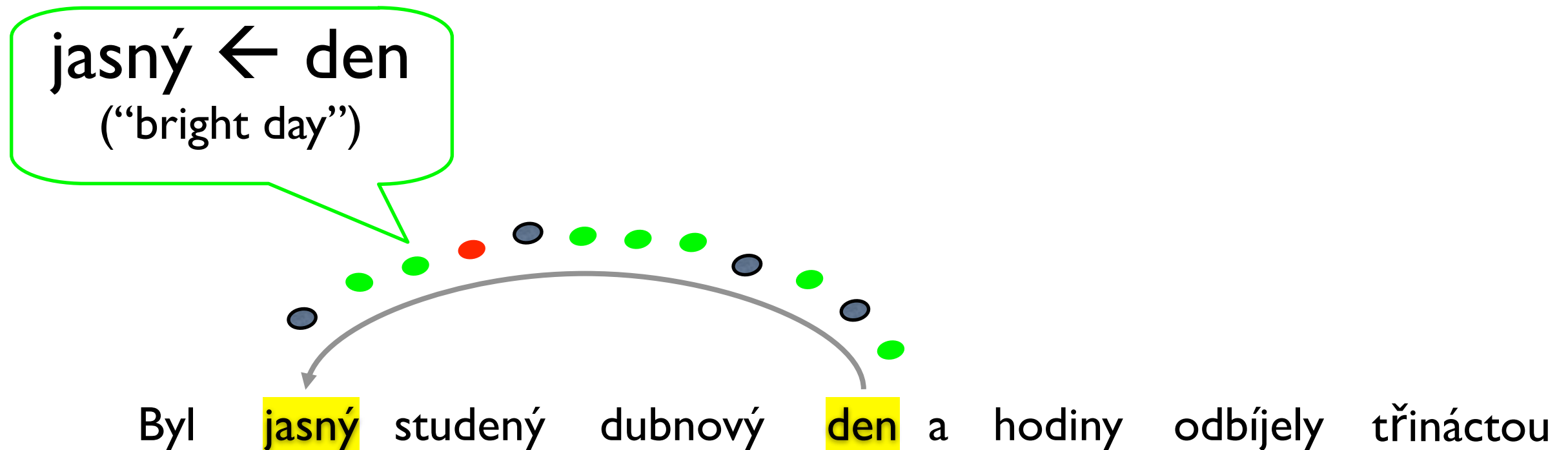
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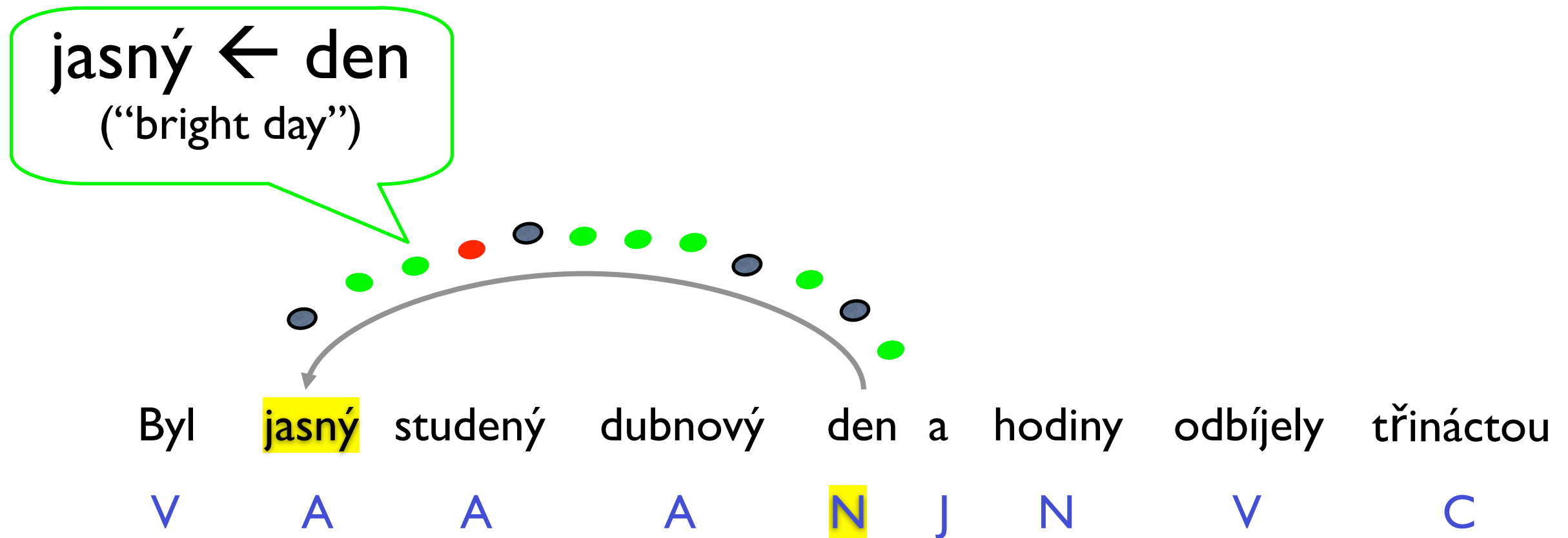
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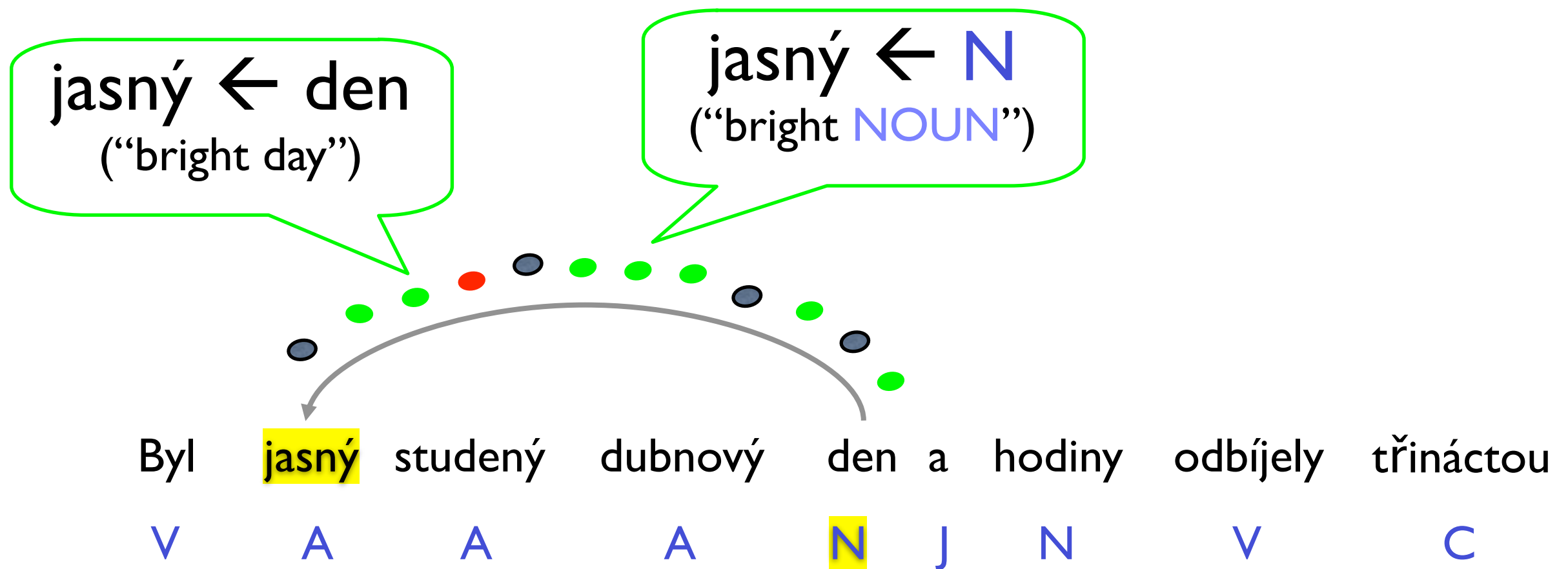
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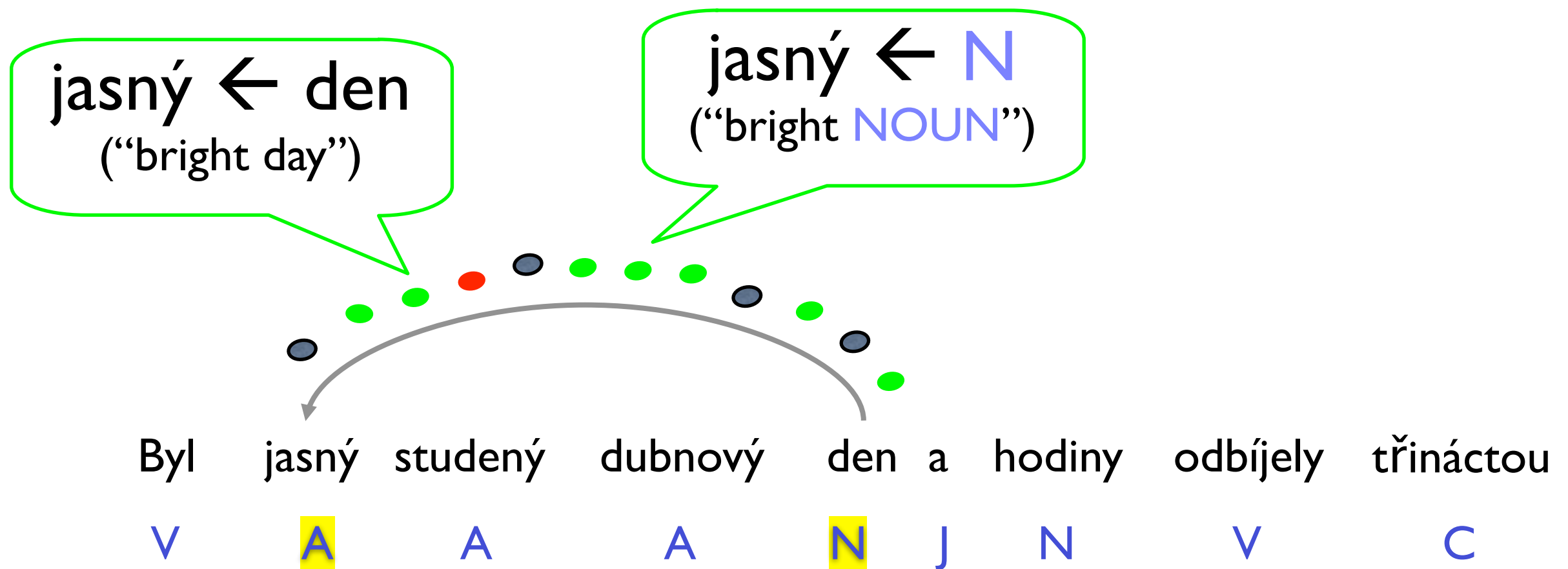
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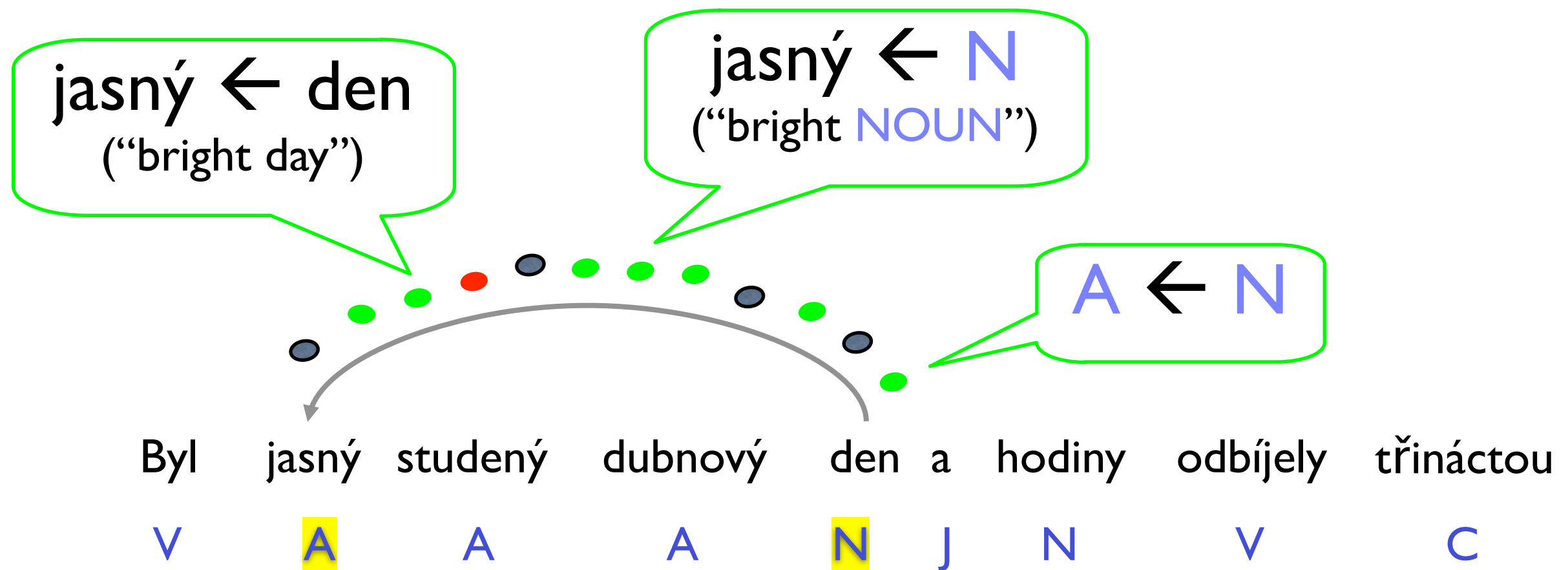
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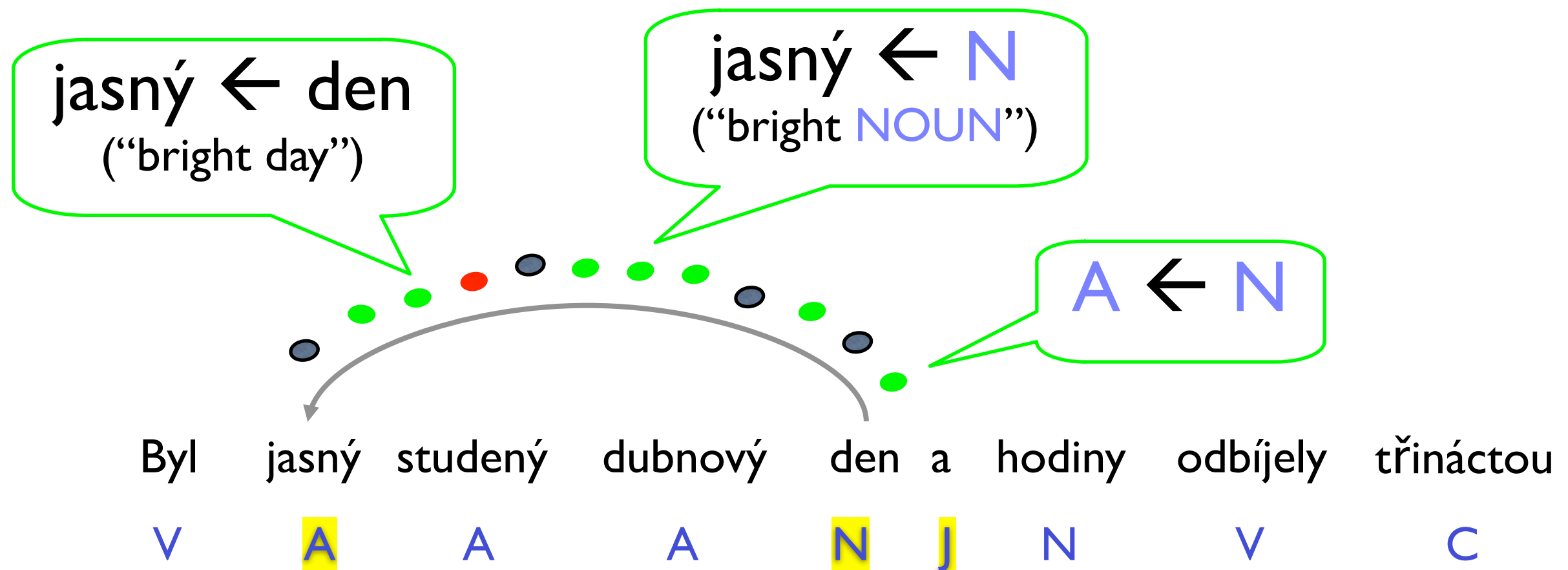
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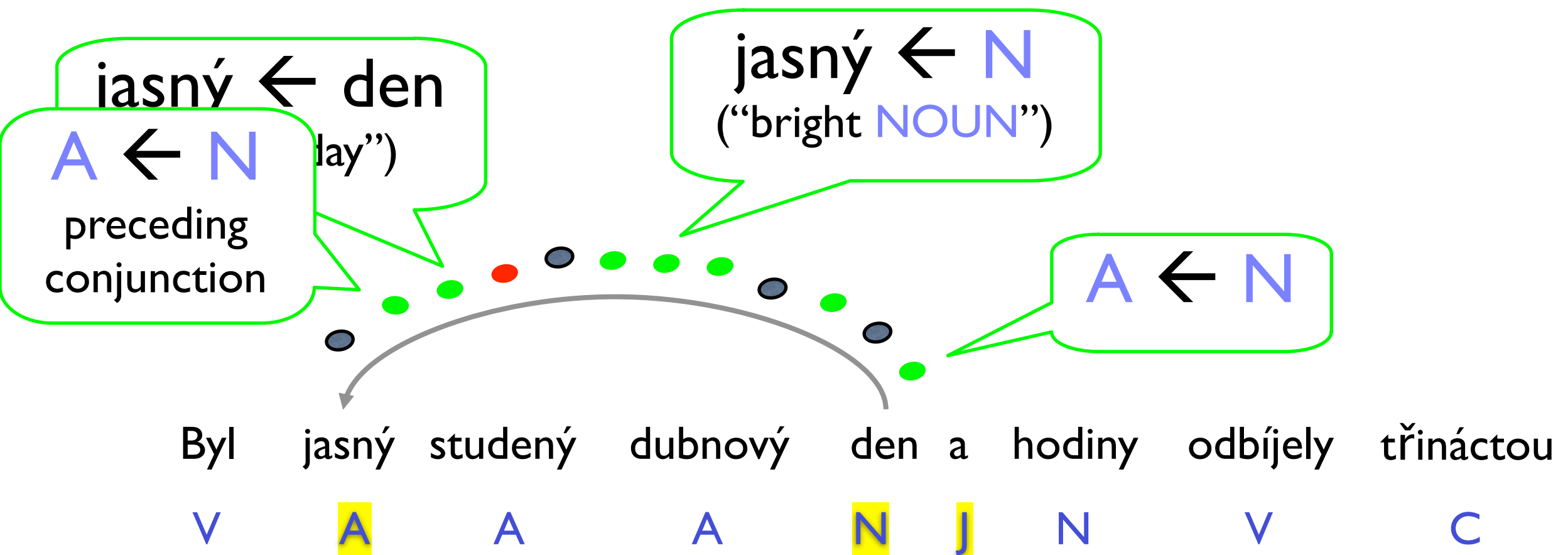
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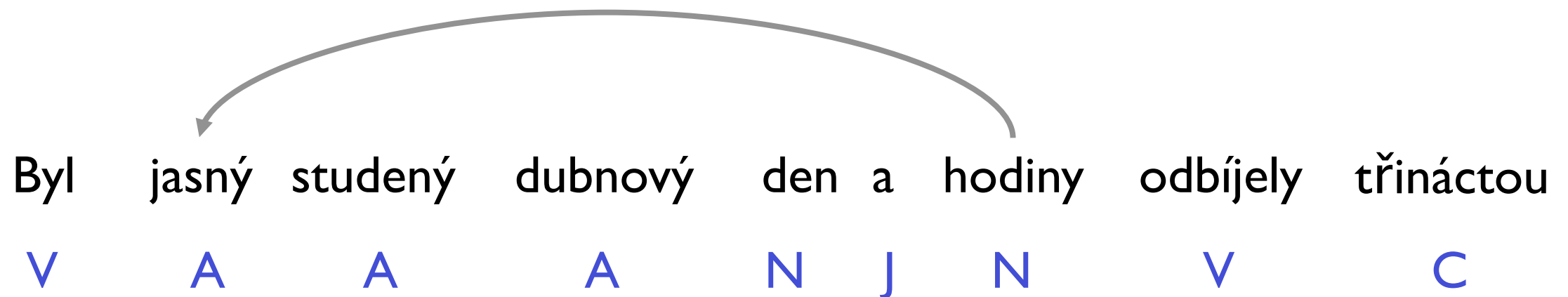
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Edge-Factored Parsers

- How about this competing edge?

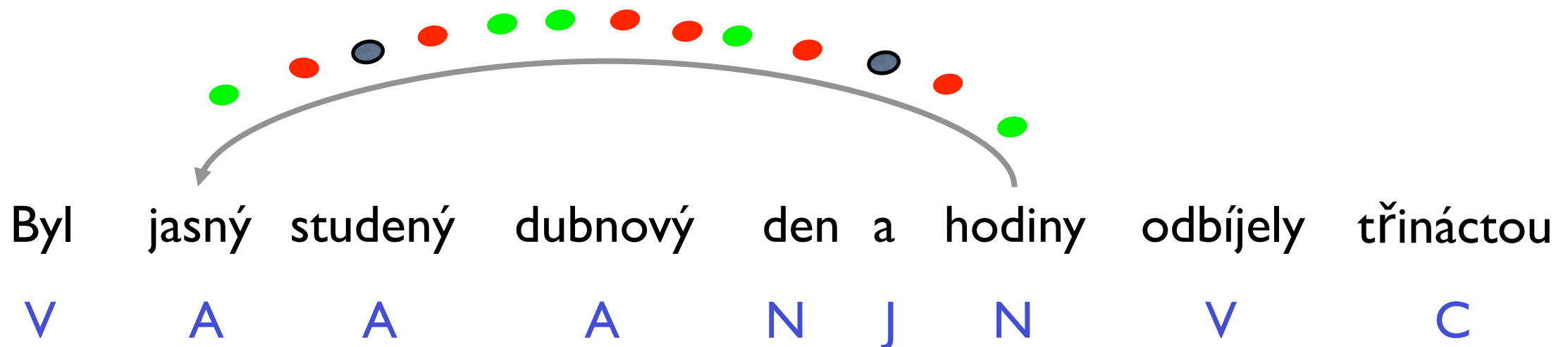


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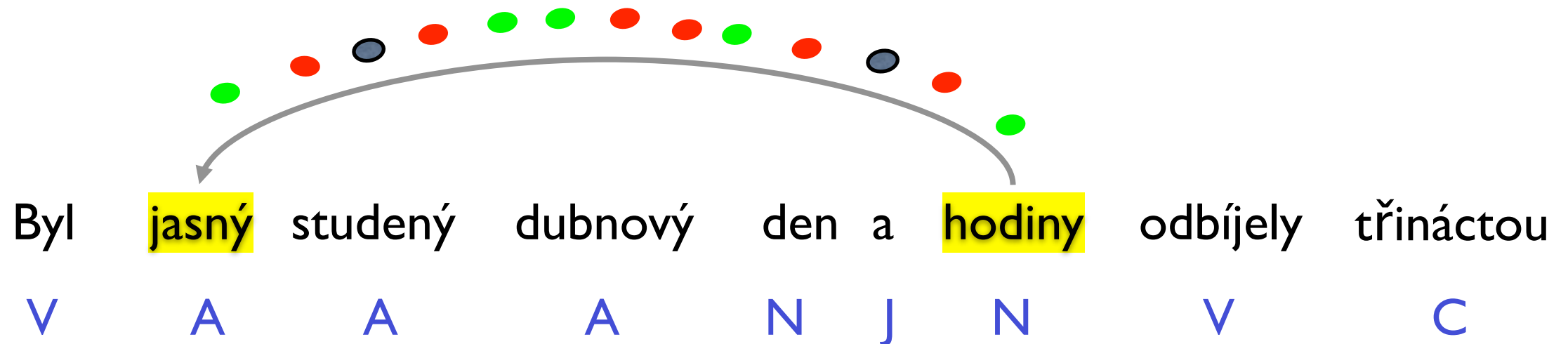
not as good, lots of red ...



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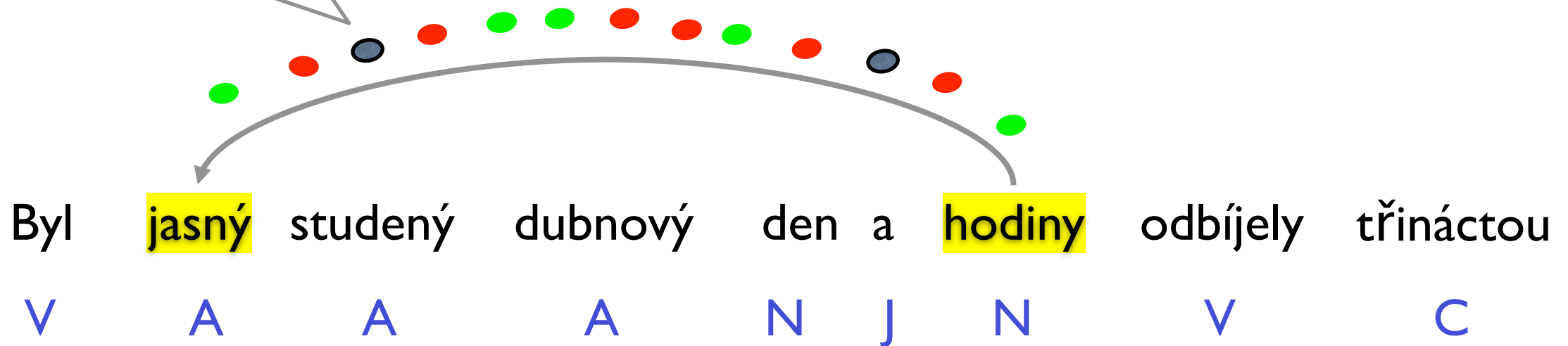
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C

"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers

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... undertrained ...



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jasn ← hodi
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stems only)

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byl jasn stud dubn den a hodi odbí třin

"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers

- How about this competing edge?

jasný ← hodiny
("bright clocks")
... *undertrained* ...

jasn ← hodi
("bright clock,"
stems only)

A_{singular} ← N_{plural}

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"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers

- How about this competing edge?

jasný ← hodiny

A ← N

where N follows
a conjunction

jasn ← hodi

(“bright clock,”
stems only)

A_{singular} ← N_{plural}

Byl jasný studený dubnový den a hodiny odbíjely třináctou

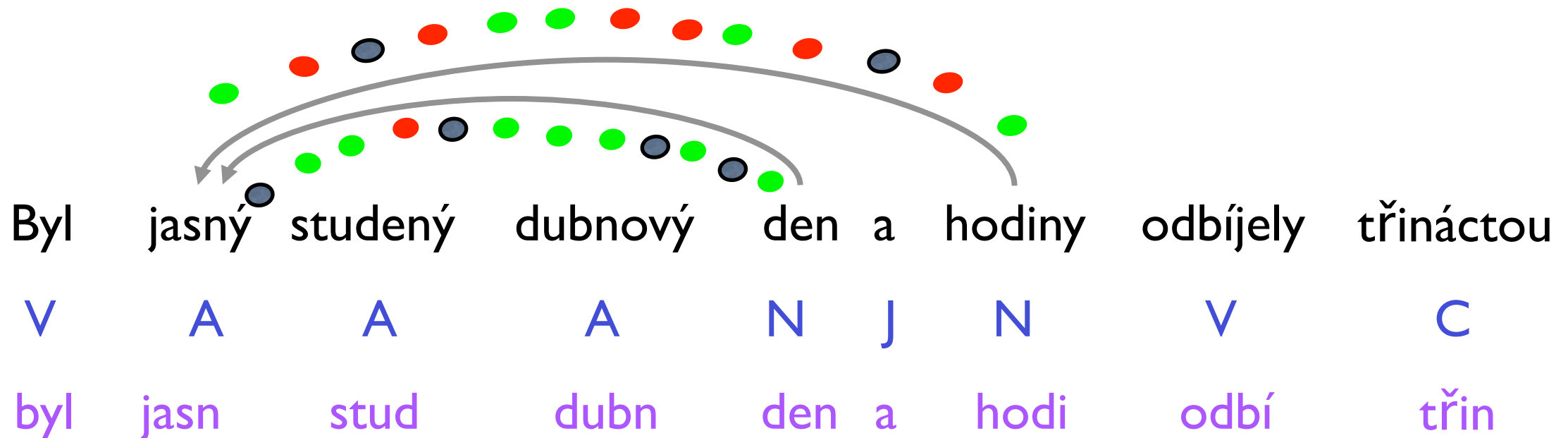
V A A A N J N V C

byl jasn stud dubn den a hodi odbí třin

“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

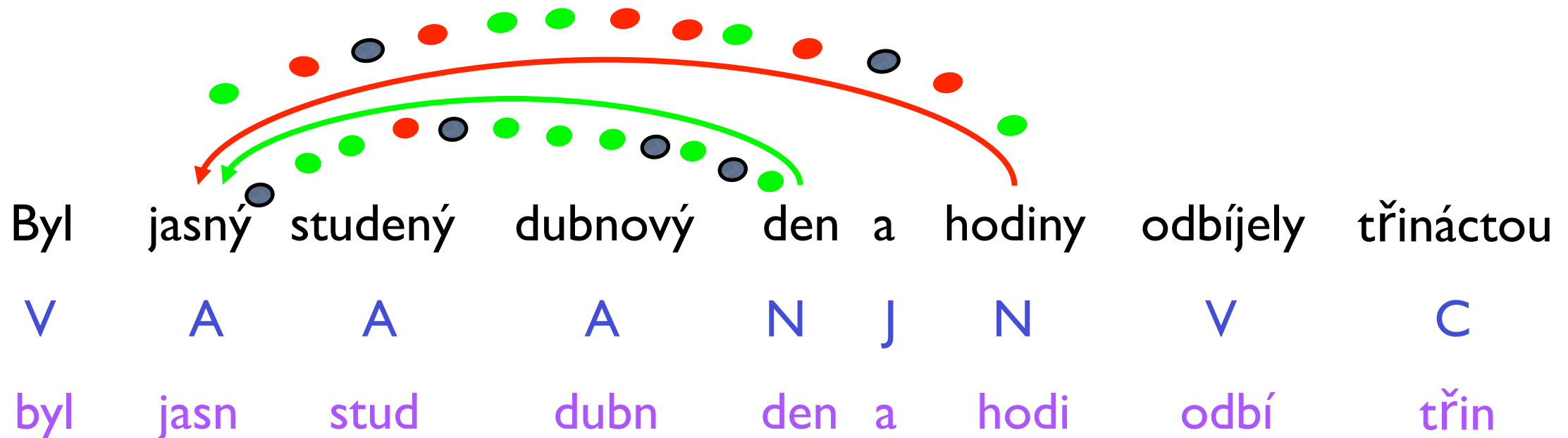
- Which edge is better?
 - “bright day” or “bright clocks”?



“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

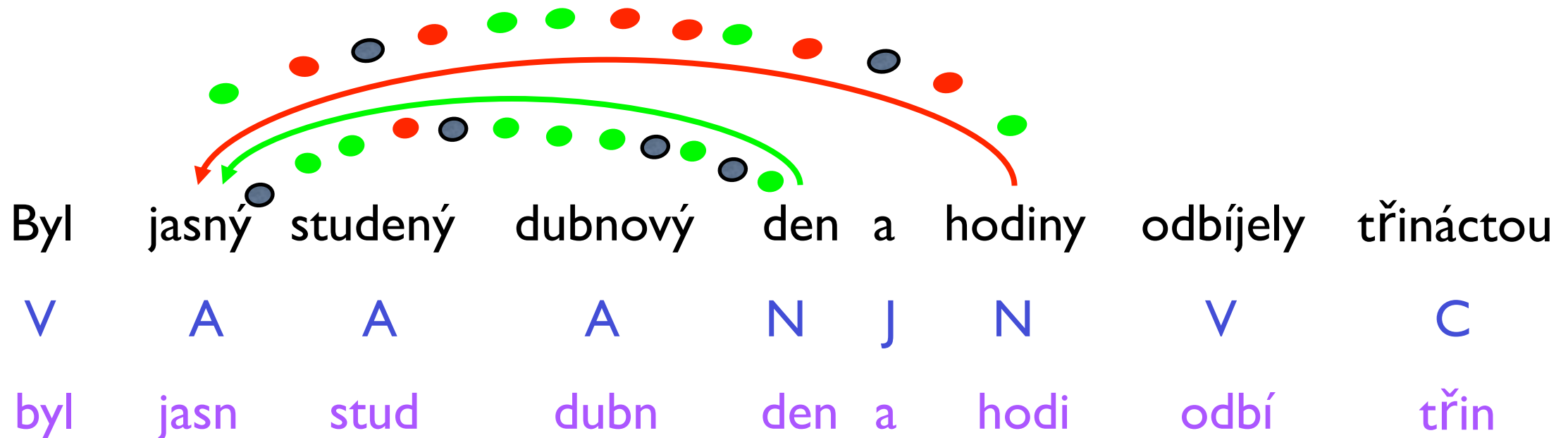
- Which edge is better?
- Score of an edge $e = \theta \cdot \mathbf{features}(e)$
- Standard algos $\rightarrow$ valid parse with max total score



“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers

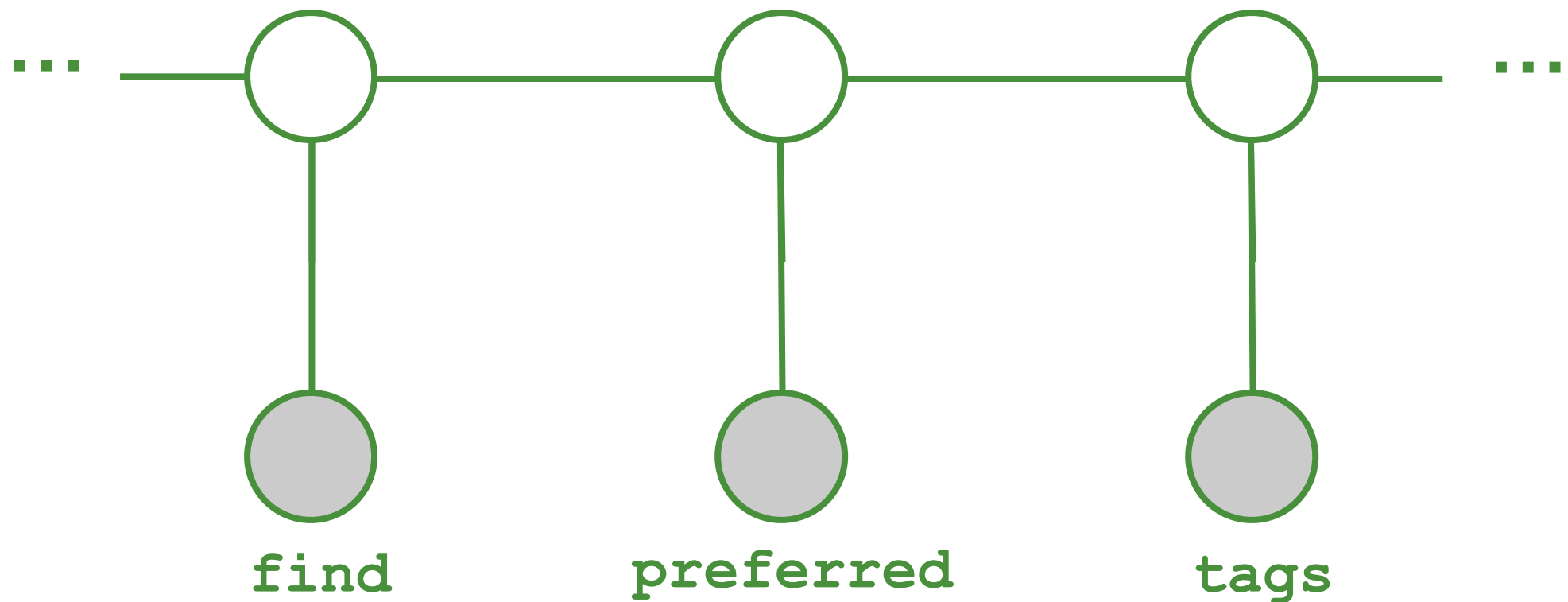
- Which edge is better? *our current weight vector*
- Score of an edge $e = \theta \cdot \text{features}(e)$
- Standard algos $\rightarrow$ valid parse with max total score



“It was a bright cold day in April and the clocks were striking thirteen”

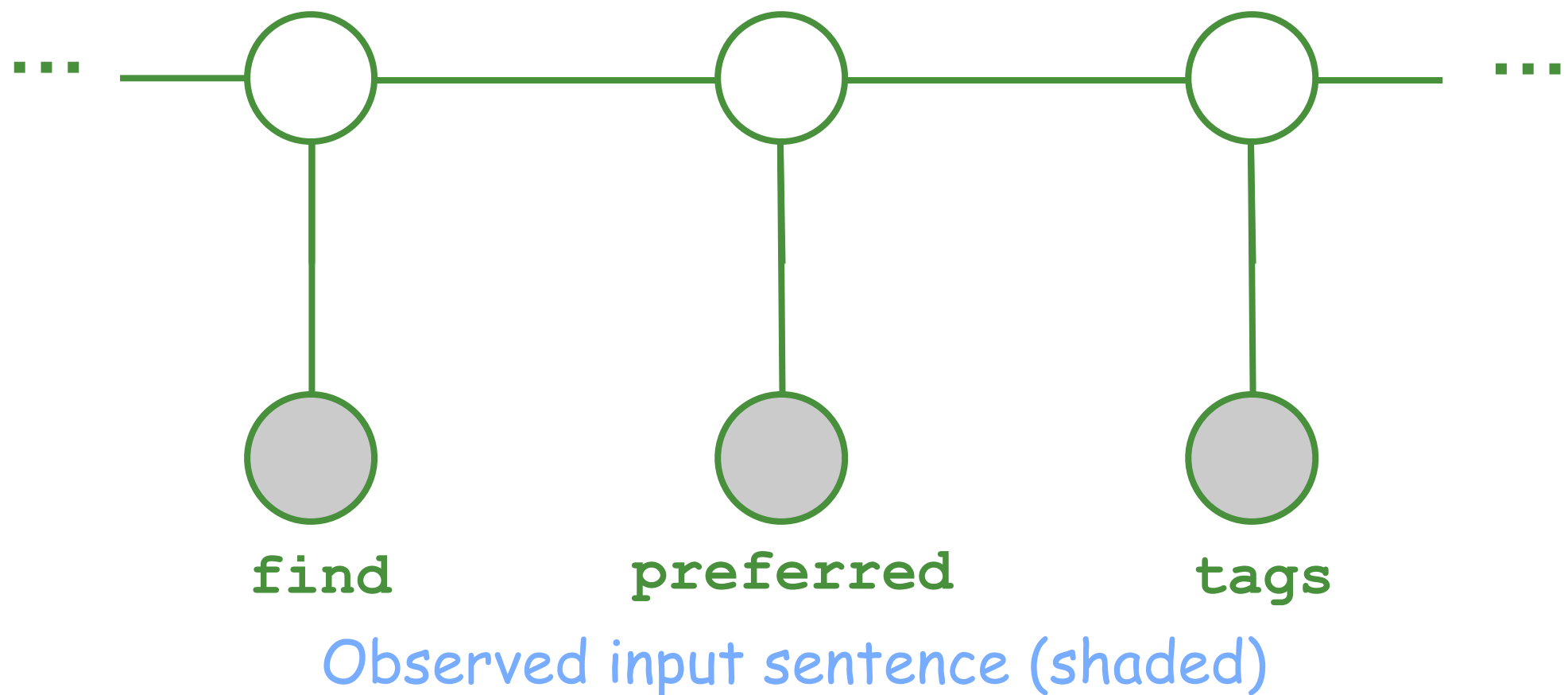
Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging



Local factors in a graphical model

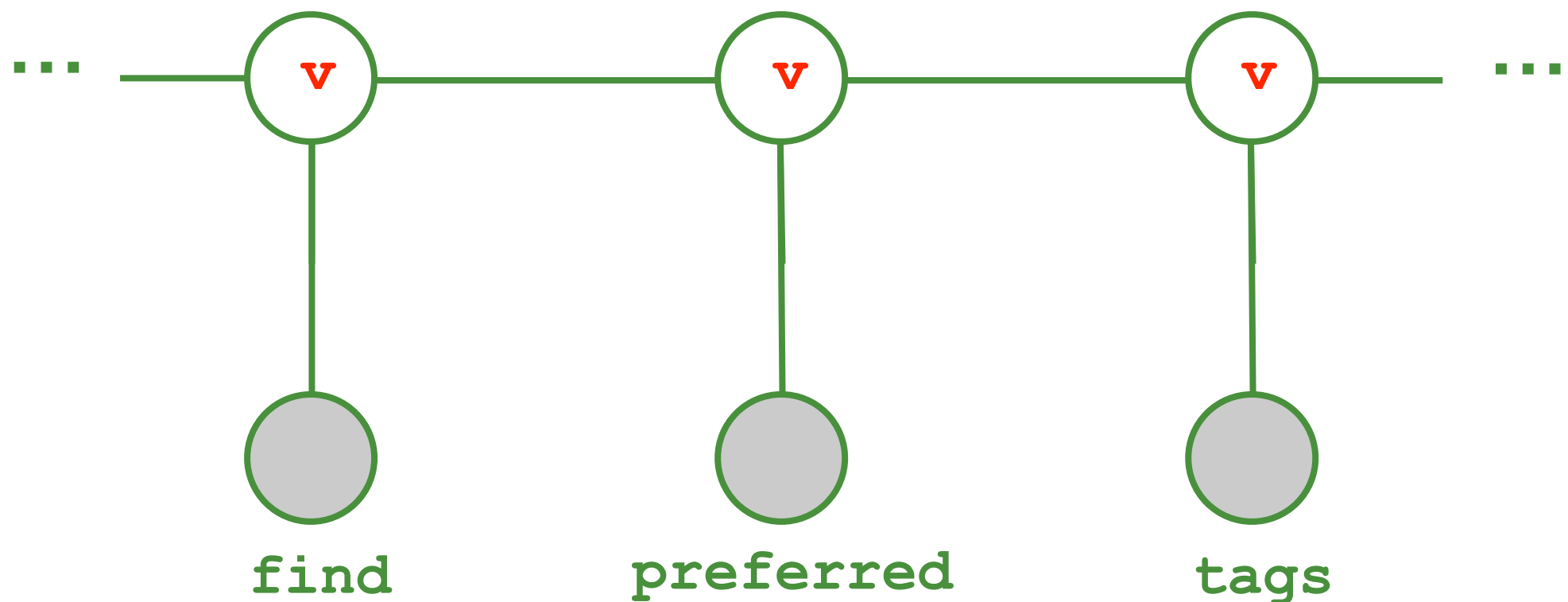
- First, a familiar example
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Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

Possible tagging (i.e., assignment to remaining variables)

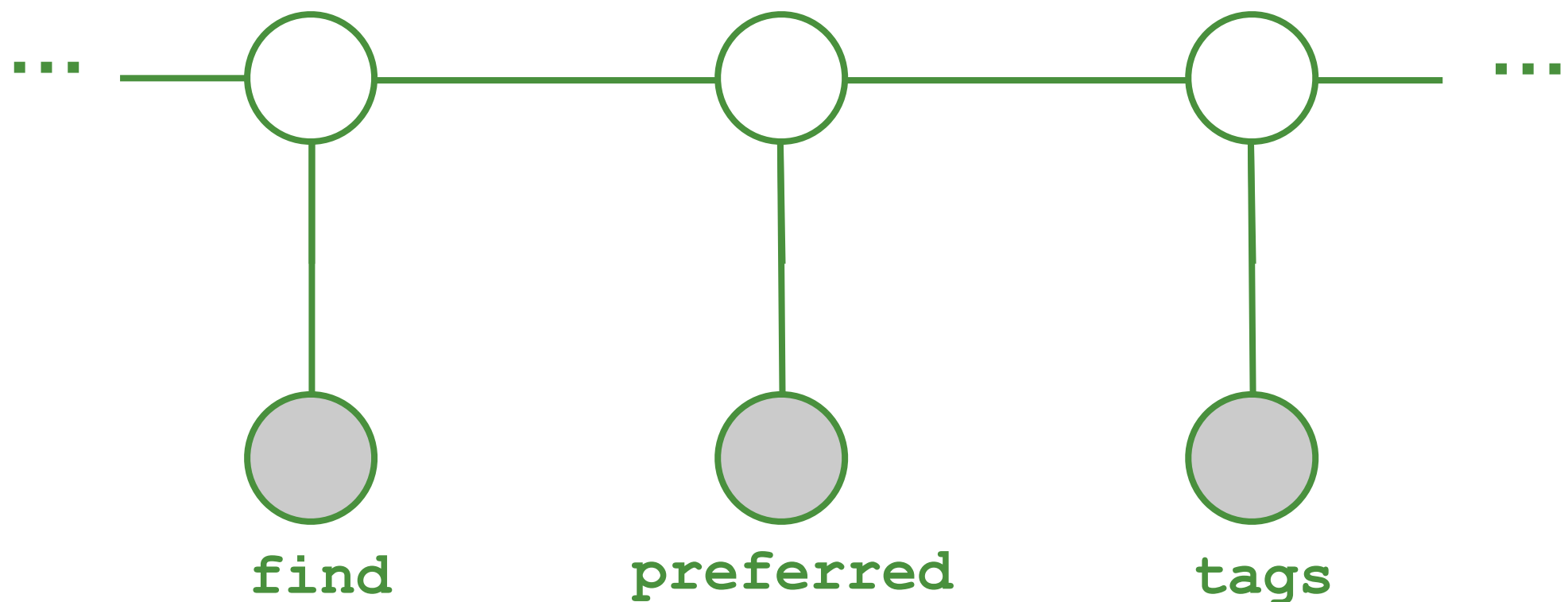


Observed input sentence (shaded)

Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

Possible tagging (i.e., assignment to remaining variables)
Another possible tagging

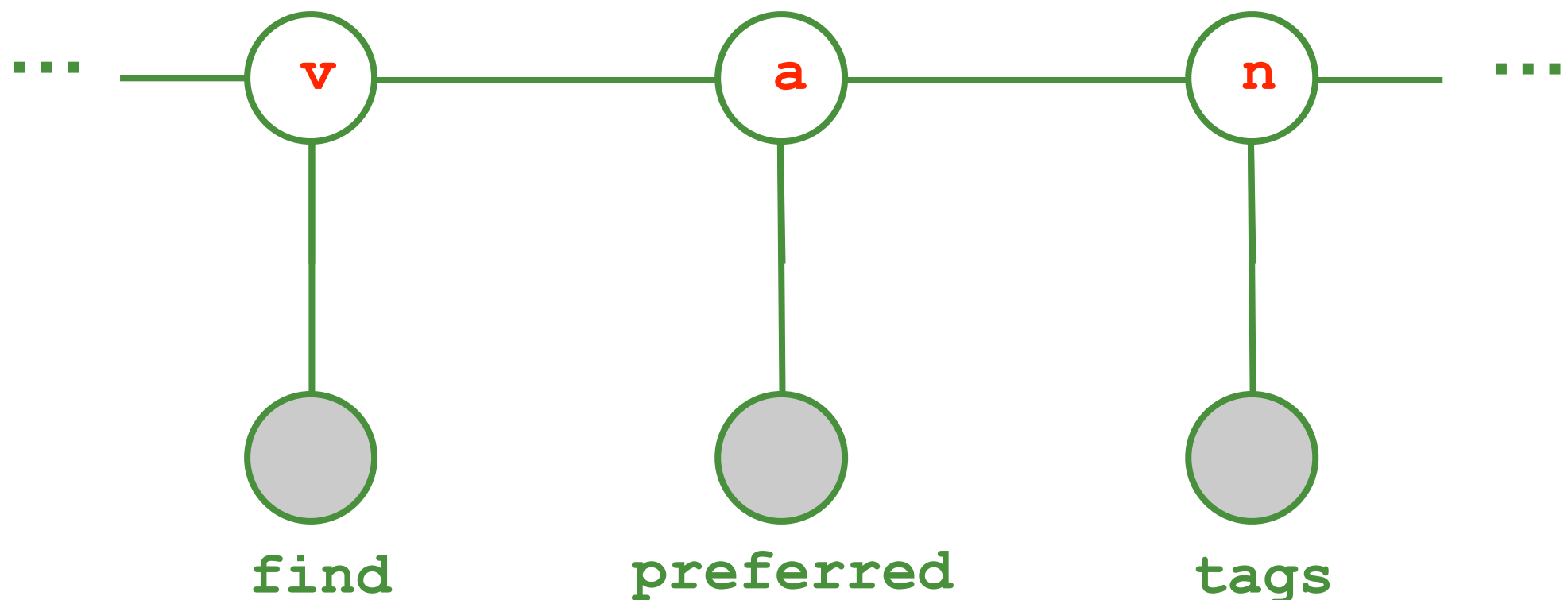


Observed input sentence (shaded)

Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

Possible tagging (i.e., assignment to remaining variables)
Another possible tagging



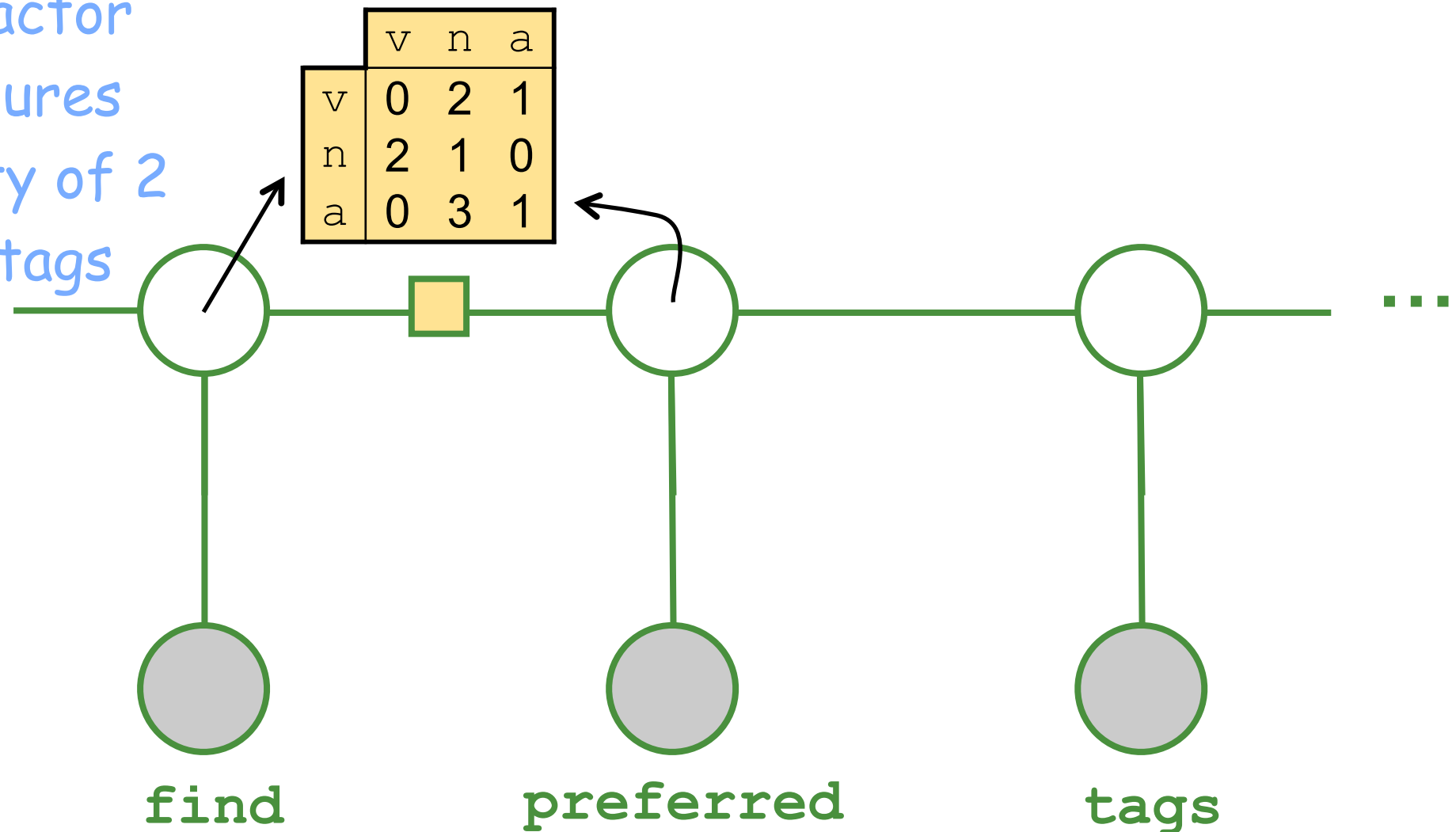
Observed input sentence (shaded)

Local factors in a graphical model

- First, a familiar example

- Conditional Random Field (CRF) for POS tagging

"Binary" factor
that measures
compatibility of 2
adjacent tags
...



Local factors in a graphical model

- First, a familiar example

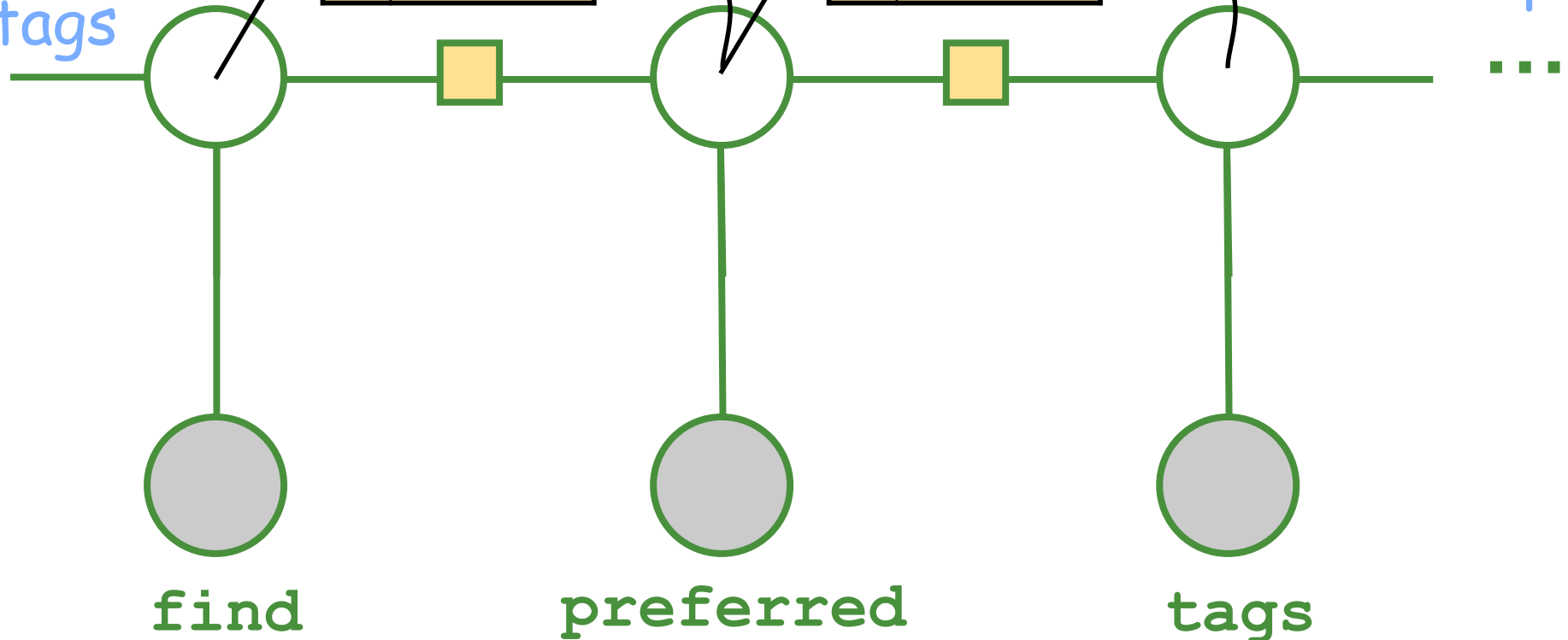
- Conditional Random Field (CRF) for POS tagging

"Binary" factor
that measures
compatibility of 2
adjacent tags
...

	v	n	a
v	0	2	1
n	2	1	0
a	0	3	1

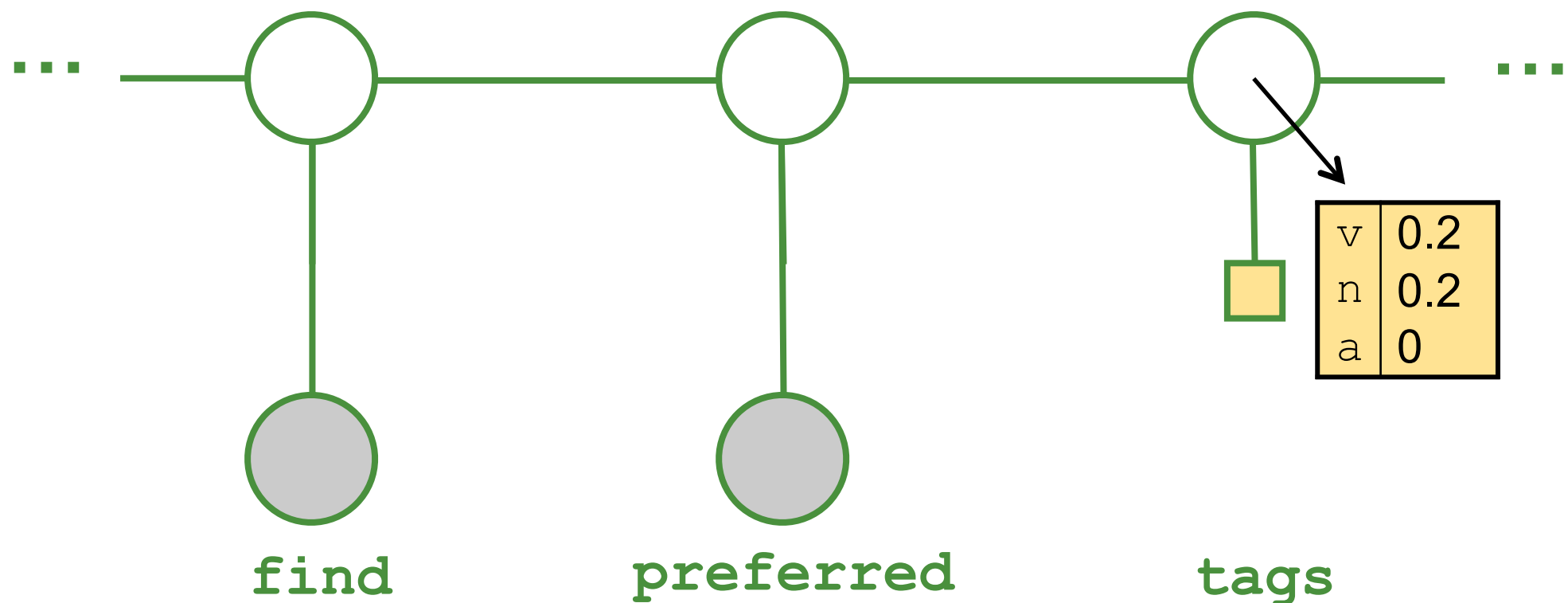
	v	n	a
v	0	2	1
n	2	1	0
a	0	3	1

Model reuses
same parameters
at this position
...



Local factors in a graphical model

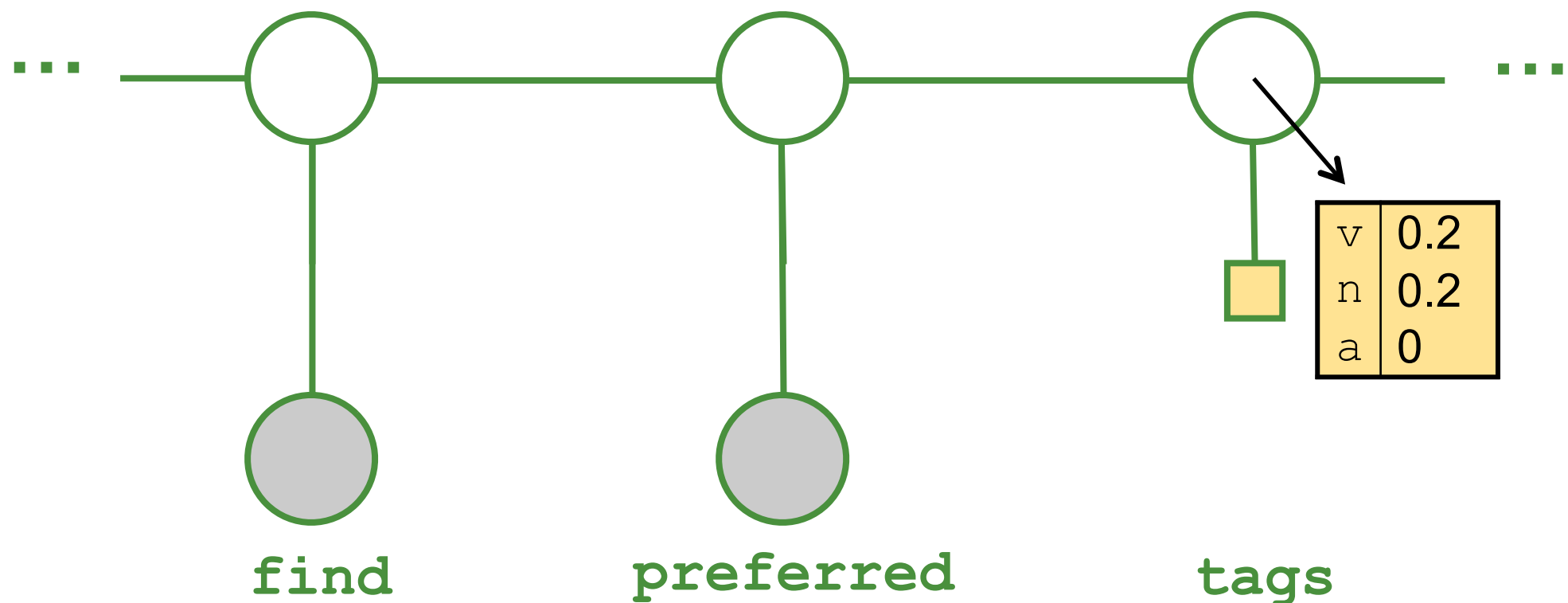
- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging



Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

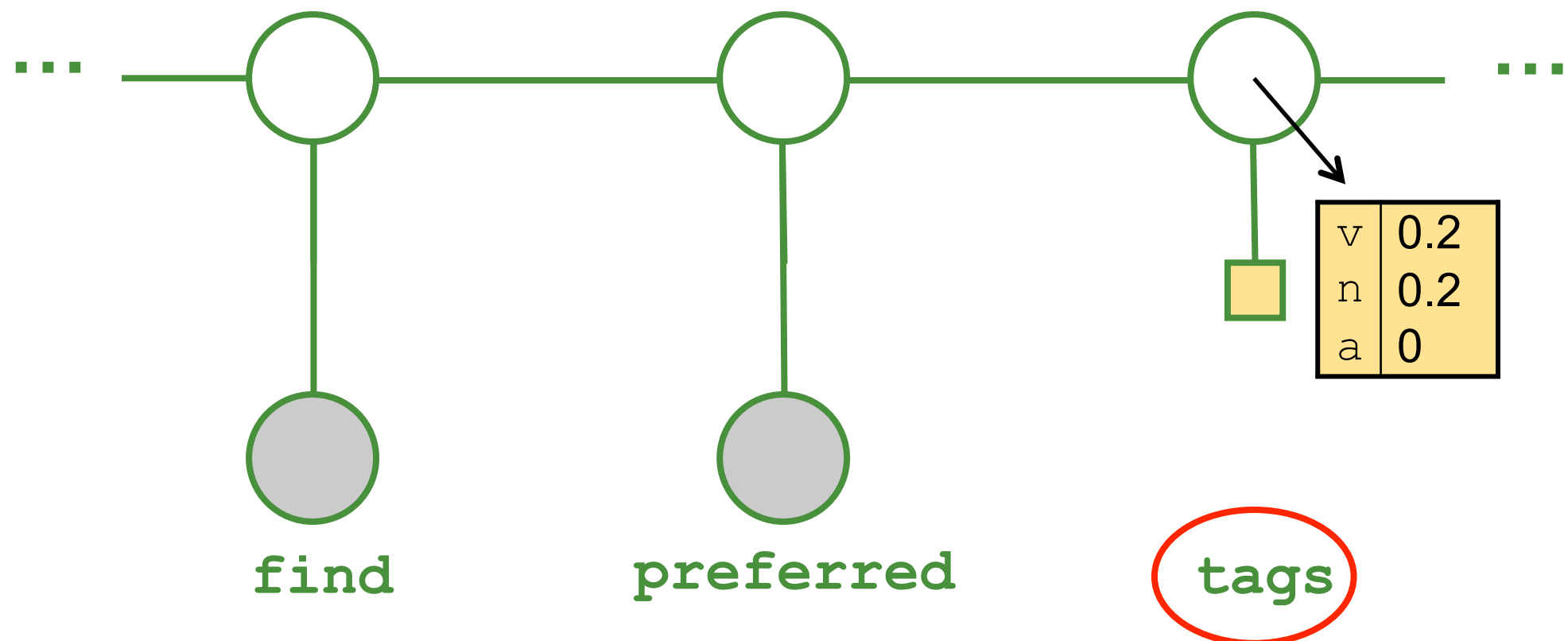
"Unary" factor evaluates this tag



Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

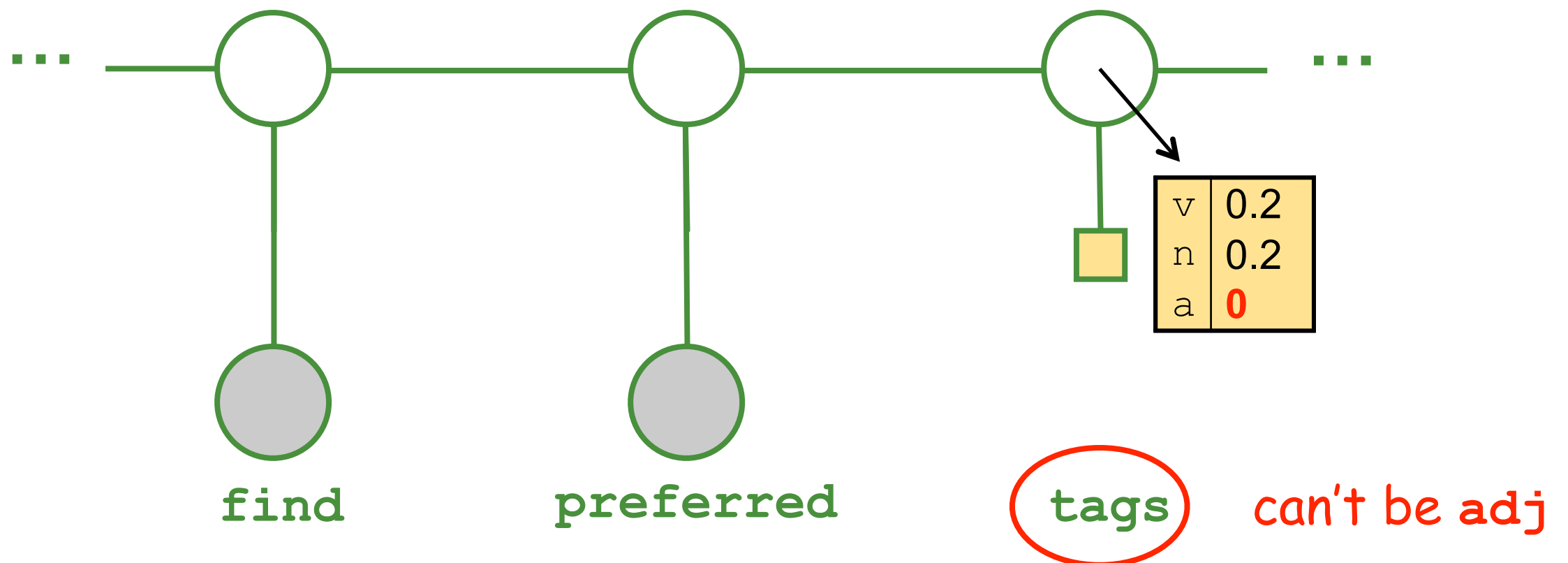
"Unary" factor evaluates this tag
Its values depend on corresponding word



Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

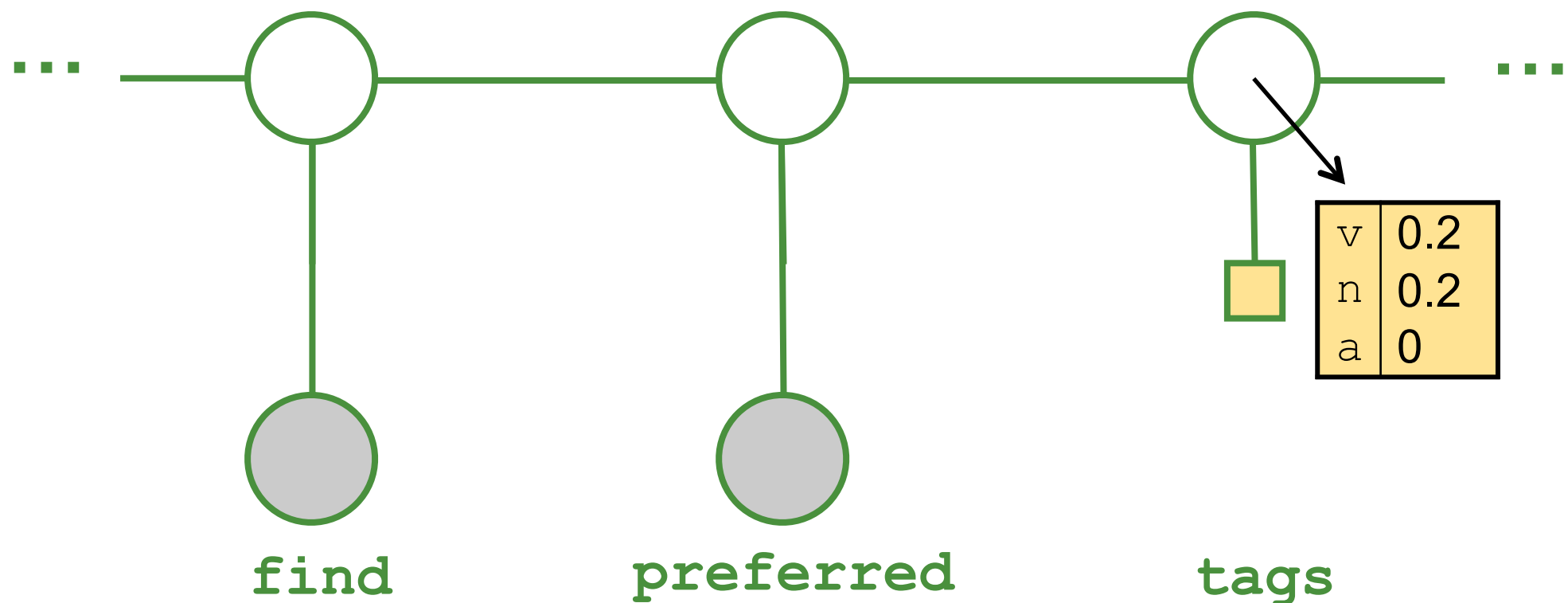
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Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

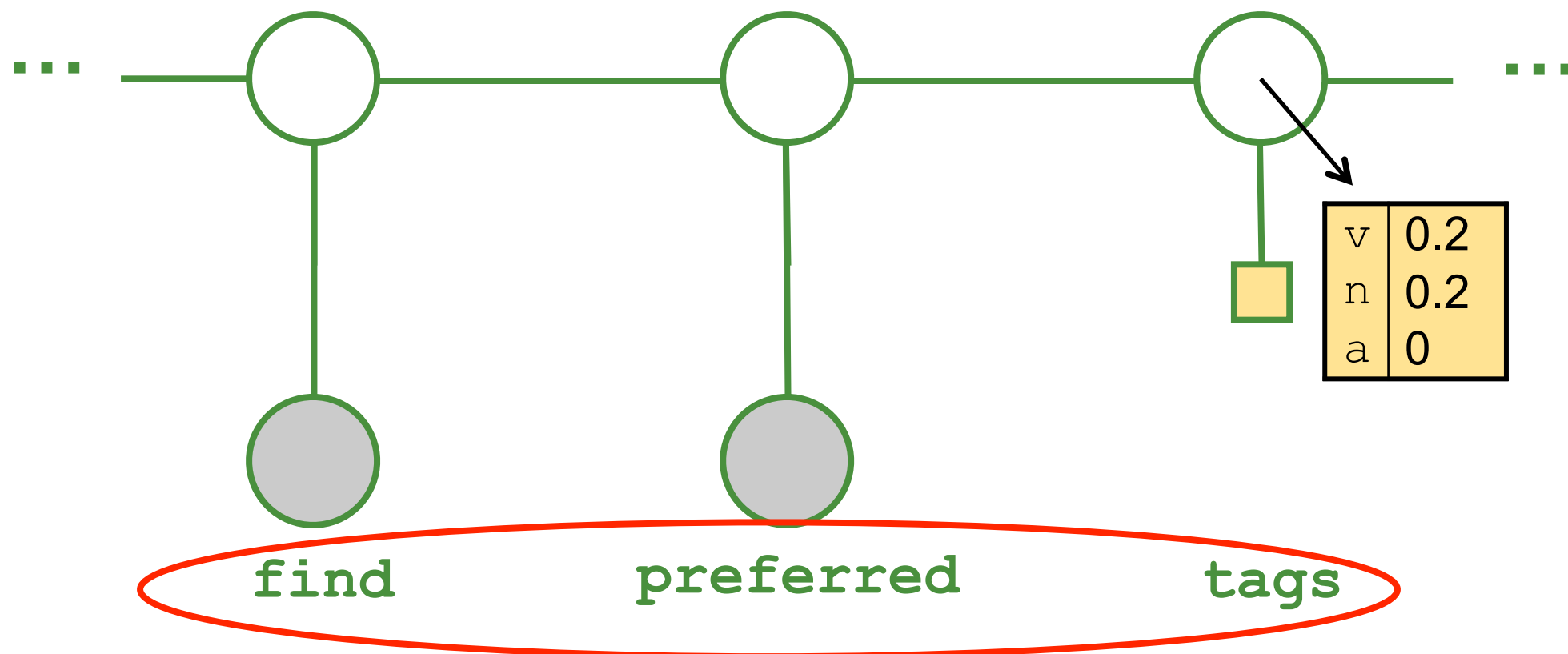
"Unary" factor evaluates this tag
Its values depend on corresponding word



Local factors in a graphical model

- First, a familiar example
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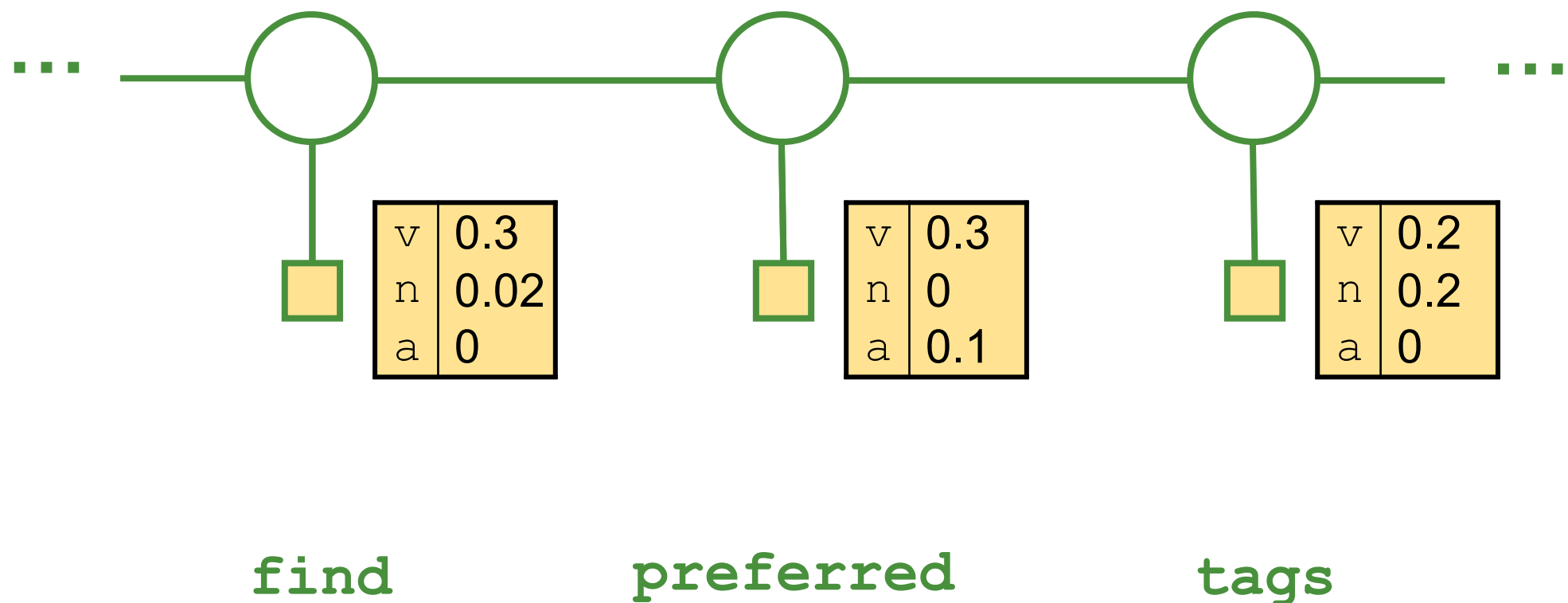
"Unary" factor evaluates this tag
Its values depend on corresponding word



(could be made to depend on entire observed sentence)

Local factors in a graphical model

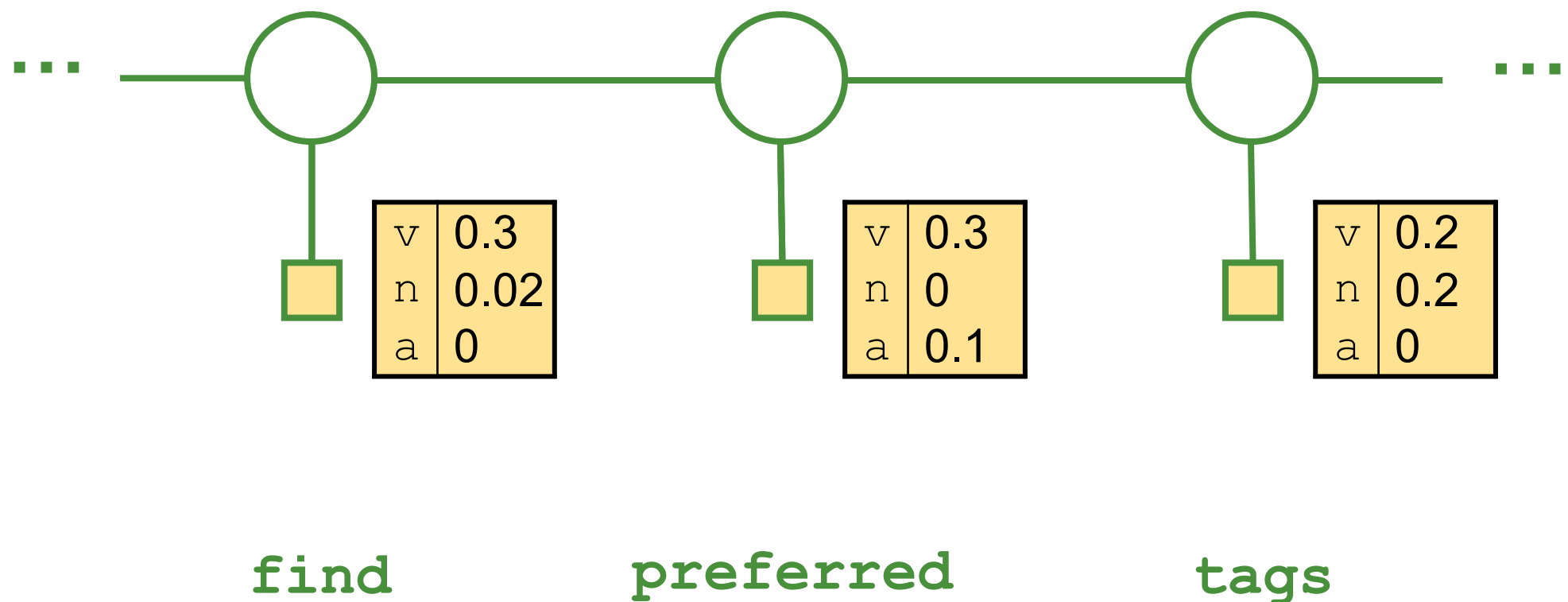
- First, a familiar example
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Local factors in a graphical model

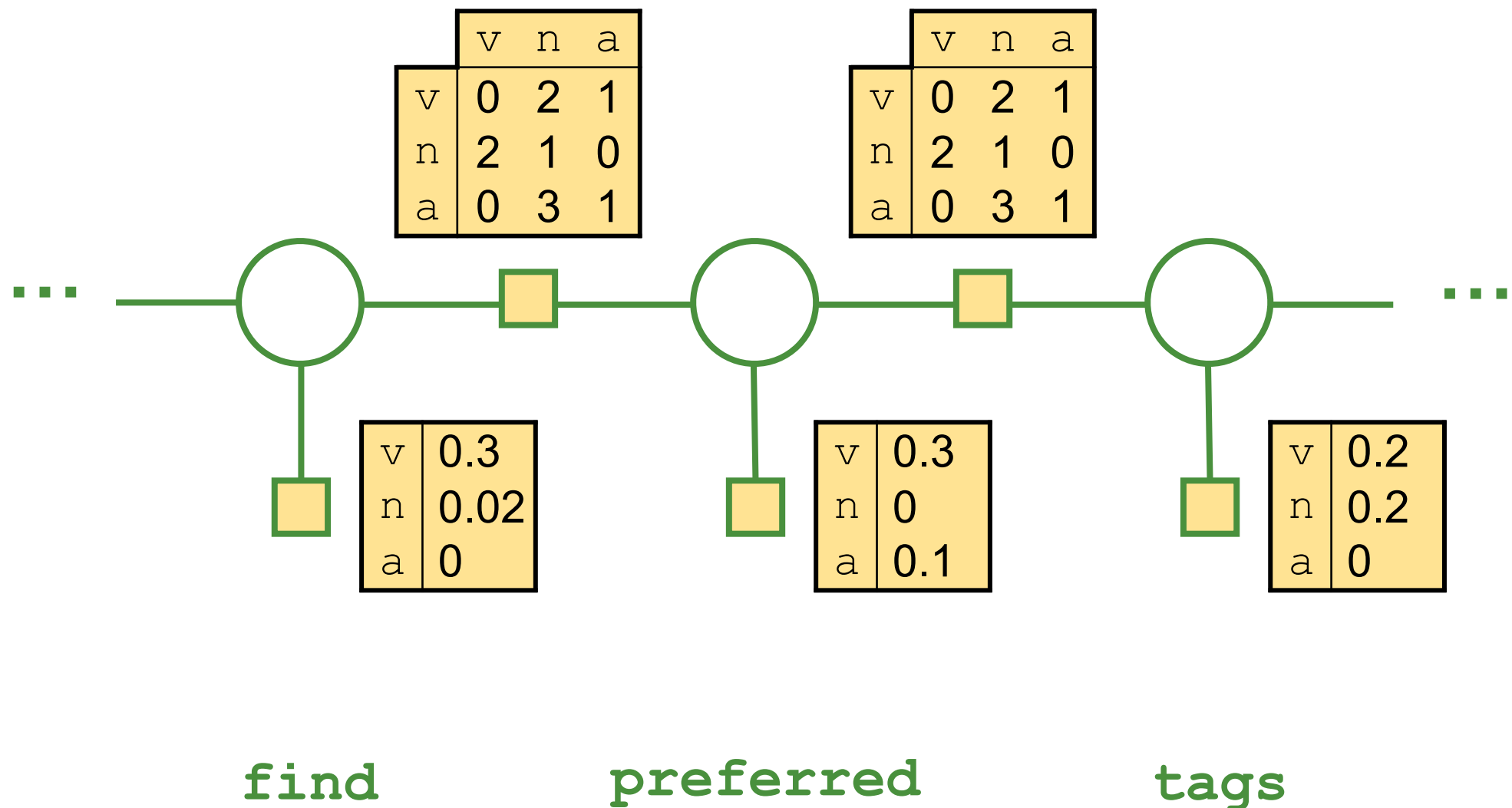
- First, a familiar example
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"Unary" factor evaluates this tag
Different unary factor at each position



Local factors in a graphical model

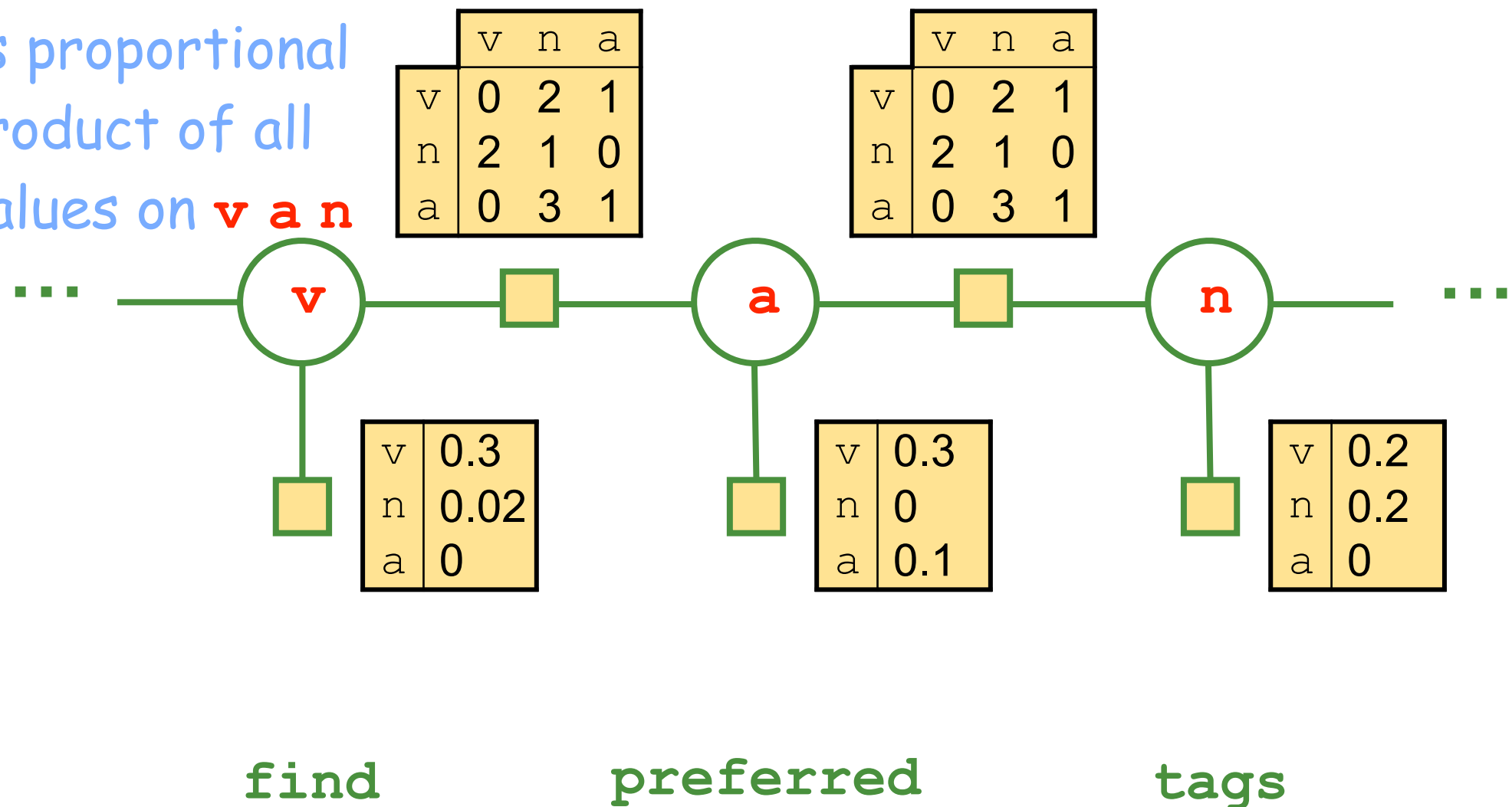
- First, a familiar example
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Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

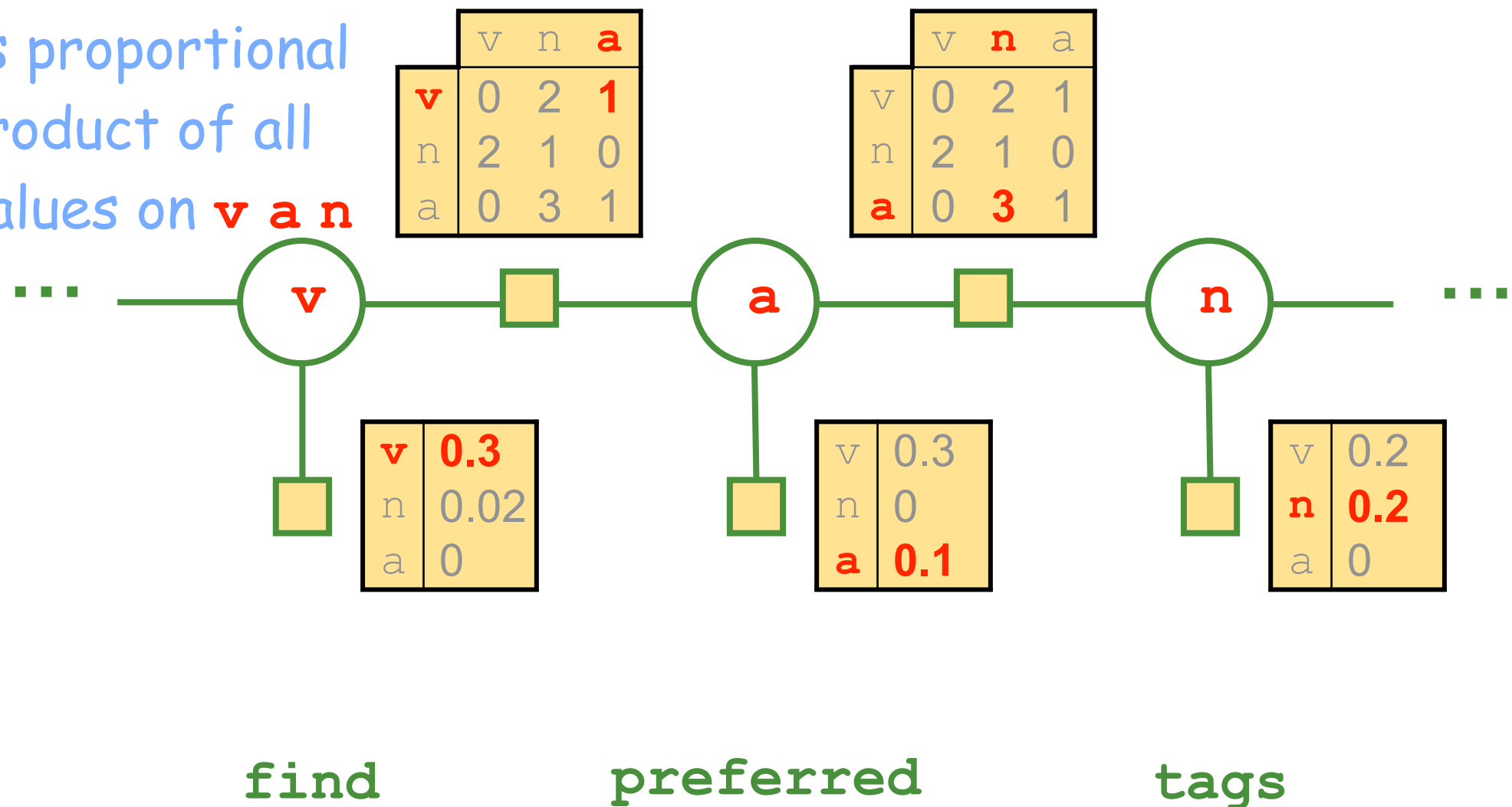
$p(\mathbf{v} \ \mathbf{a} \ \mathbf{n})$ is proportional
to the product of all
factors' values on $\mathbf{v} \ \mathbf{a} \ \mathbf{n}$



Local factors in a graphical model

- First, a familiar example
 - Conditional Random Field (CRF) for POS tagging

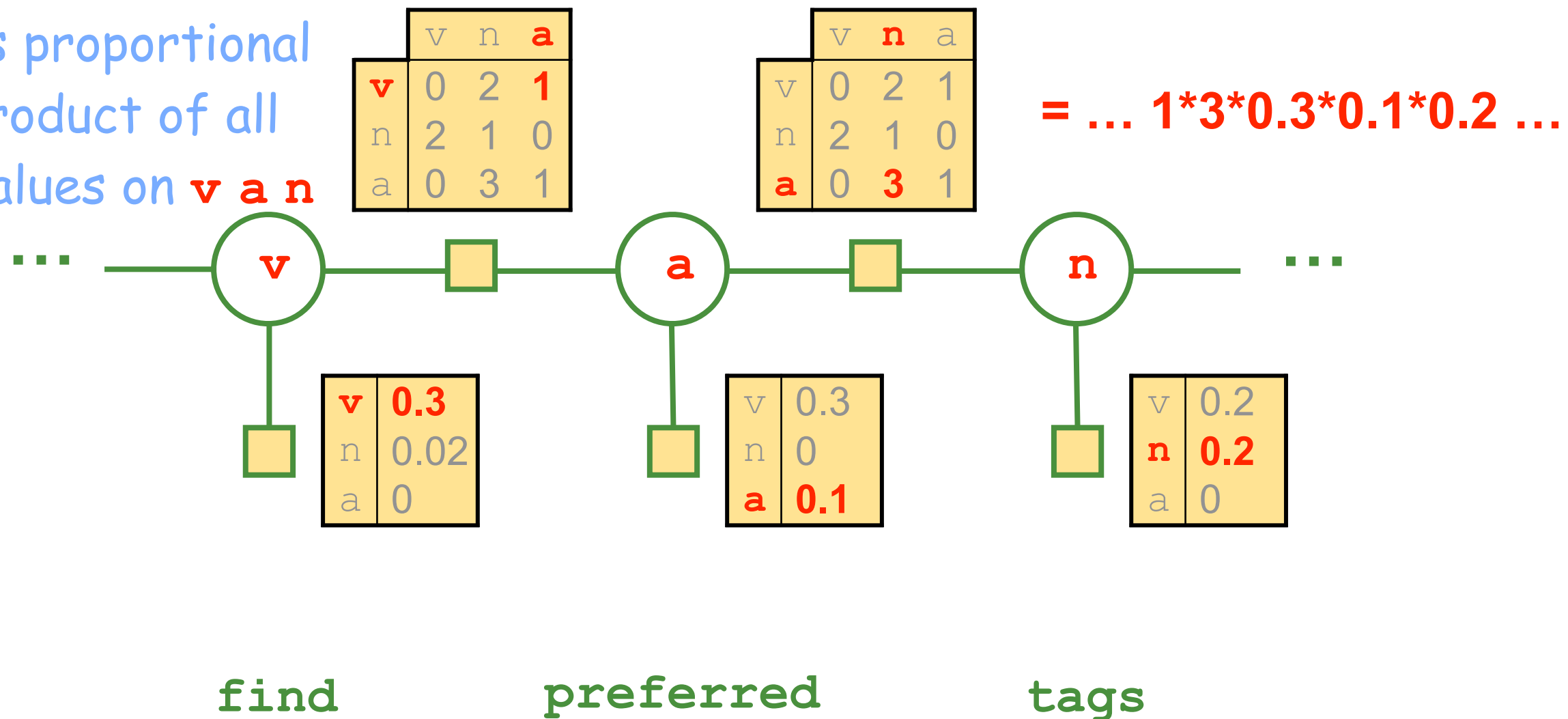
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Local factors in a graphical model

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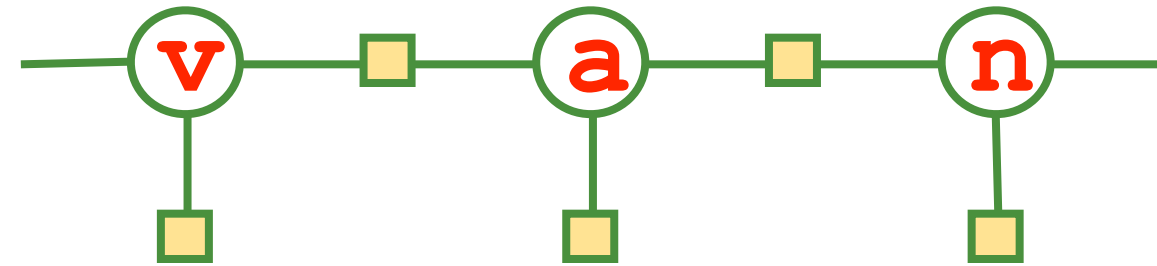
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to the product of all
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Graphical Models for Parsing

- First, a labeling example

- ✧ CRF for POS tagging



- Now let's do dependency parsing!

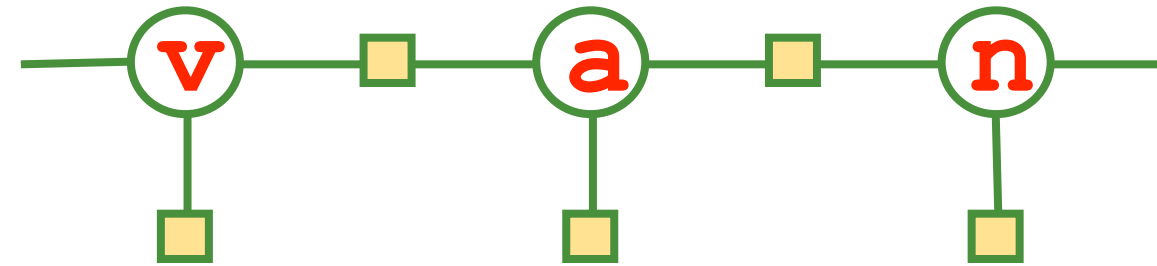
- ✧ $O(n^2)$ boolean variables for the possible links

... find preferred links ...

Graphical Models for Parsing

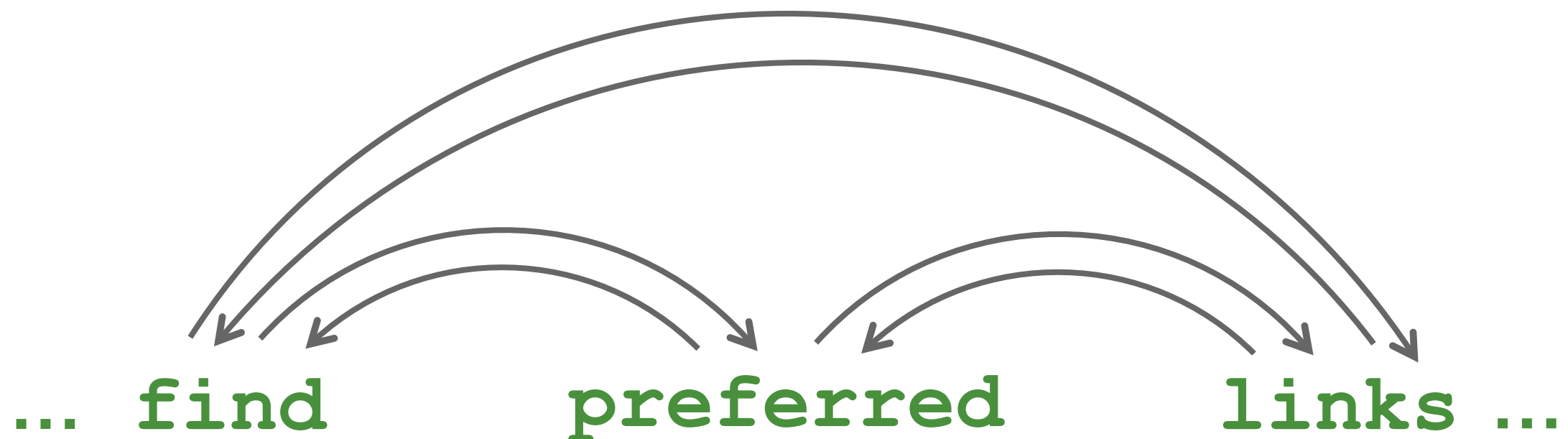
- First, a labeling example

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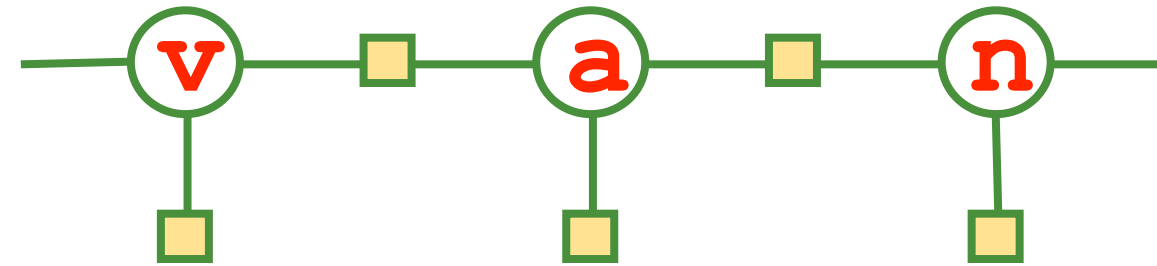
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Graphical Models for Parsing

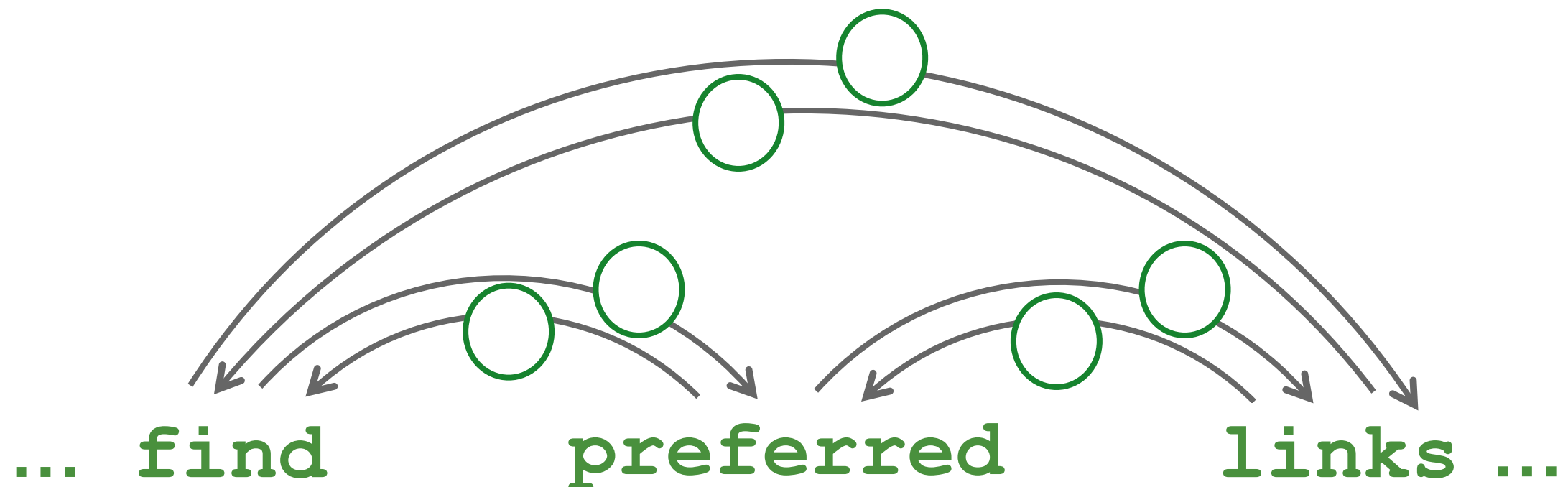
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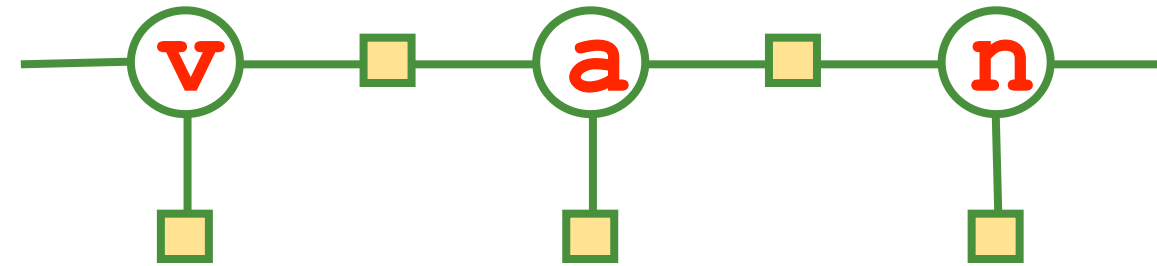
- ✦ $O(n^2)$ boolean variables for the possible links



Graphical Models for Parsing

- First, a labeling example

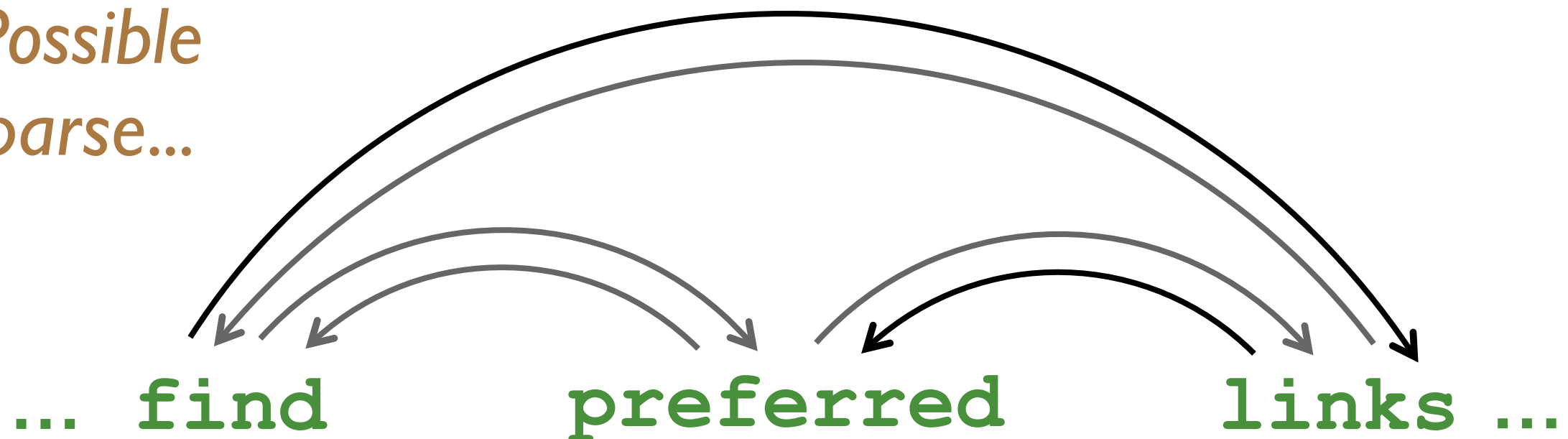
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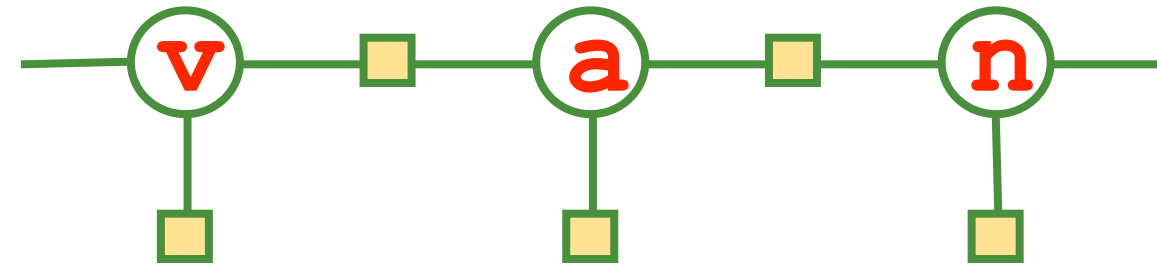
*Possible
parse...*



Graphical Models for Parsing

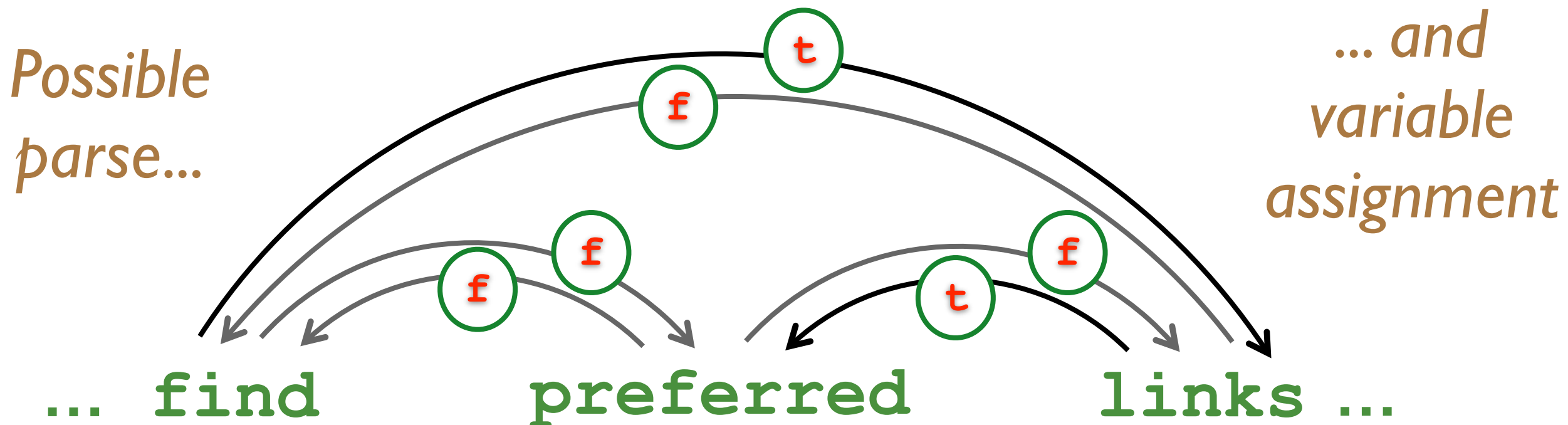
- First, a labeling example

- ✧ CRF for POS tagging



- Now let's do dependency parsing!

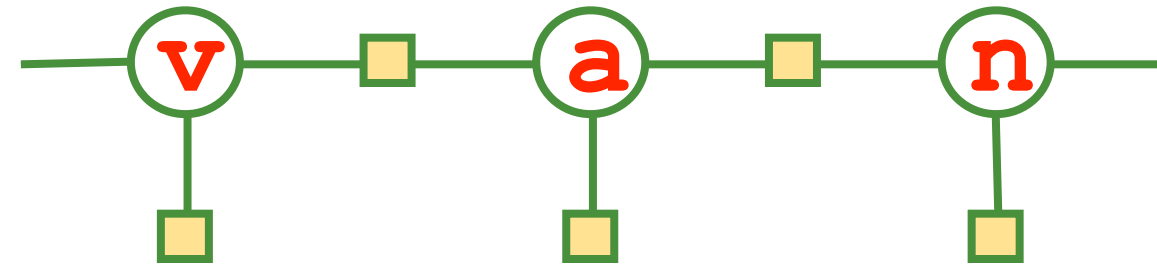
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Graphical Models for Parsing

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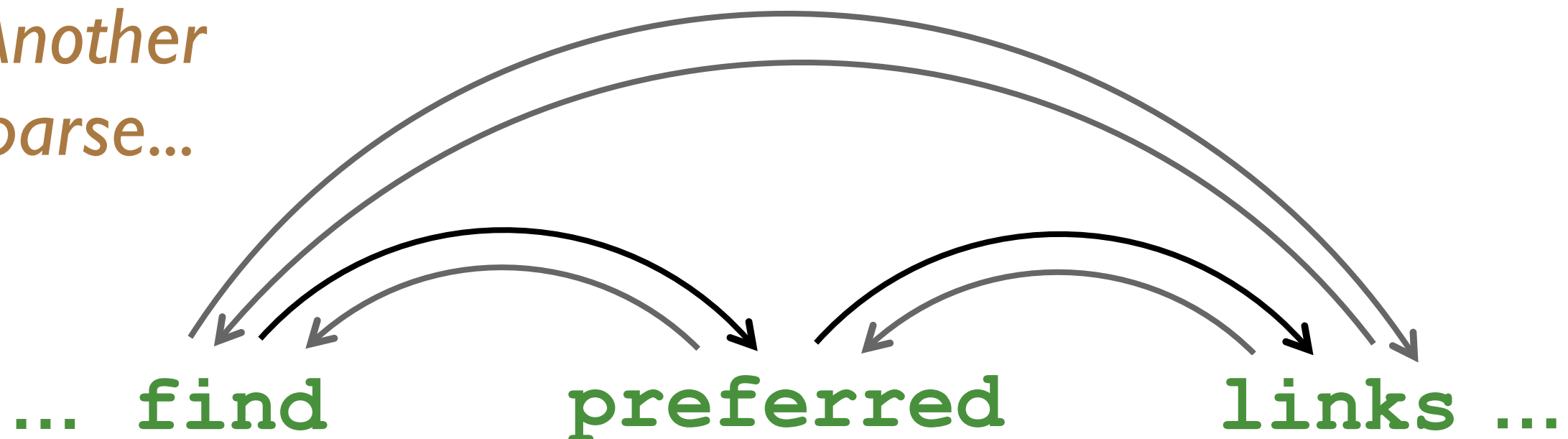
- ✧ CRF for POS tagging



- Now let's do dependency parsing!

- ✧ $O(n^2)$ boolean variables for the possible links

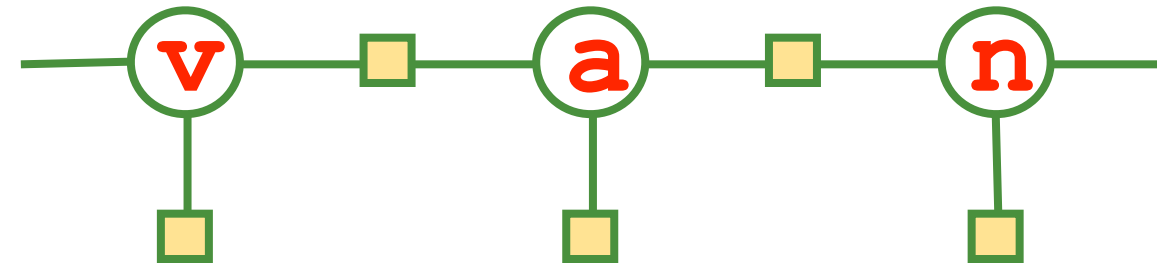
*Another
parse...*



Graphical Models for Parsing

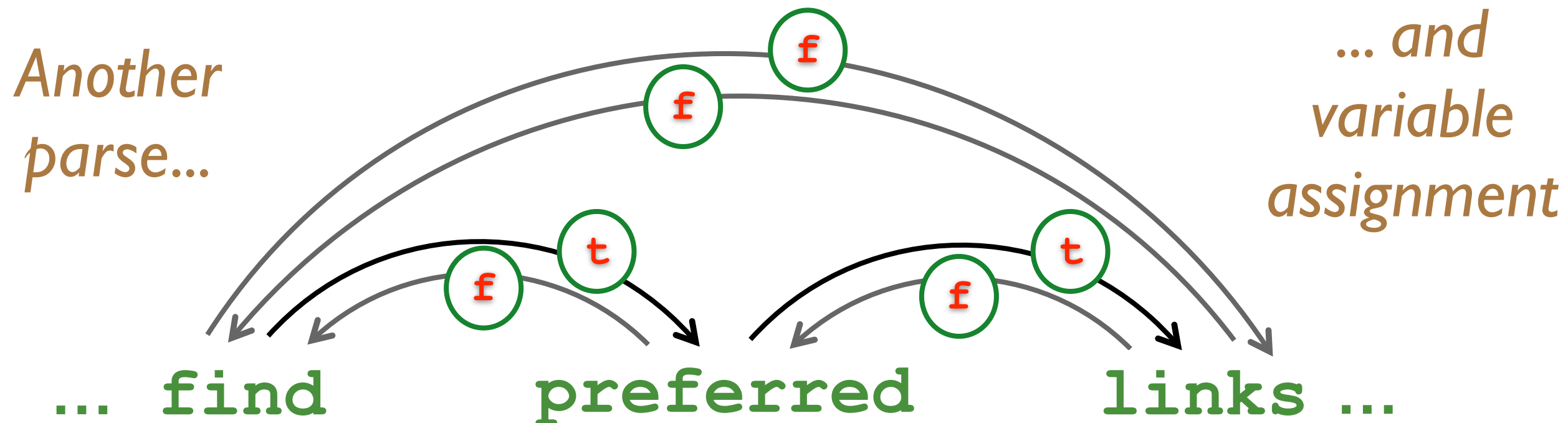
- First, a labeling example

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- Now let's do dependency parsing!

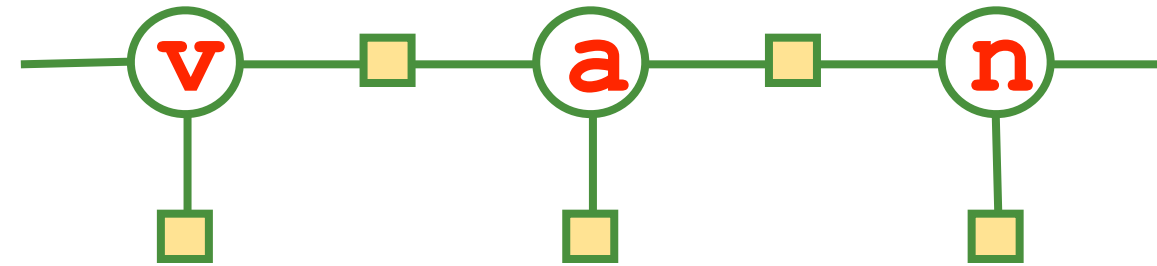
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Graphical Models for Parsing

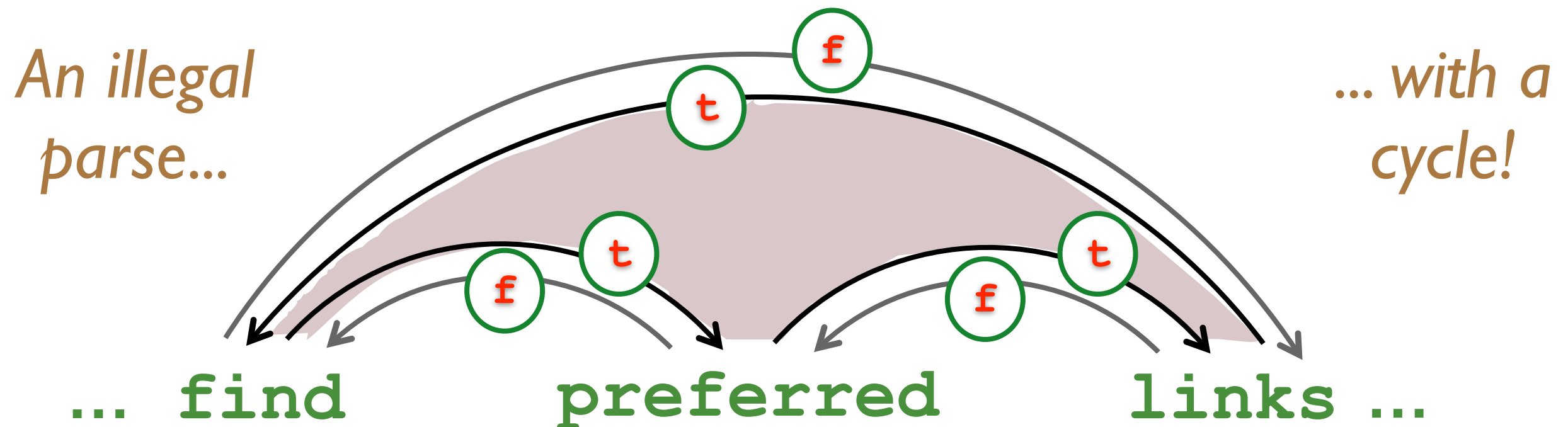
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- Now let's do dependency parsing!

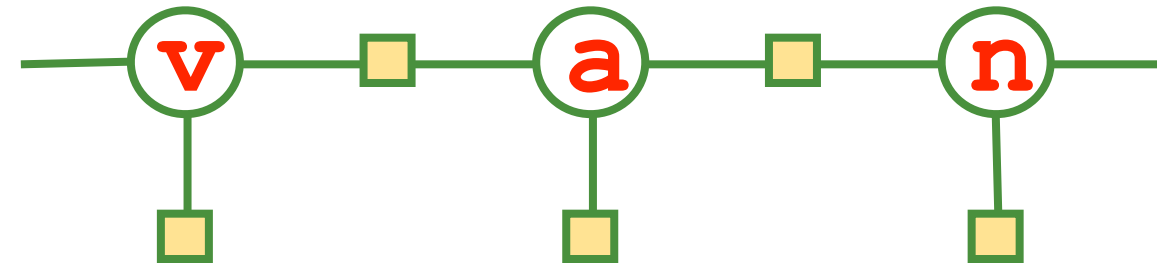
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Graphical Models for Parsing

- First, a labeling example

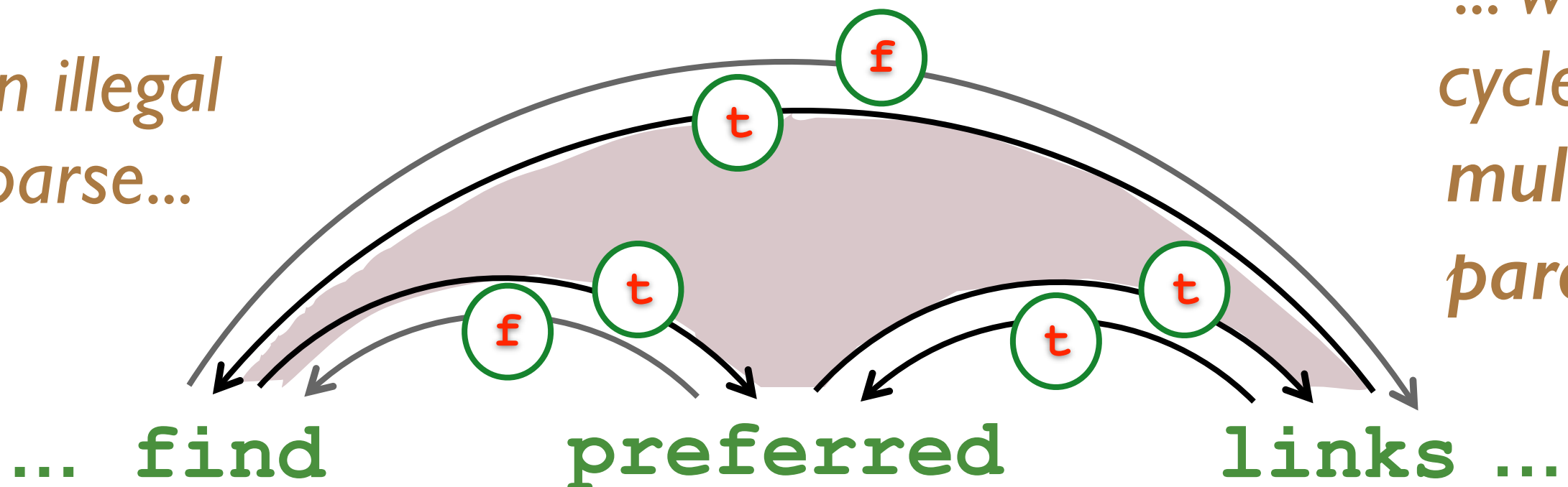
- ✧ CRF for POS tagging



- Now let's do dependency parsing!

- ✧ $O(n^2)$ boolean variables for the possible links

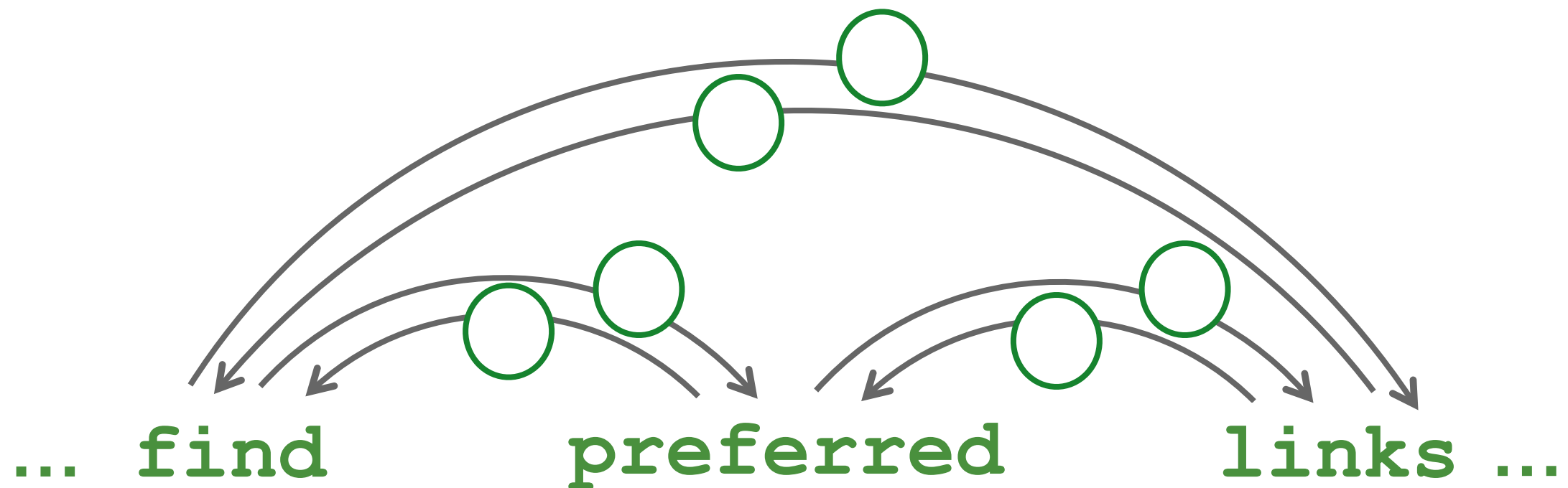
*An illegal
parse...*



*... with a
cycle and
multiple
parents!*

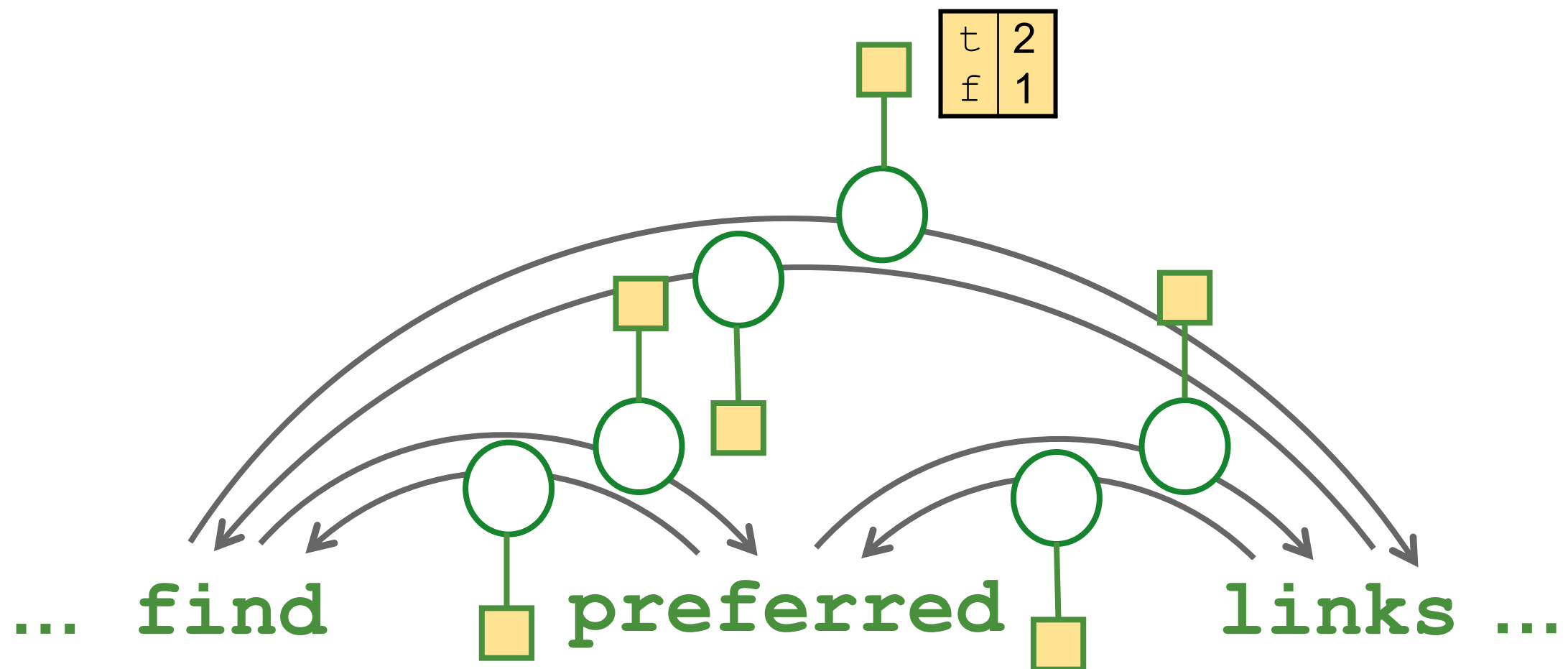
Local Factors for Parsing

- What factors determine parse probability?



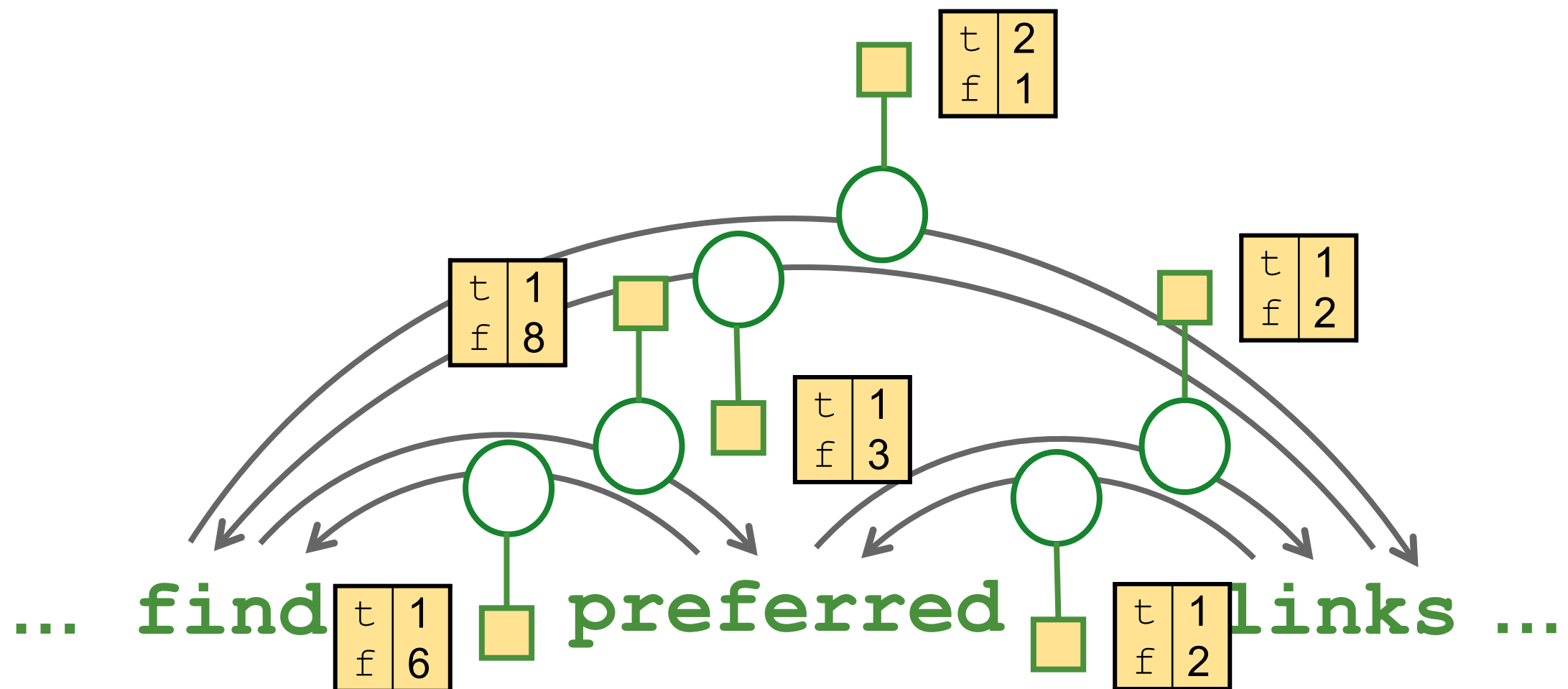
Local Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation



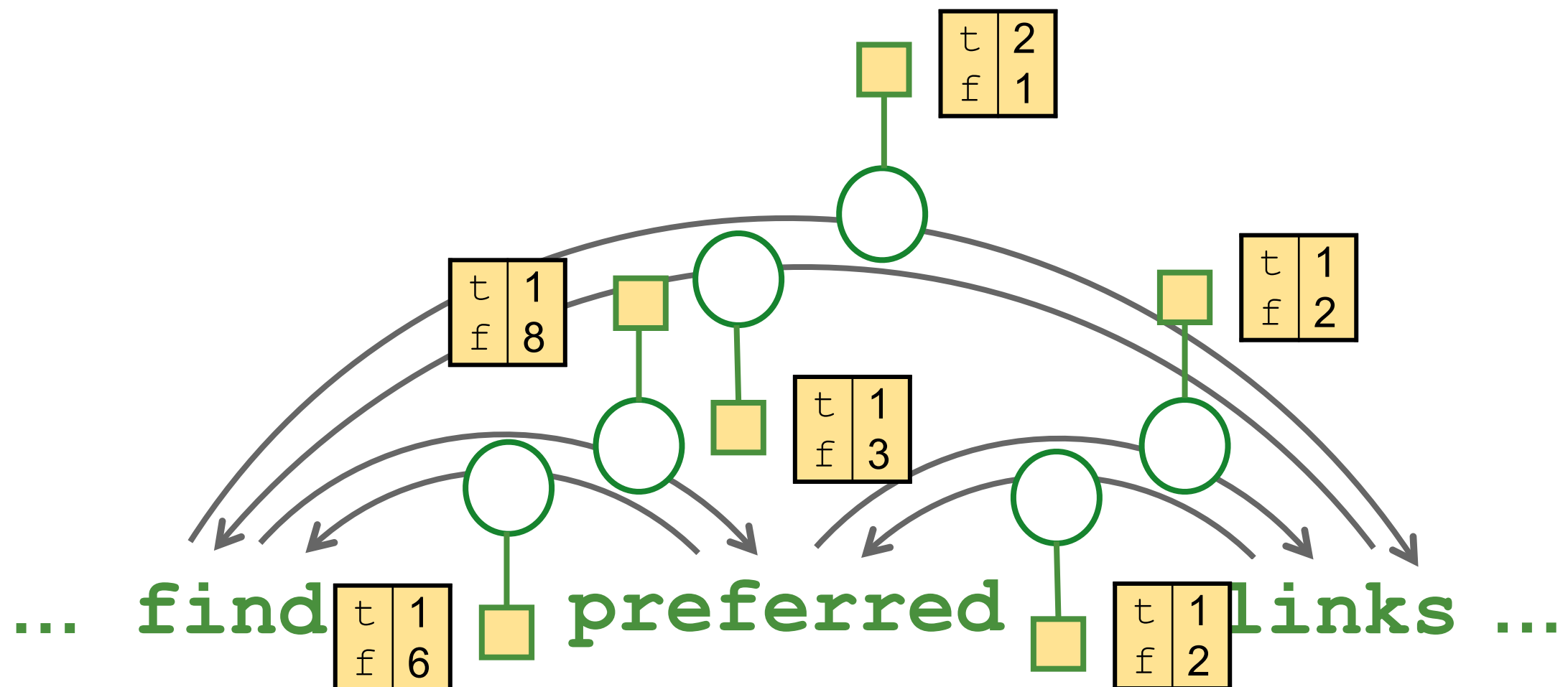
Local Factors for Parsing

- What factors determine parse probability?
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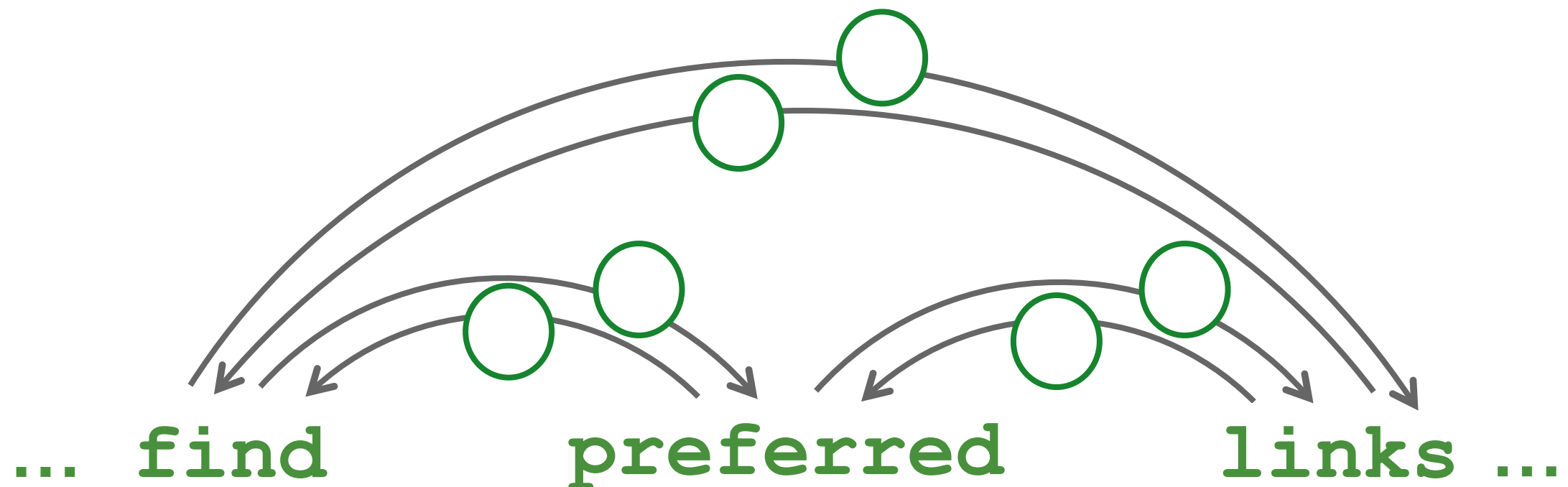
Local Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation
- But what if the best assignment isn't a tree?



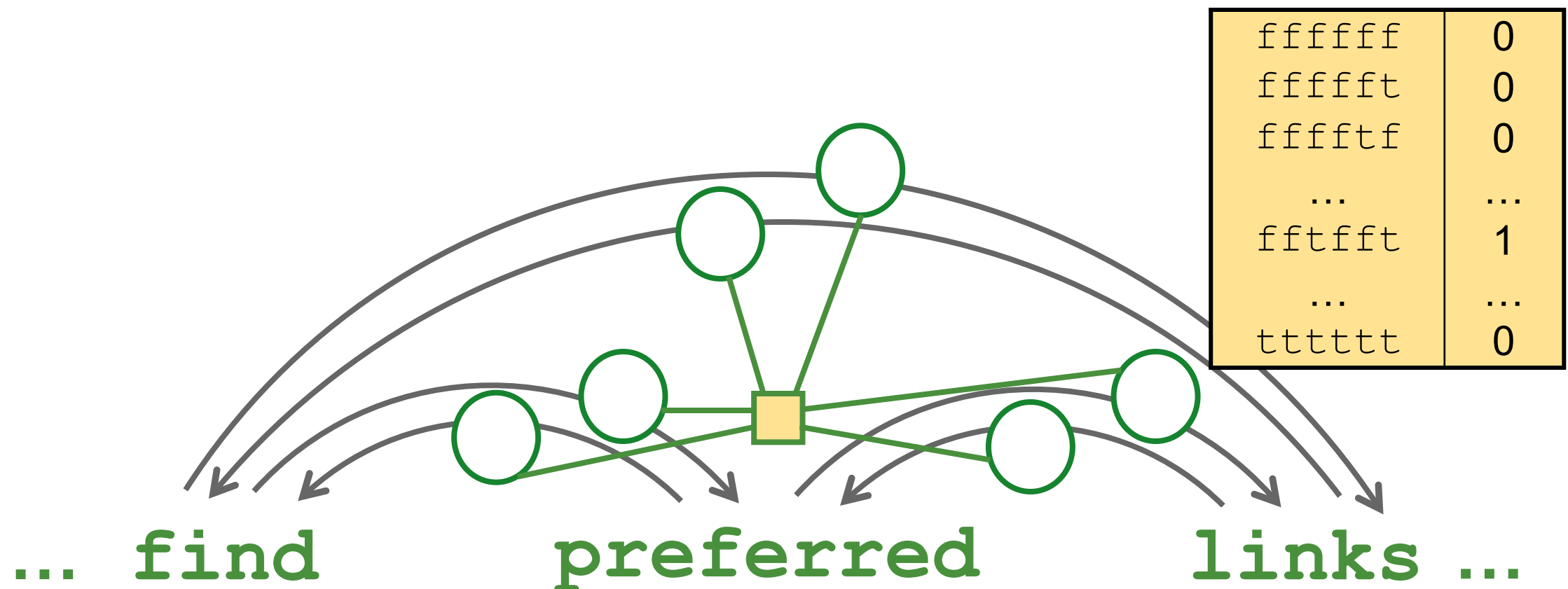
Global Factors for Parsing

- What factors determine parse probability?
 - ✧ Unary factors to score each link in isolation



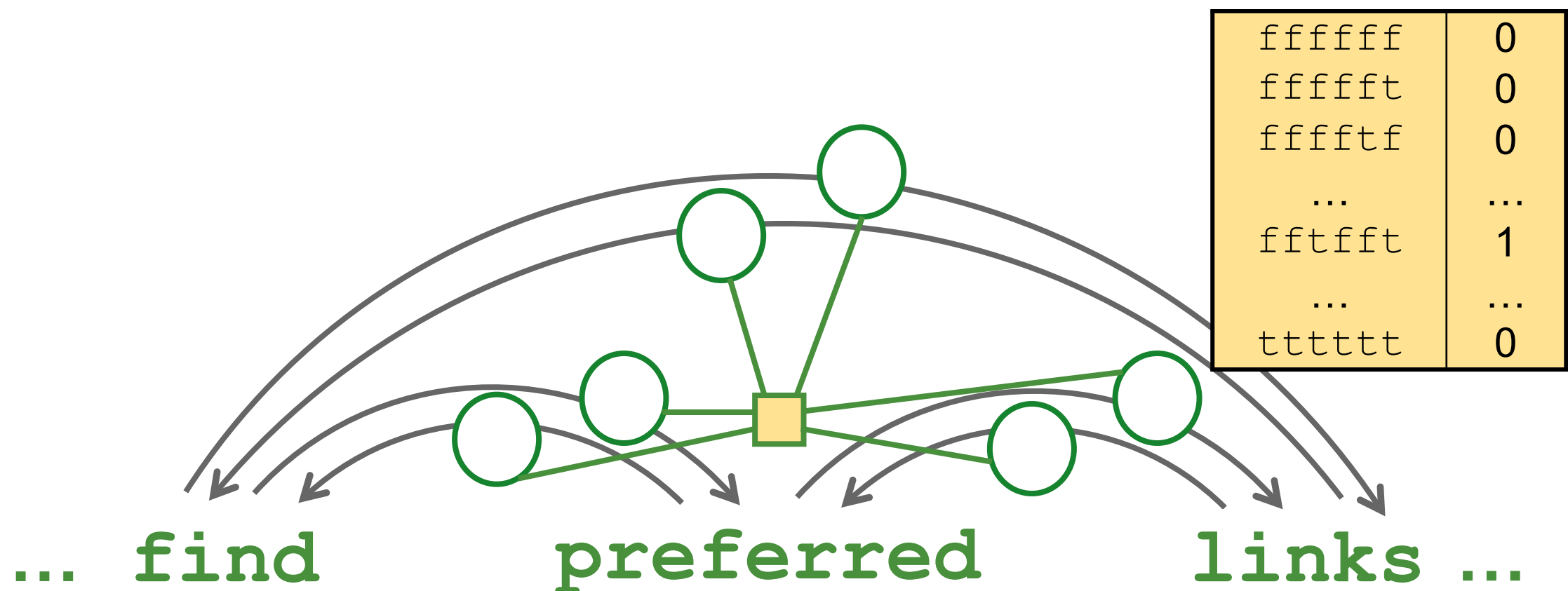
Global Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation
 - ❖ Global TREE factor to *require* links to form a legal tree



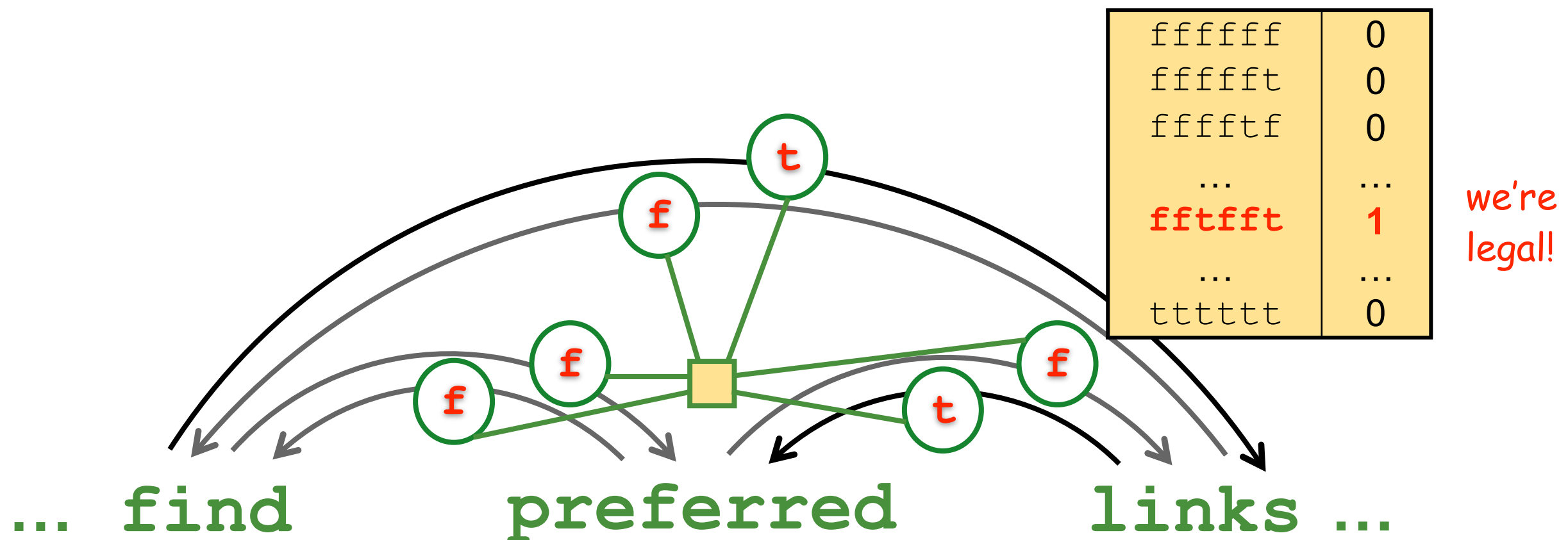
Global Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation
 - ❖ Global TREE factor to *require* links to form a legal tree
 - A *hard constraint*: potential is either 0 or 1



Global Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation
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Global Factors for Parsing

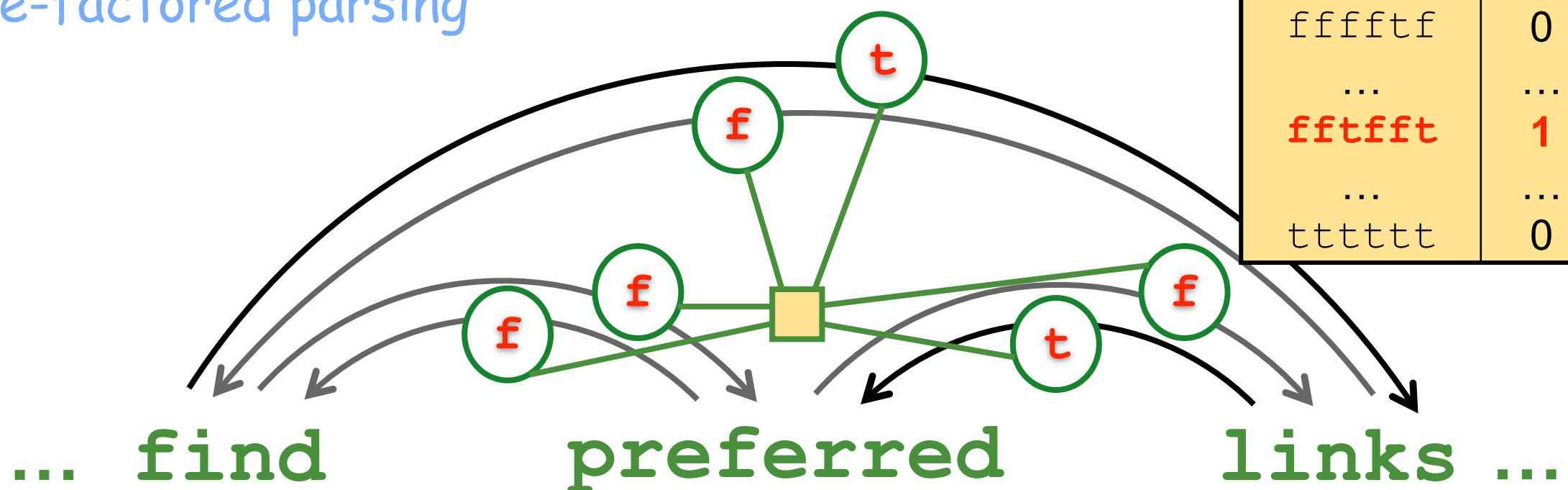
- What factors determine parse tree to be projective (no crossing links)
 - ❖ Unary factors to score each link in isolation
 - ❖ Global TREE factor to *require* links to form a legal tree
 - A *hard constraint*: potential is either 0 or 1

So far, this is equivalent to edge-factored parsing

64 entries (0/1)

ffffff	0
ffffft	0
fffftf	0
...	...
fftf ft	1
...	...
tttttt	0

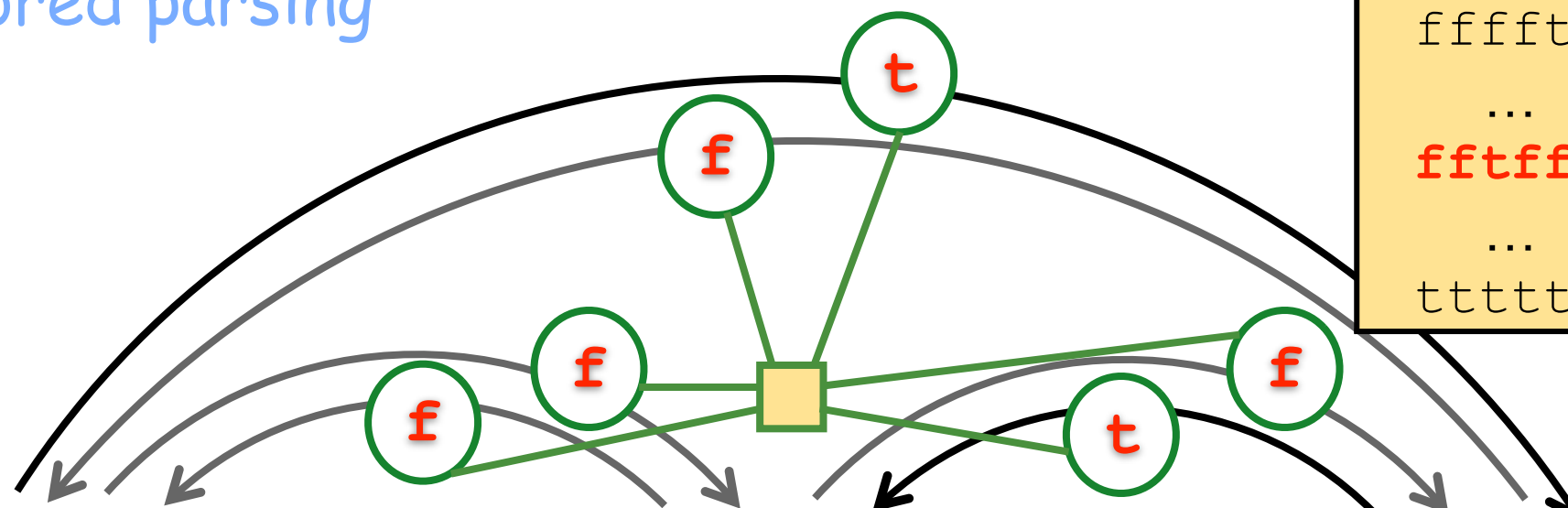
we're legal!



Global Factors for Parsing

- What factors determine parse tree to be projective (no crossing links)
 - Unary factors to score each link in isolation
 - Global TREE factor to *require* links to form a legal tree
 - A *hard constraint*: potential is either 0 or 1

So far, this is equivalent to edge-factored parsing



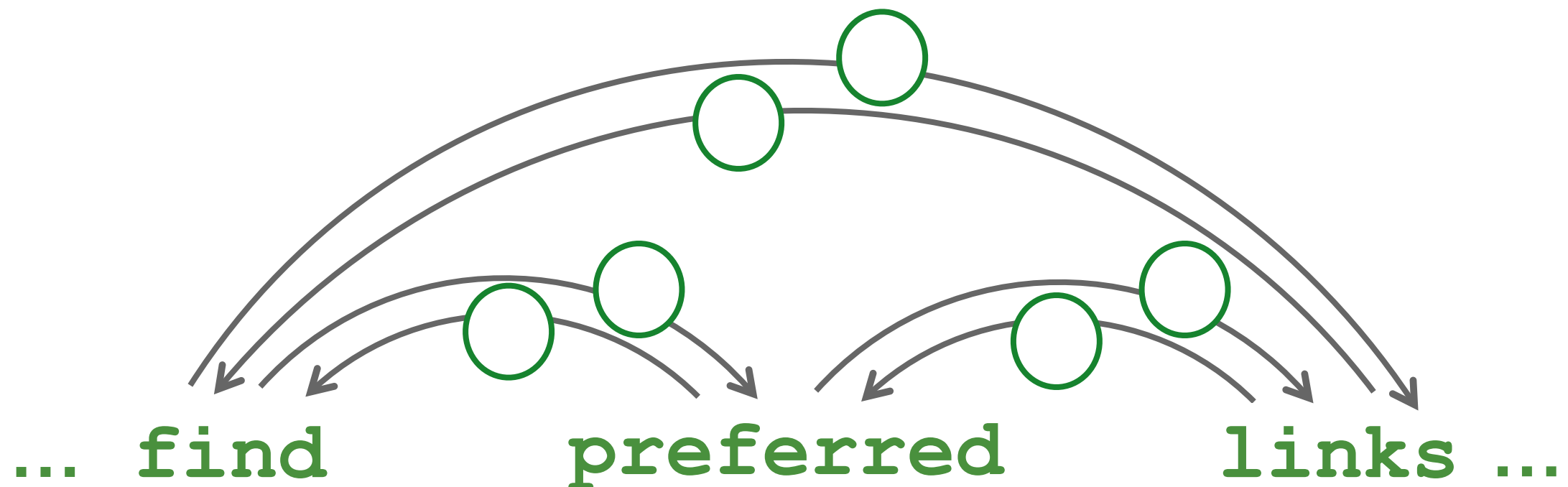
ffffff	0
ffffft	0
fffftf	0
...	...
fftf ff	1
...	...
tttttt	0

we're legal!

... **fin** Note: traditional parsers don't loop through this table to consider exponentially many trees one at a time. They use combinatorial algorithms; so should we!

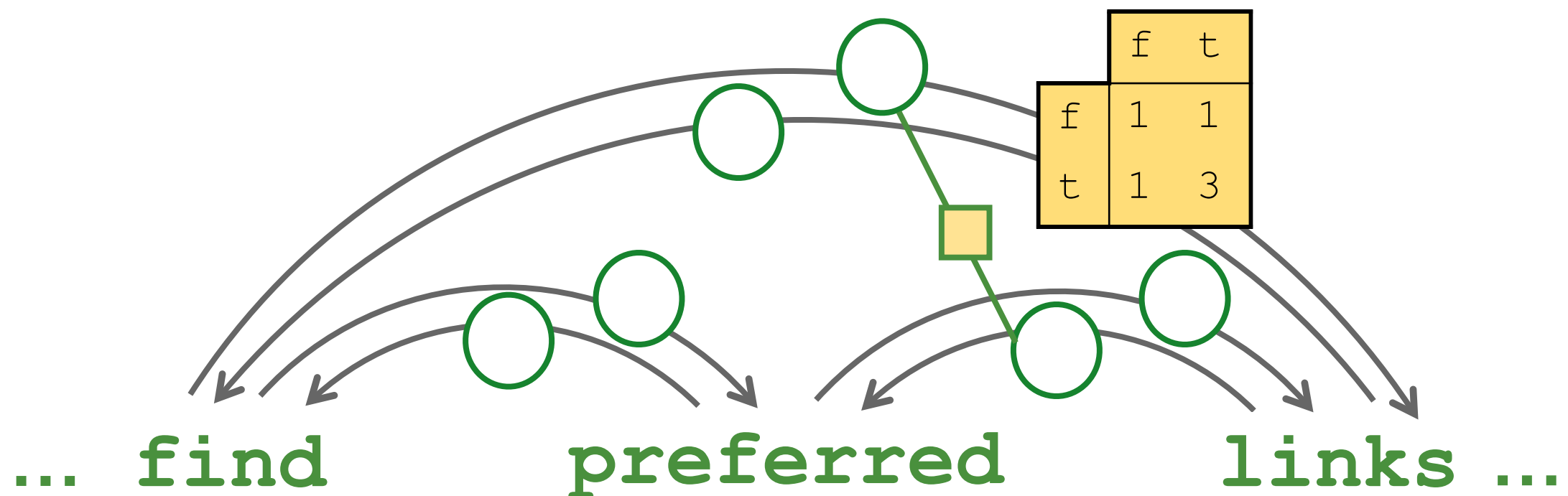
Local Factors for Parsing

- What factors determine parse probability?
 - ✧ Unary factors to score each link in isolation
 - ✧ Global TREE factor to *require* links to form a legal tree
 - A *hard constraint*: potential is either 0 or 1
 - ✧ Second order effects: factors on 2 variables
 - Grandparent–parent–child chains



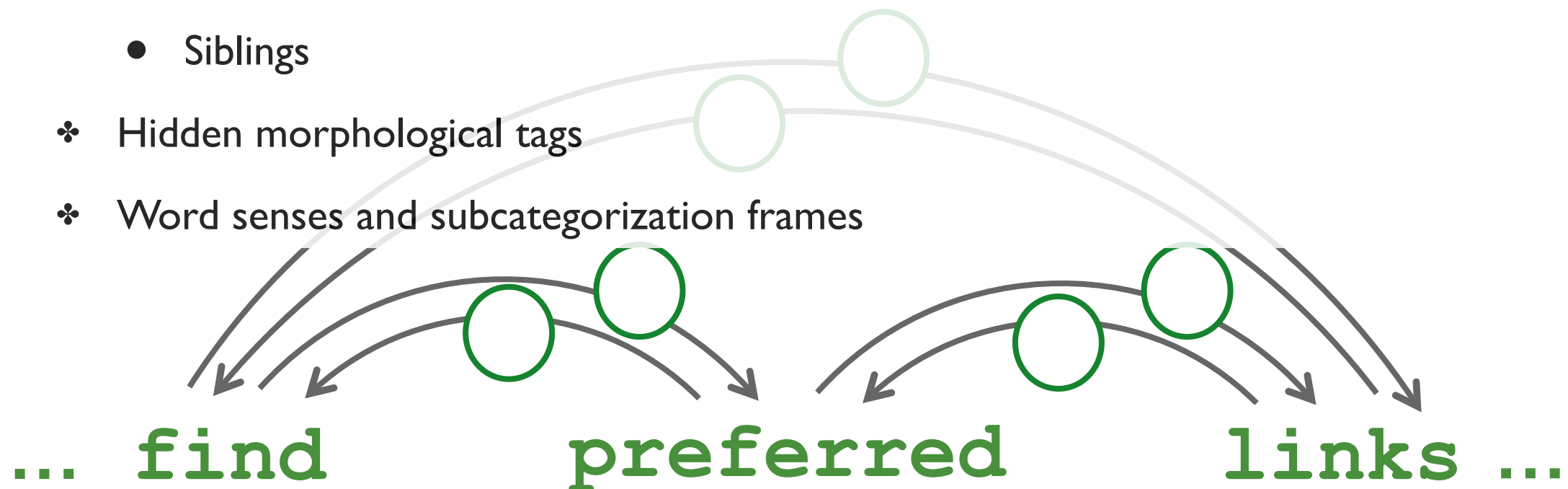
Local Factors for Parsing

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Local Factors for Parsing

- What factors determine parse probability?
 - ❖ Unary factors to score each link in isolation
 - ❖ Global TREE factor to *require* links to form a legal tree
 - A *hard constraint*: potential is either 0 or 1
 - ❖ Second order effects: factors on 2 variables
 - Grandparent–parent–child chains
 - No crossing links
 - Siblings
 - ❖ Hidden morphological tags
 - ❖ Word senses and subcategorization frames



Great Ideas in ML: Message Passing



Great Ideas in ML: Message Passing

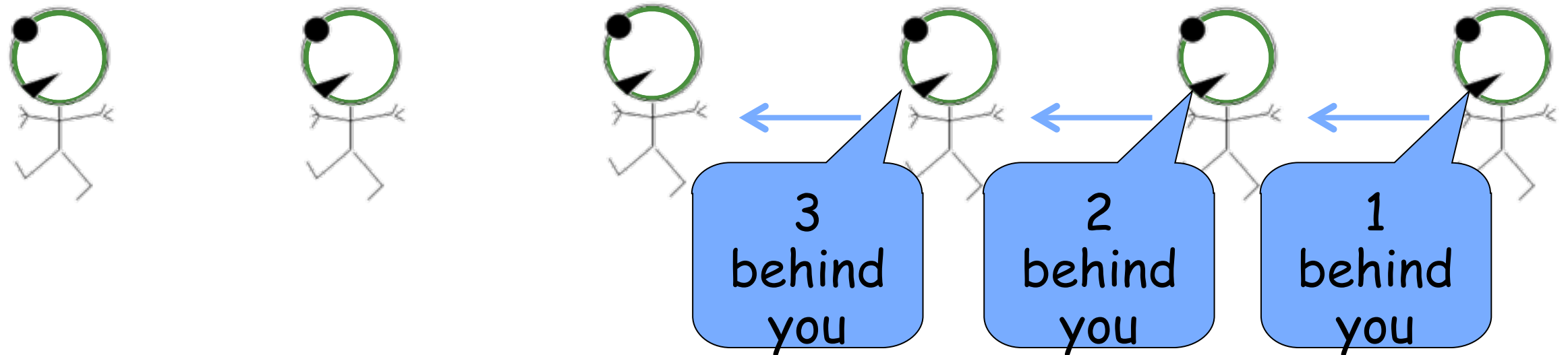
Count the soldiers



adapted from MacKay (2003) textbook

Great Ideas in ML: Message Passing

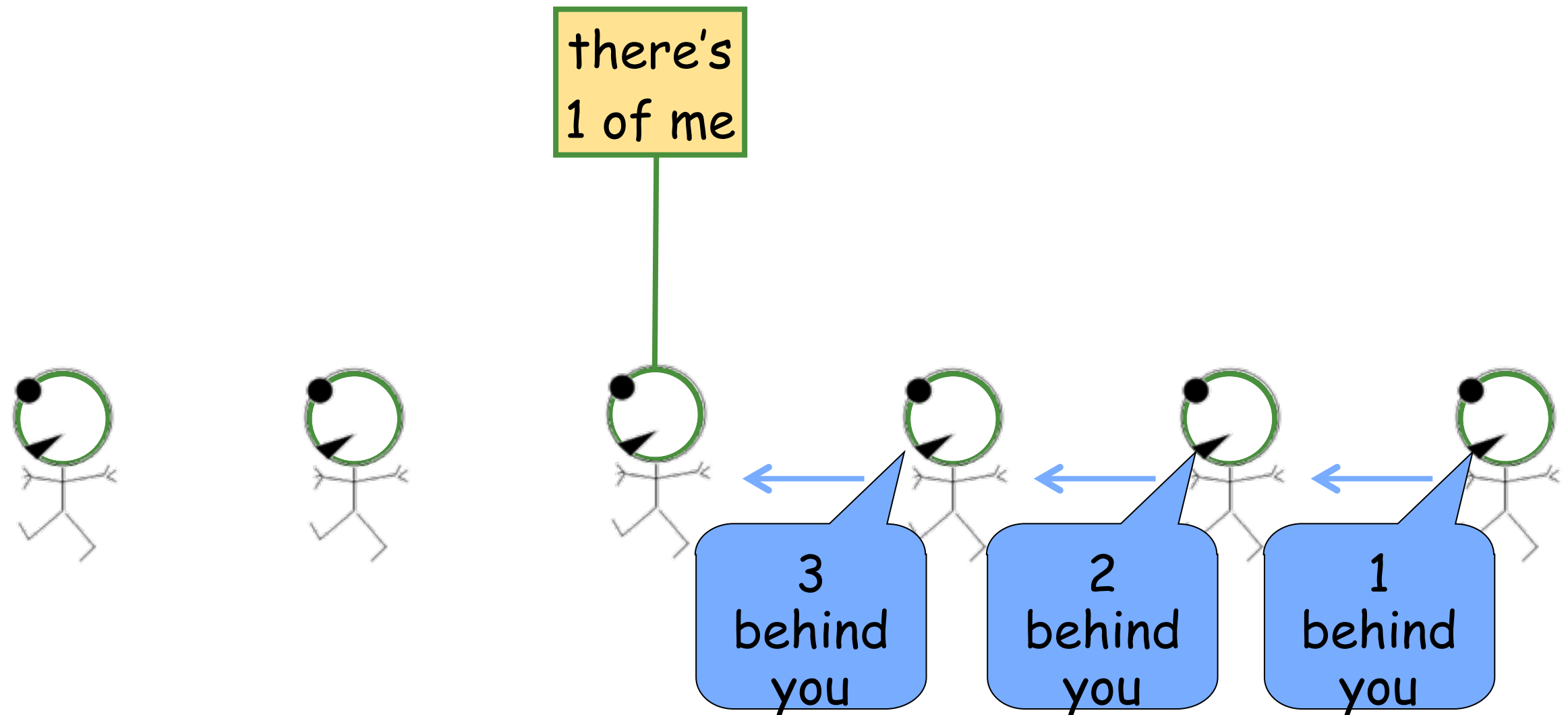
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Great Ideas in ML: Message Passing

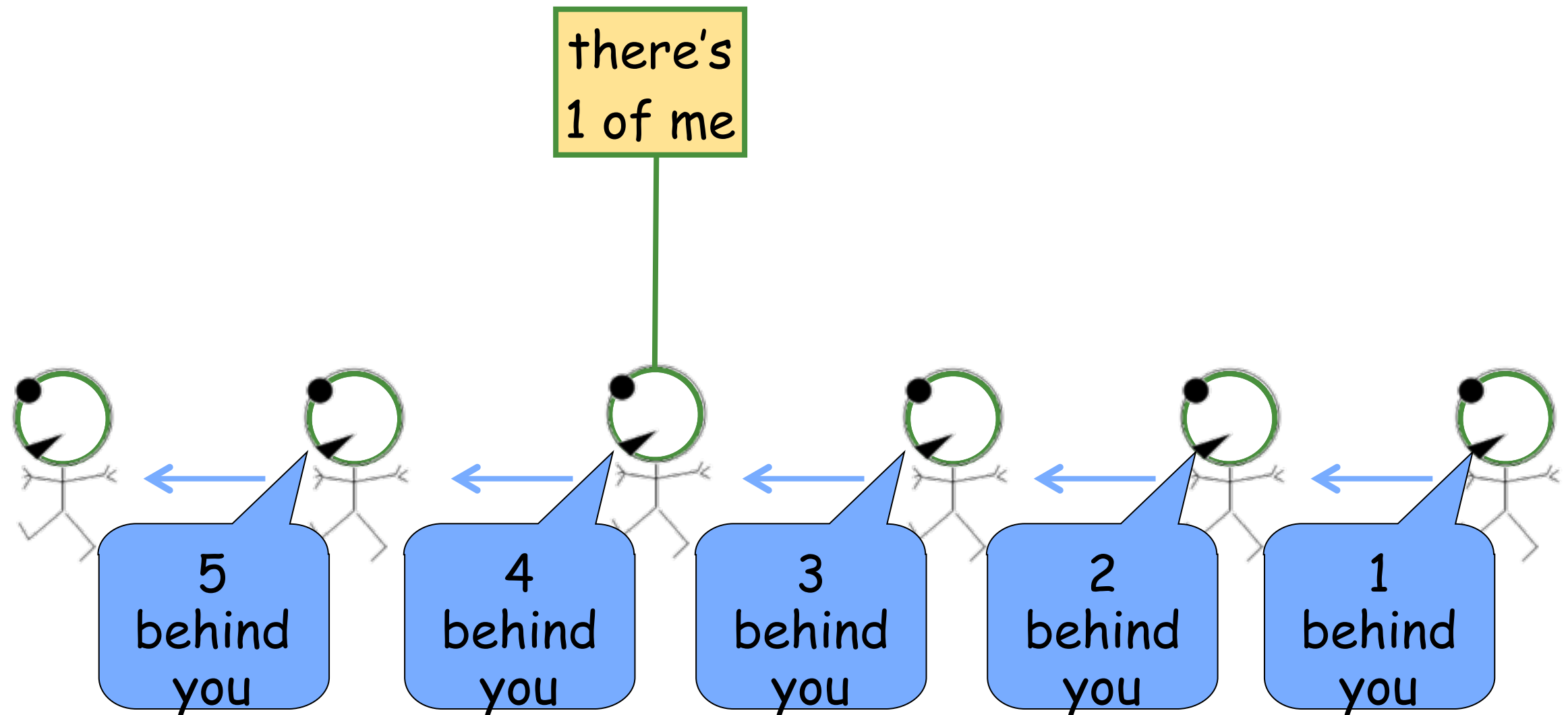
Count the soldiers



adapted from MacKay (2003) textbook

Great Ideas in ML: Message Passing

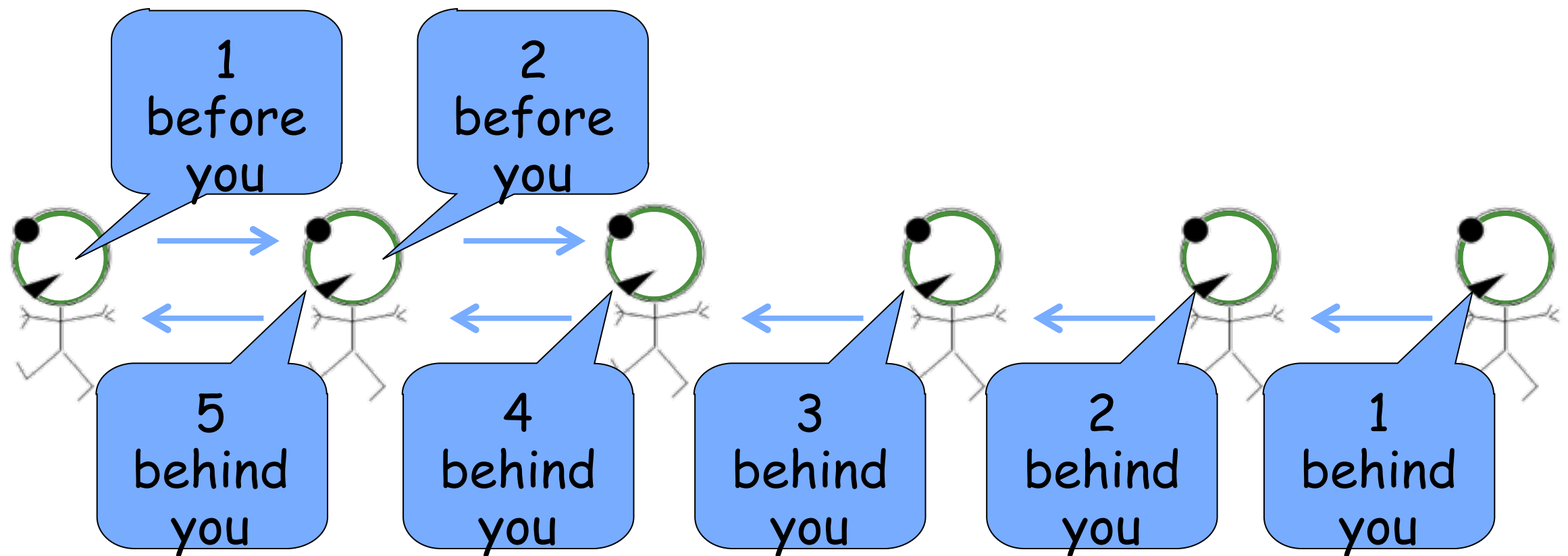
Count the soldiers



adapted from MacKay (2003) textbook

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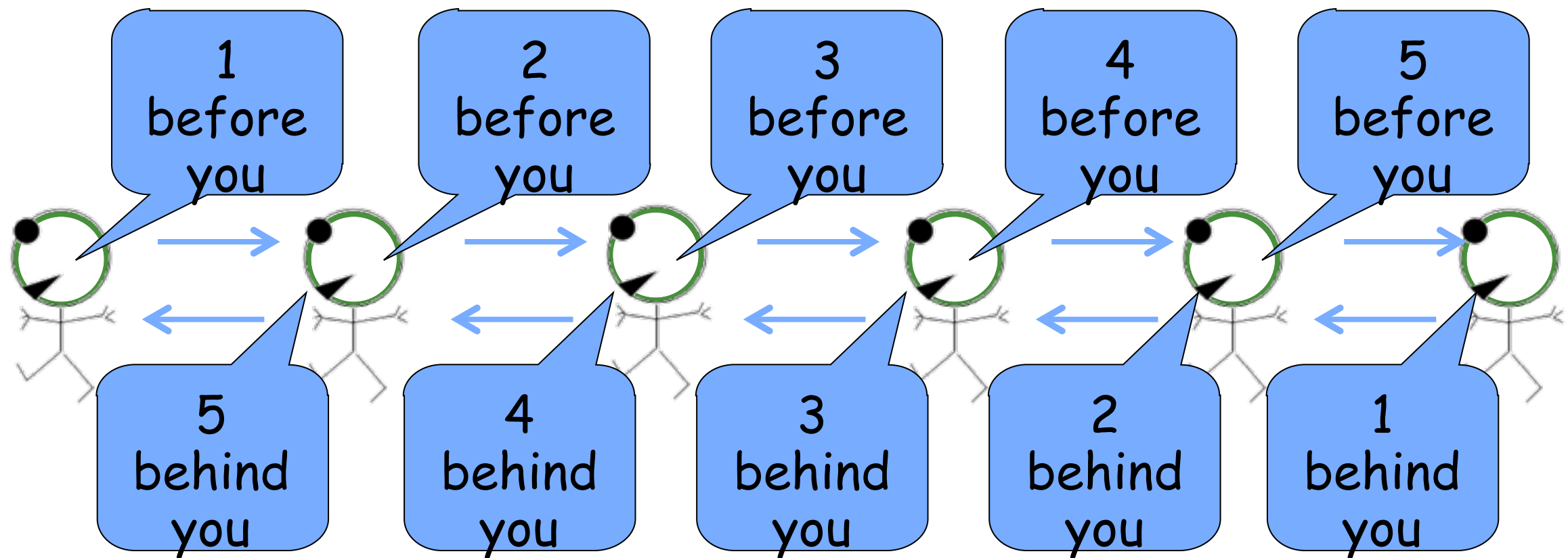
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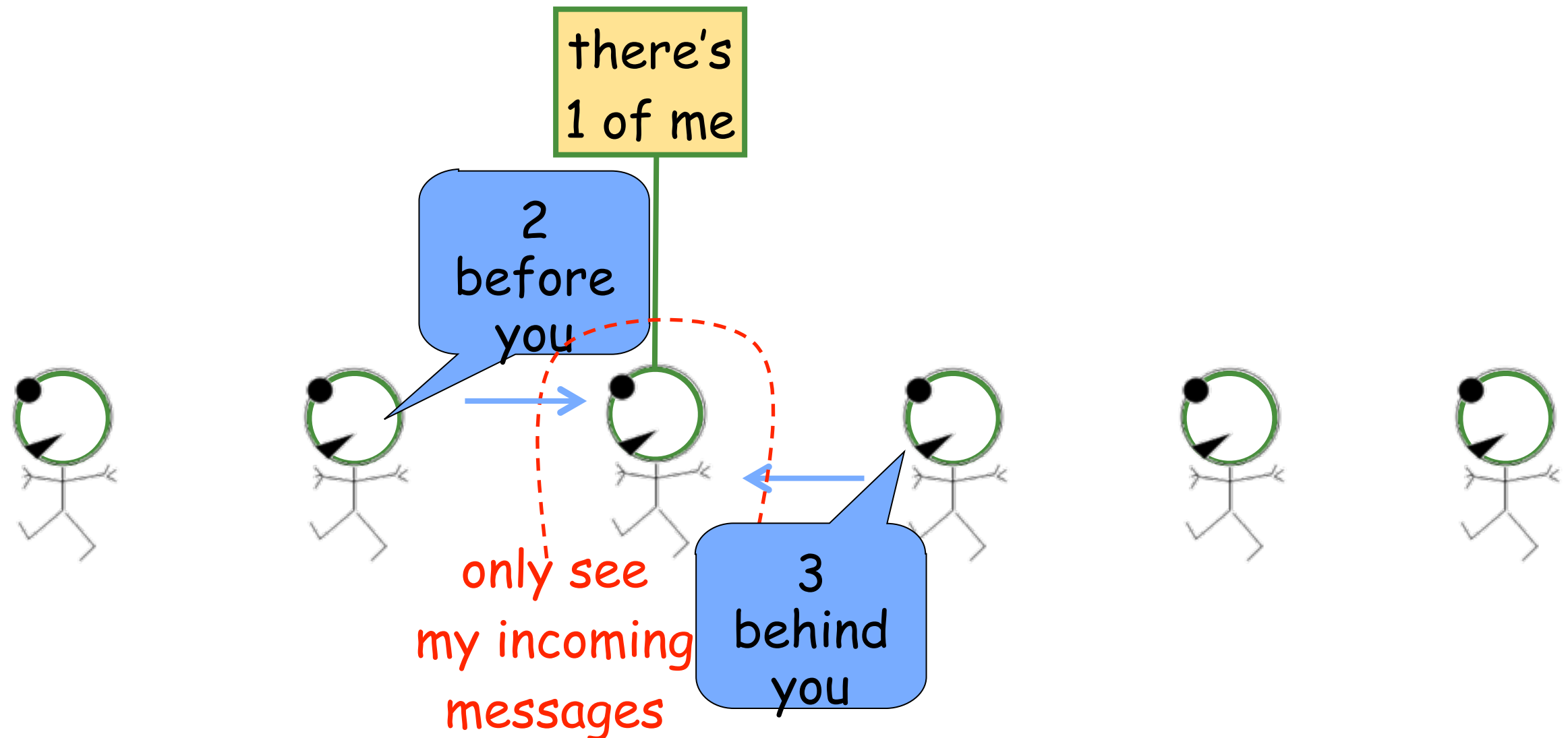
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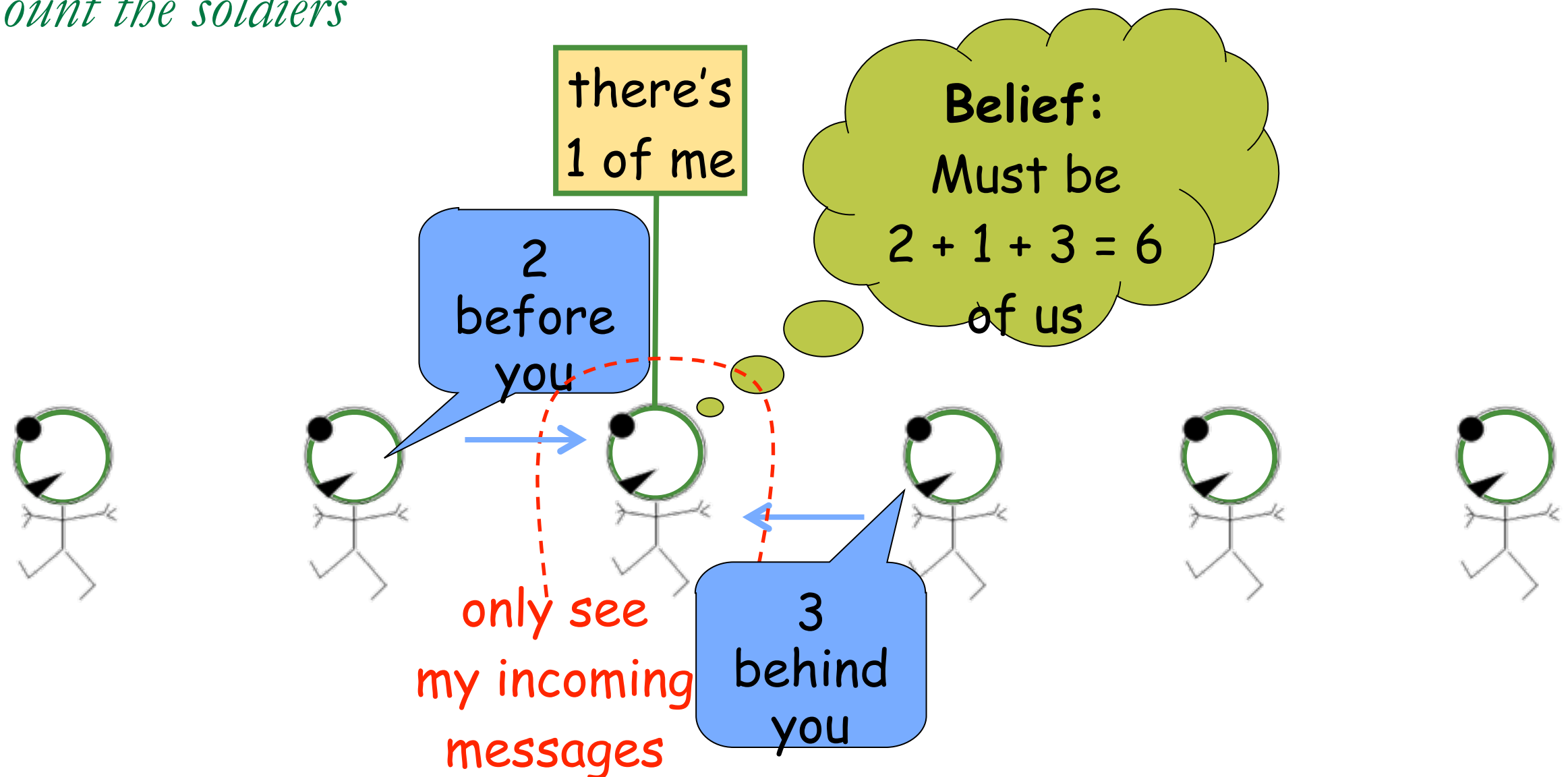
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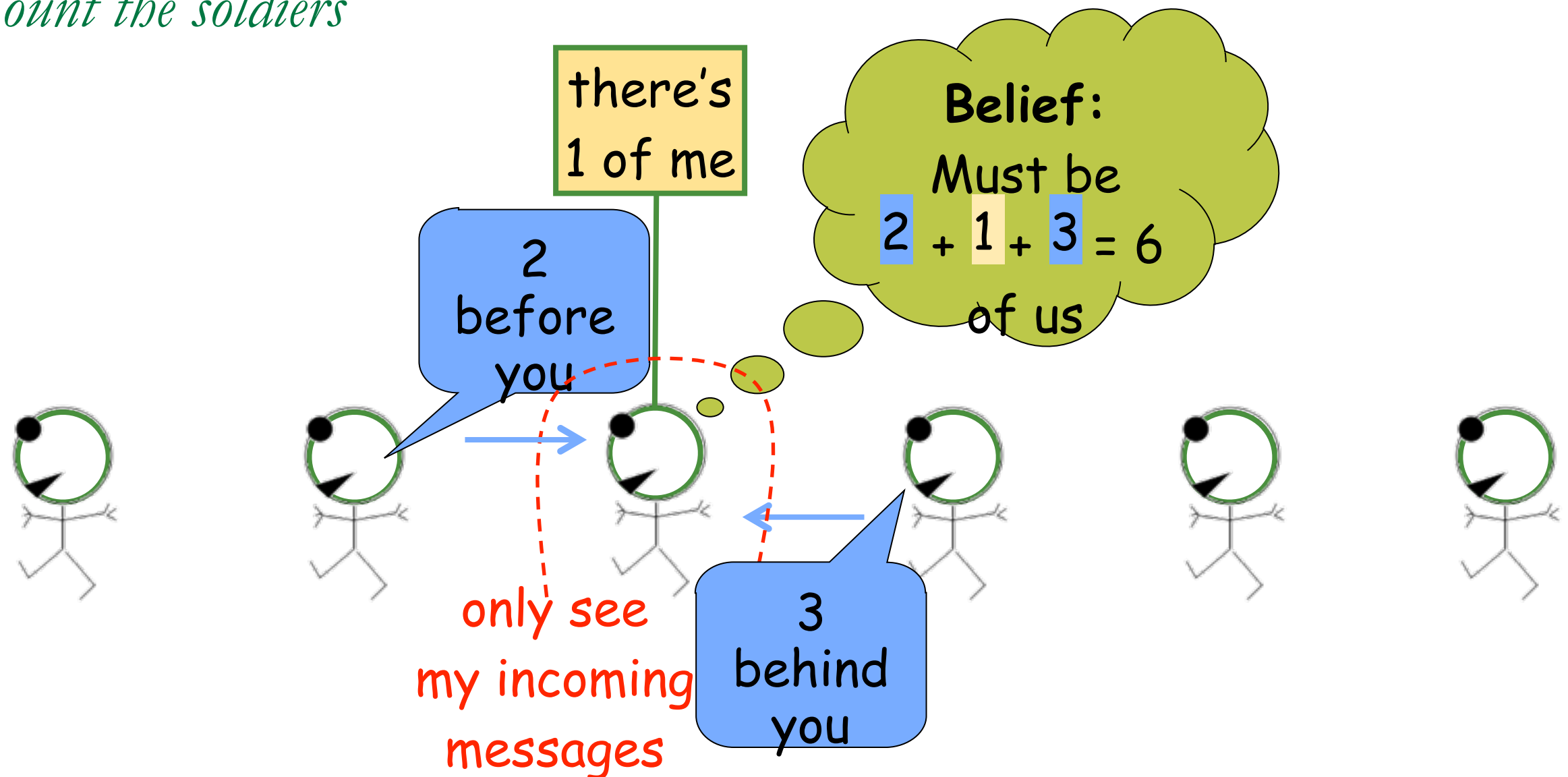
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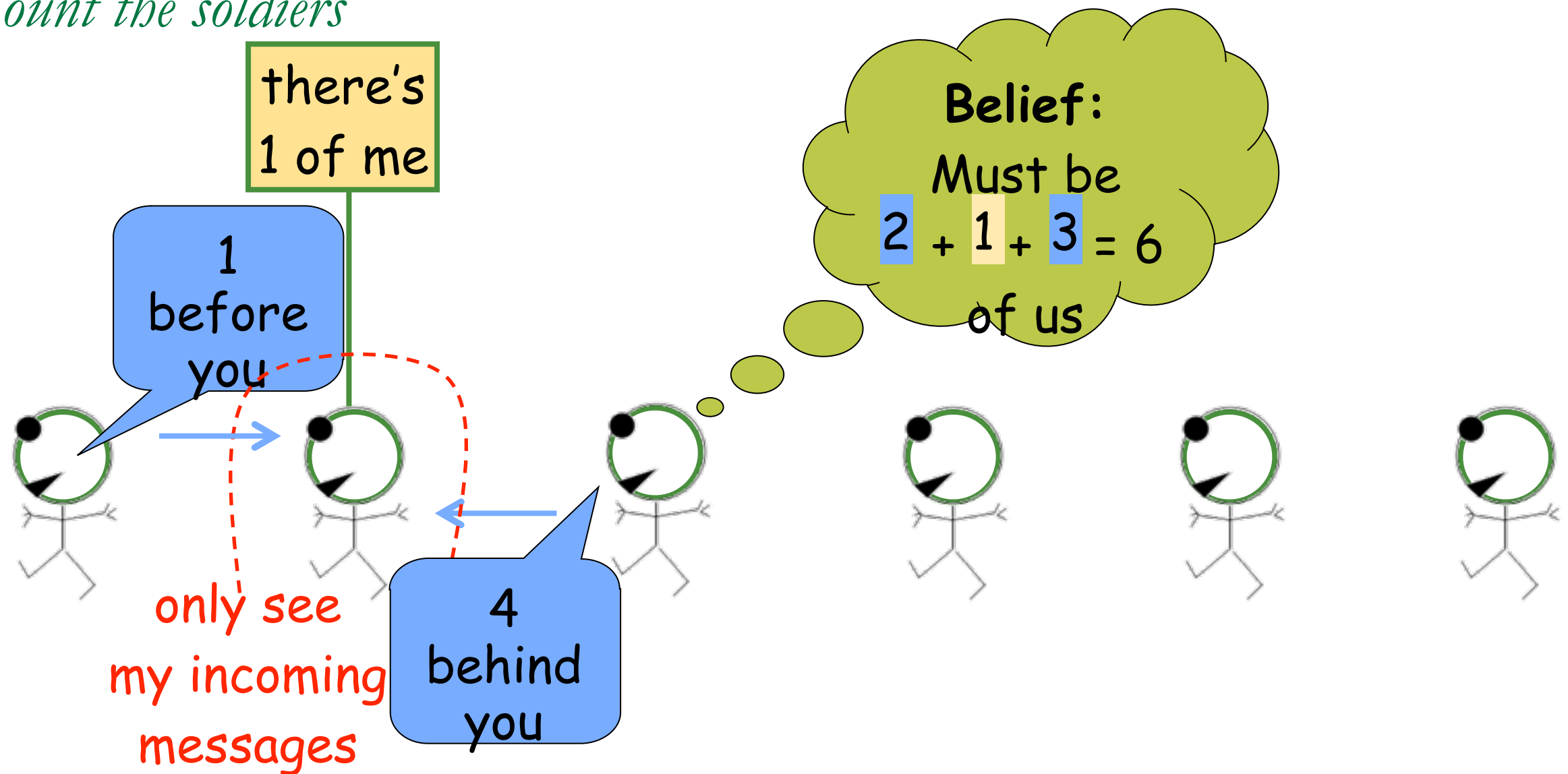
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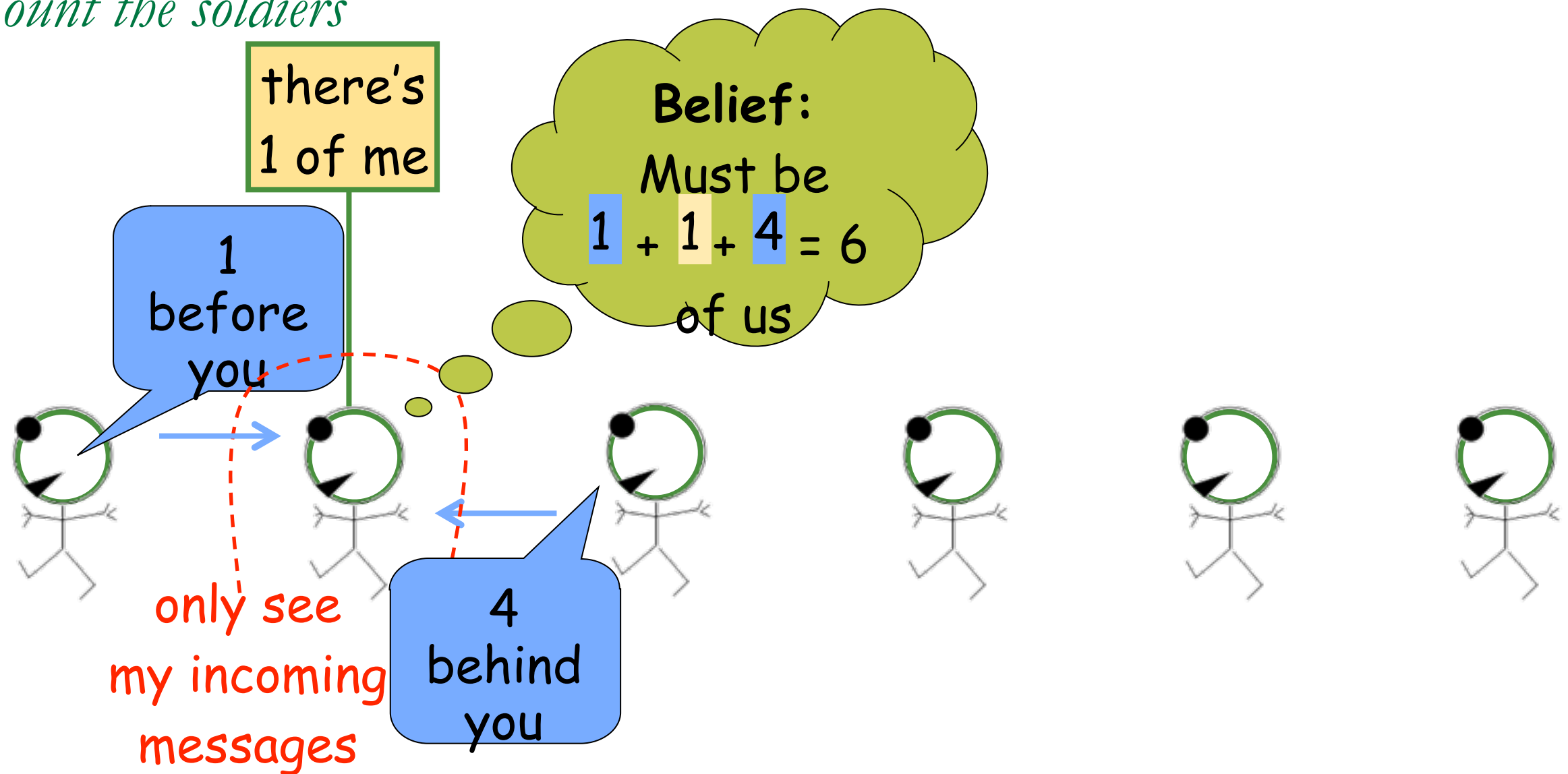
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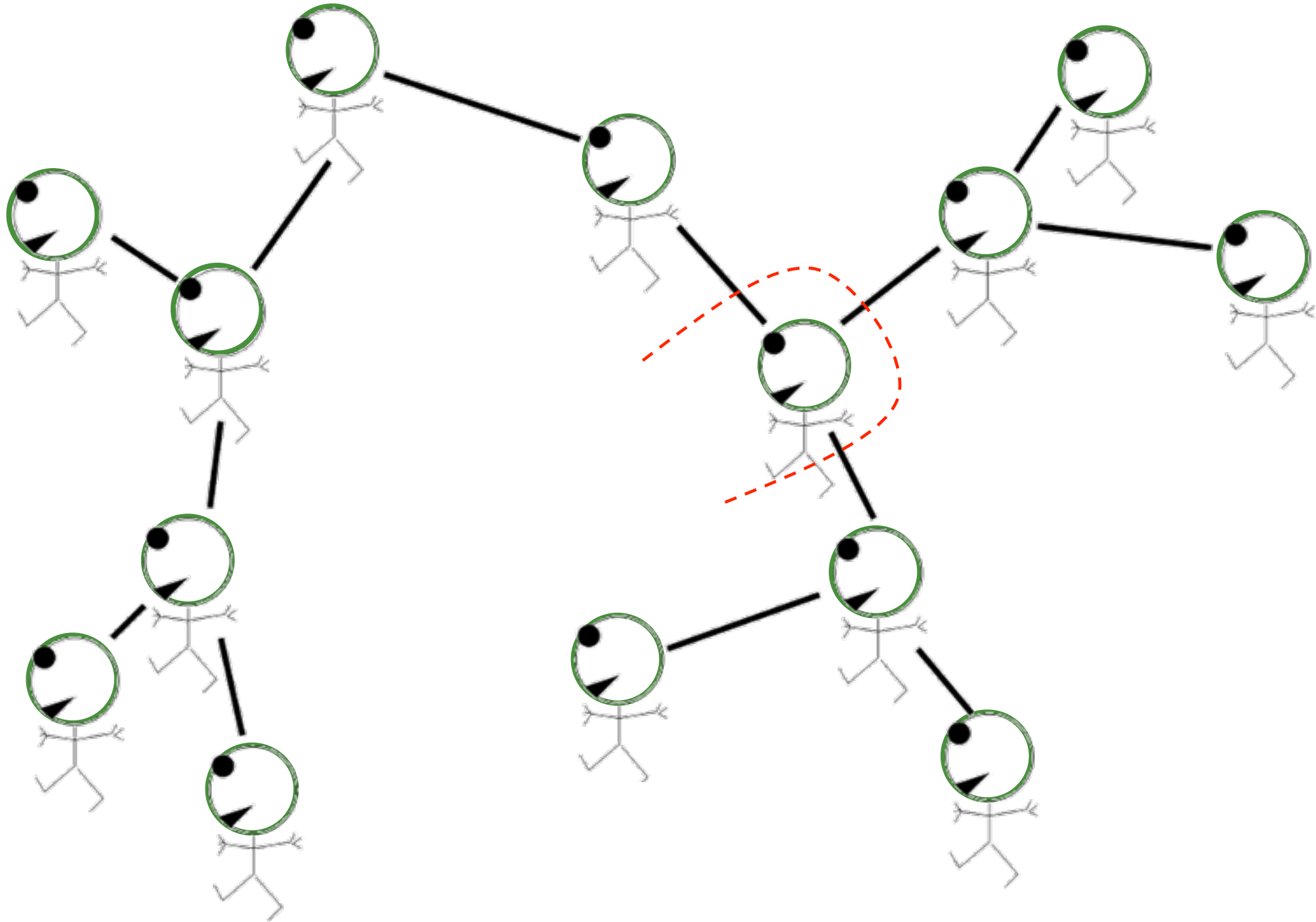
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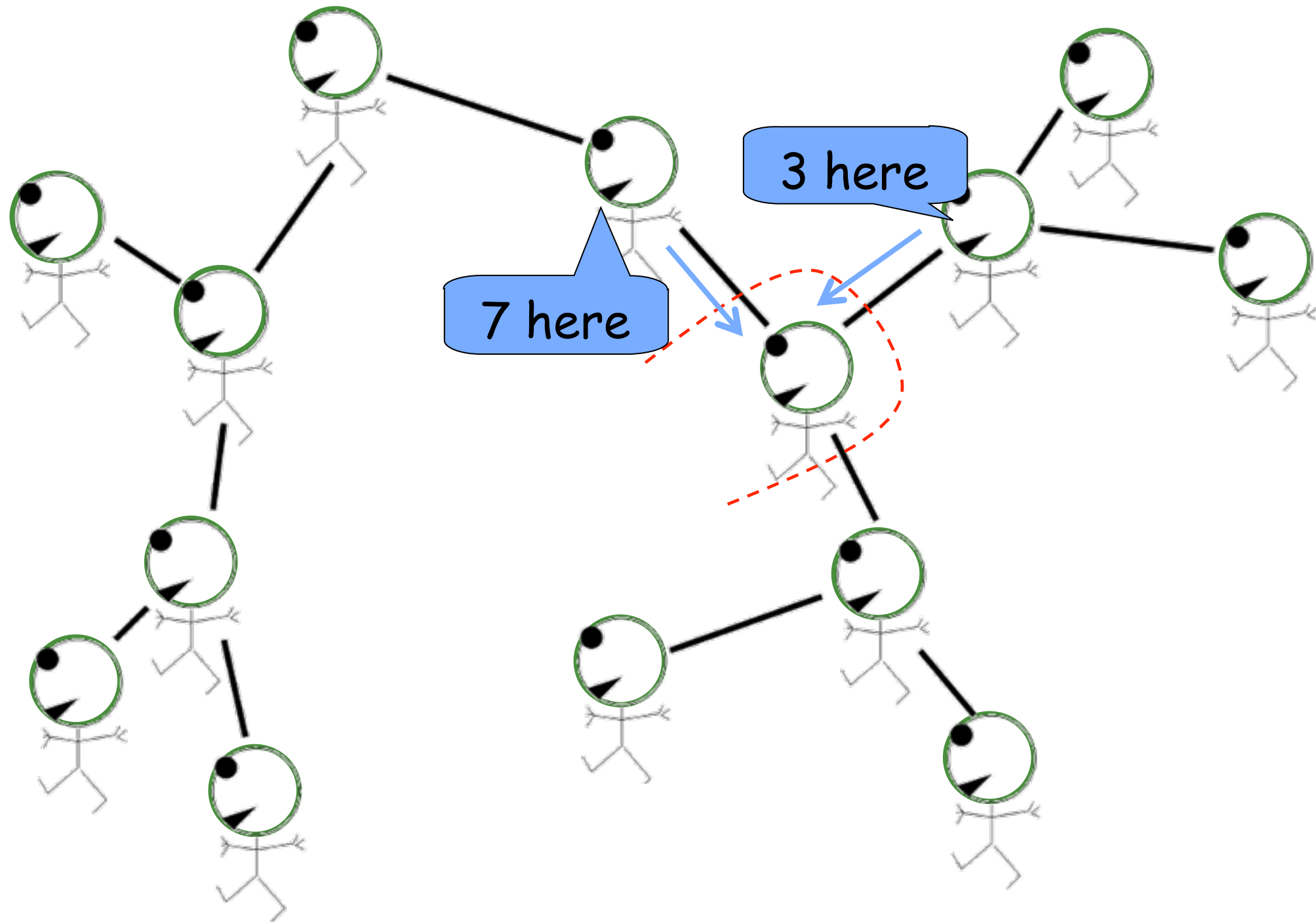
Each soldier receives reports from all branches of tree



adapted from MacKay (2003) textbook

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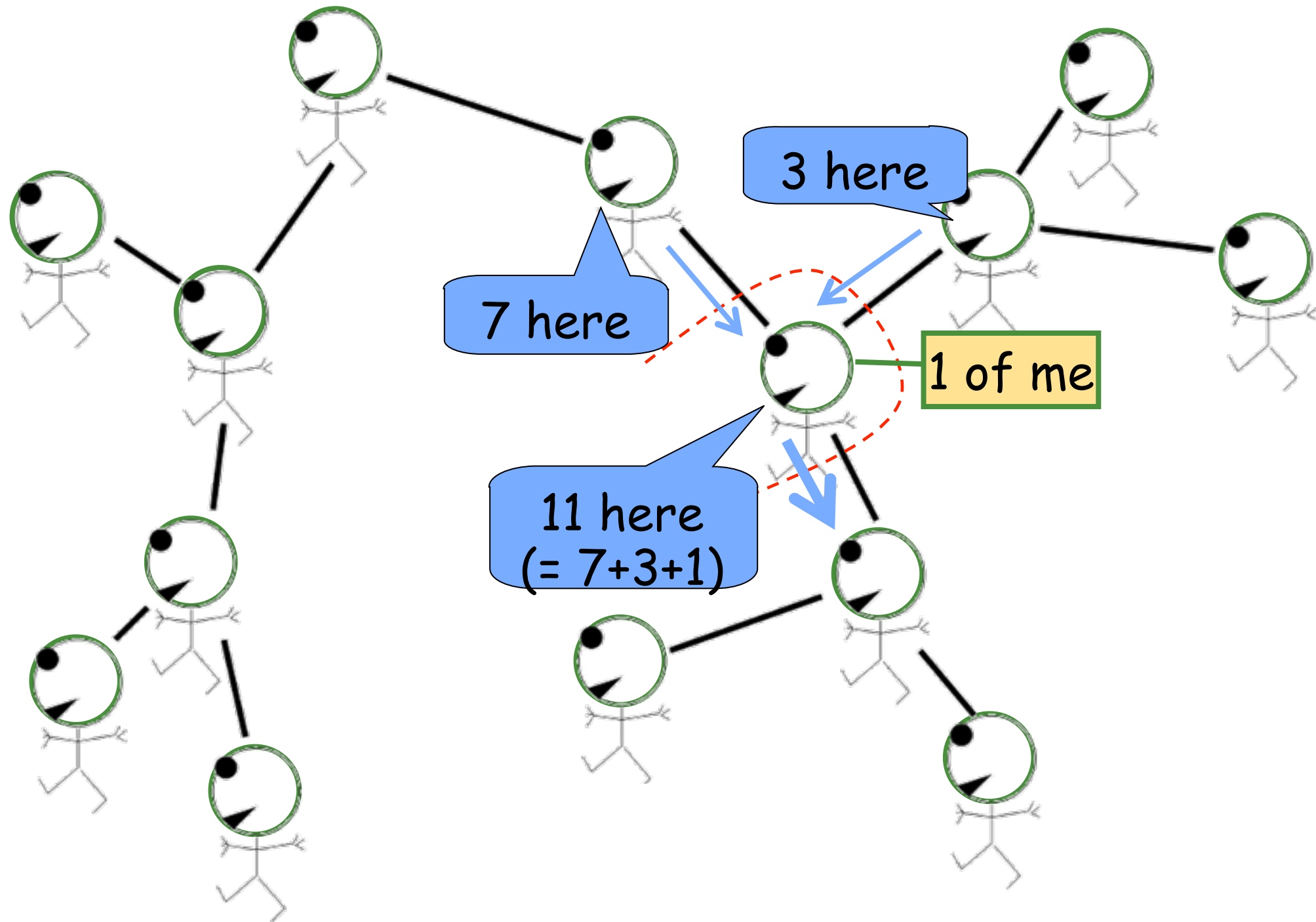
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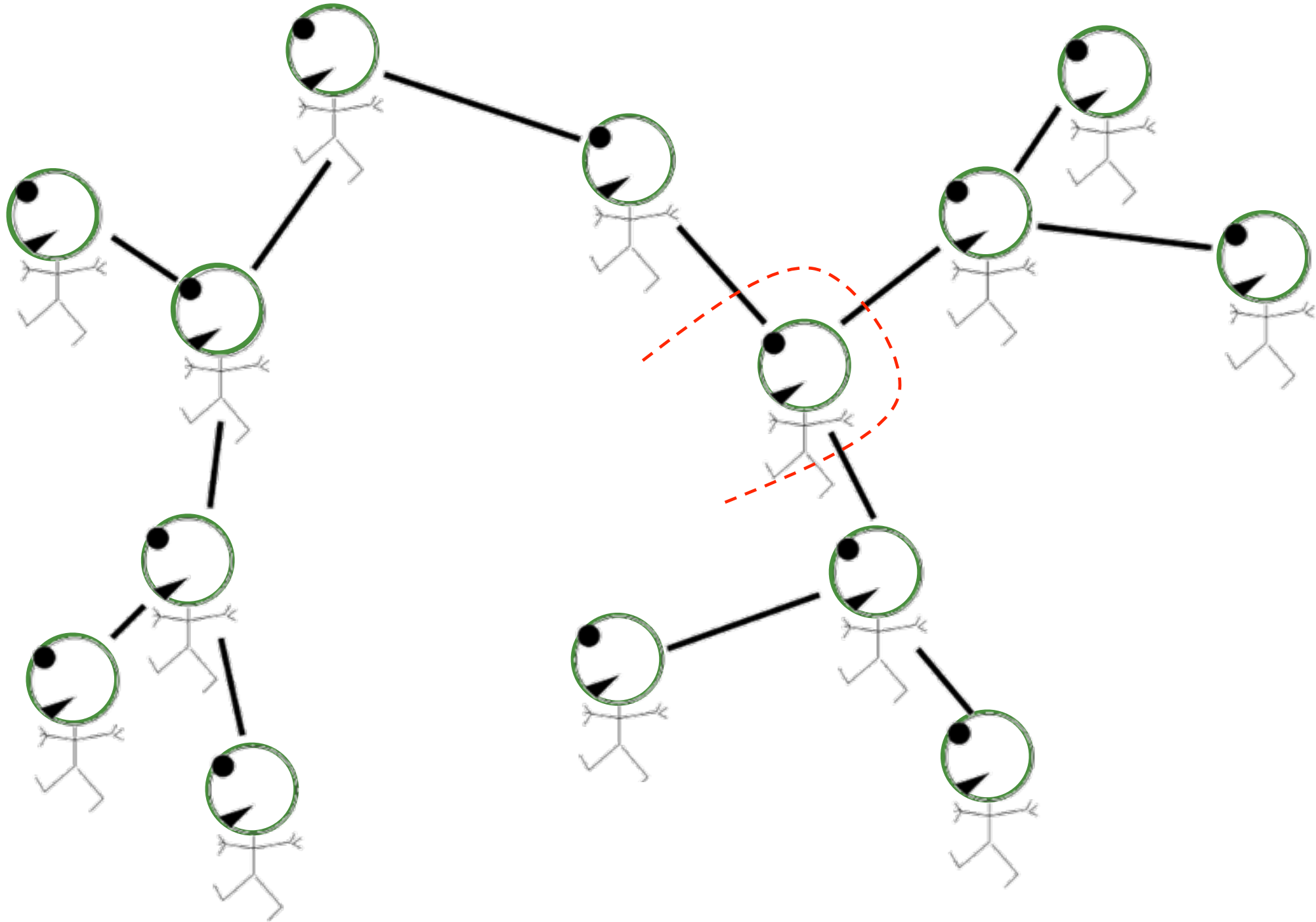
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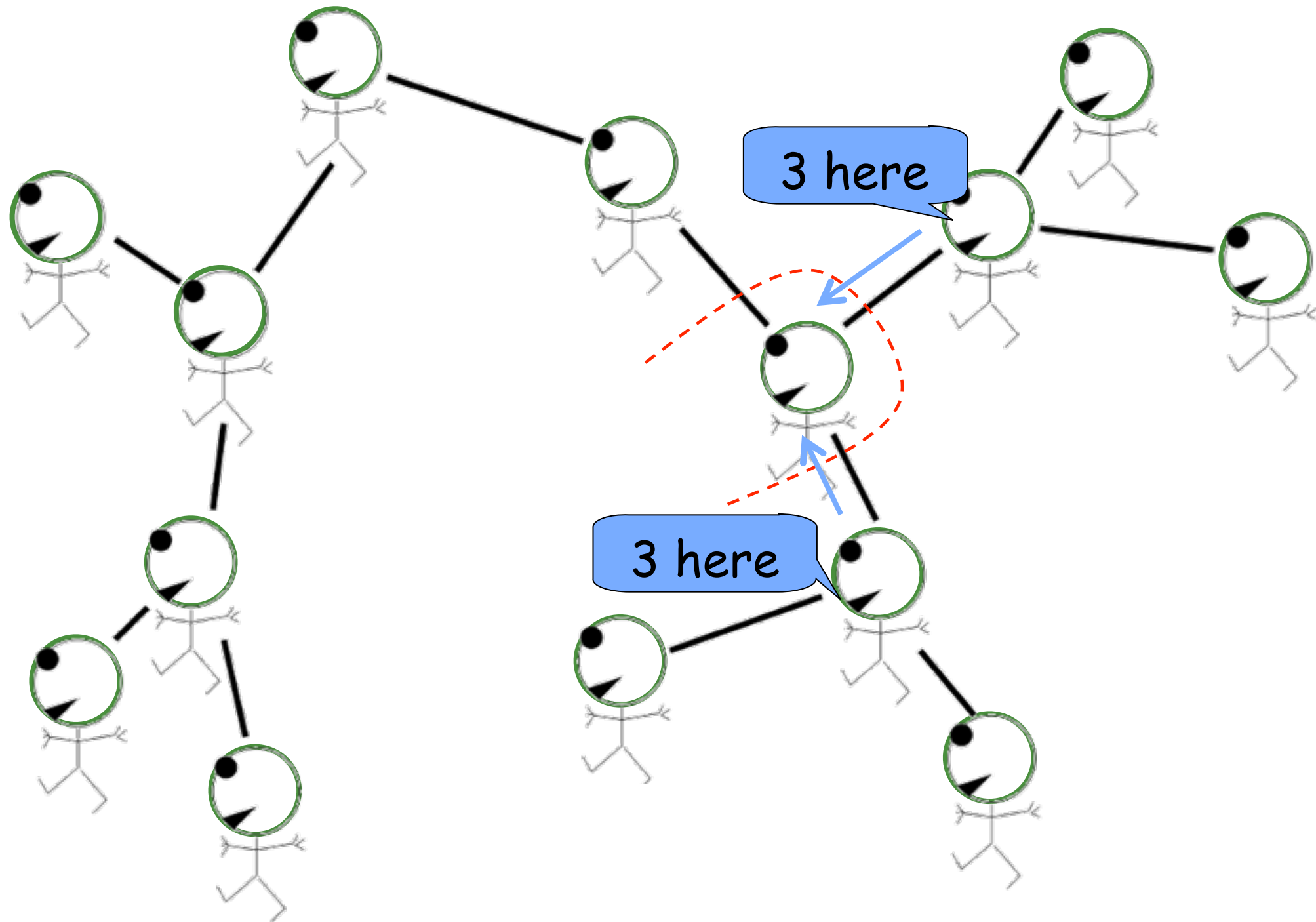
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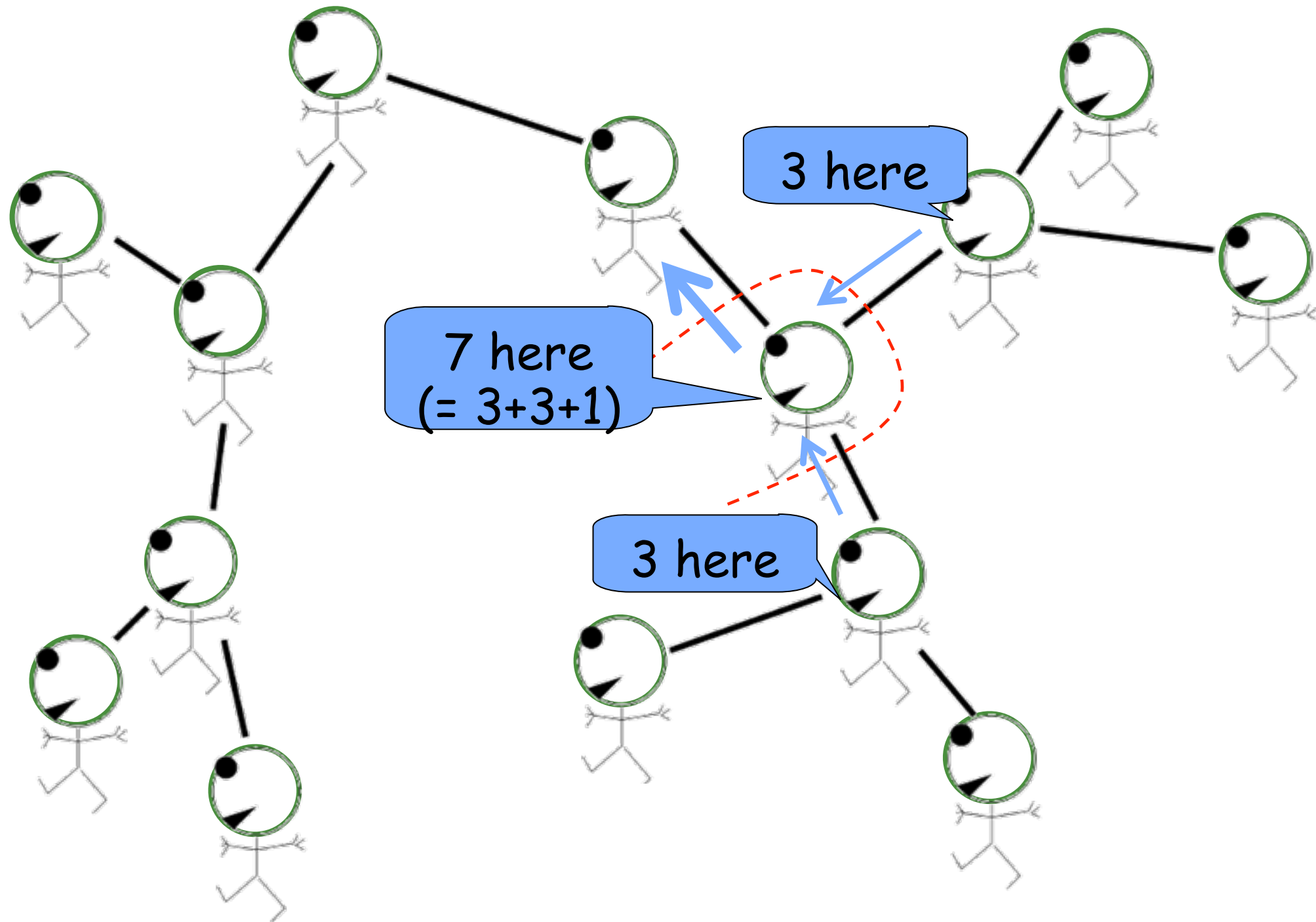
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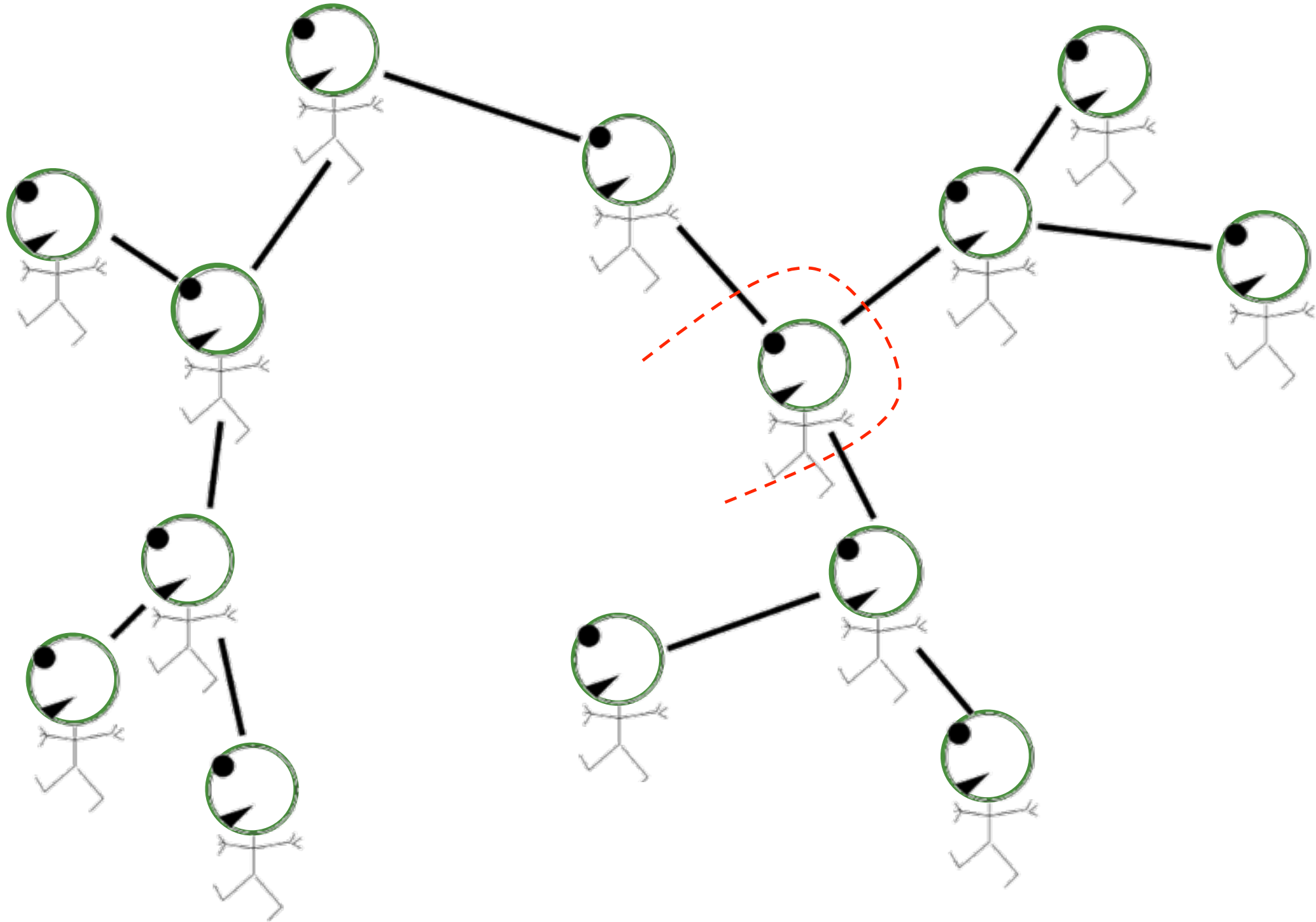
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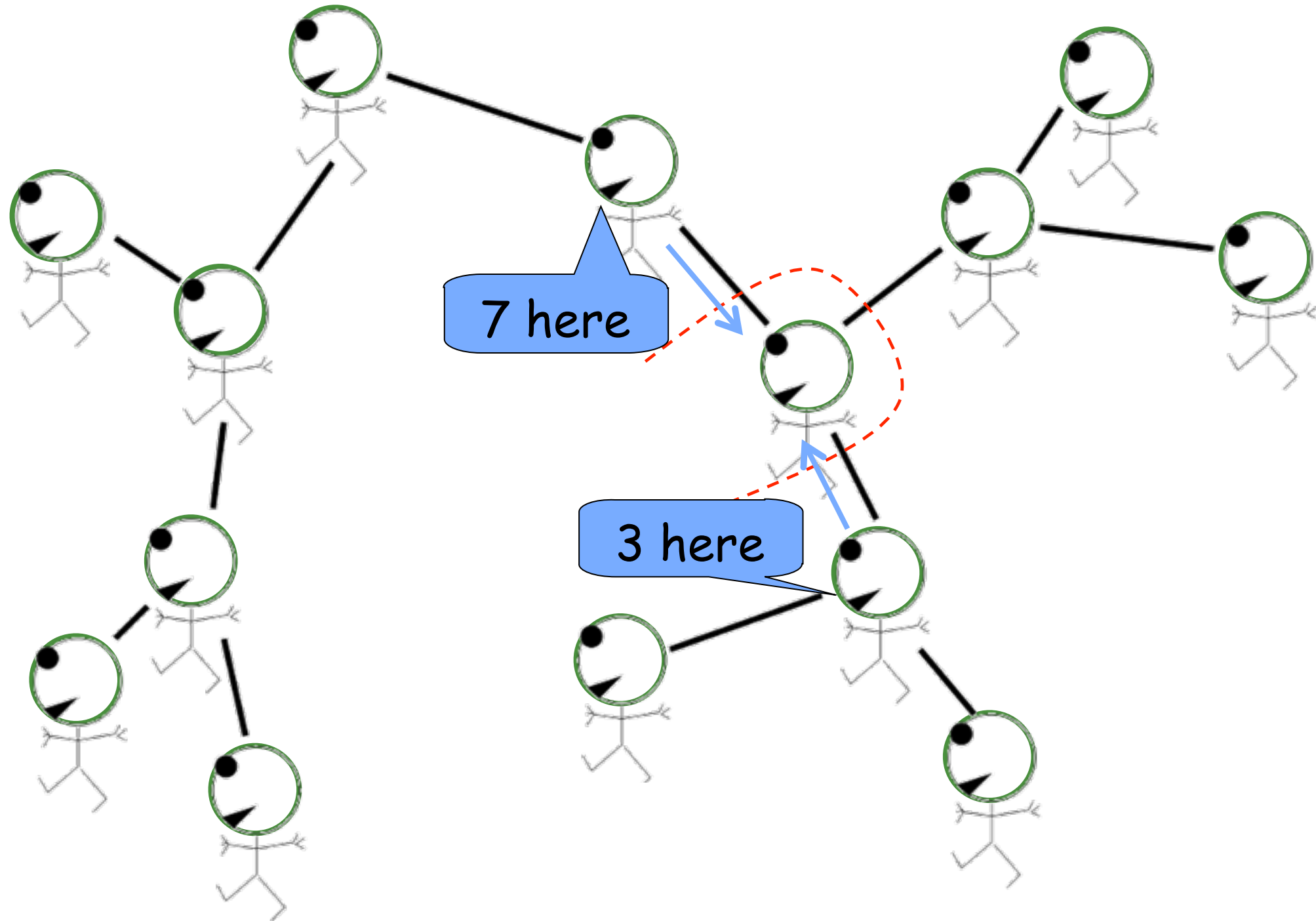
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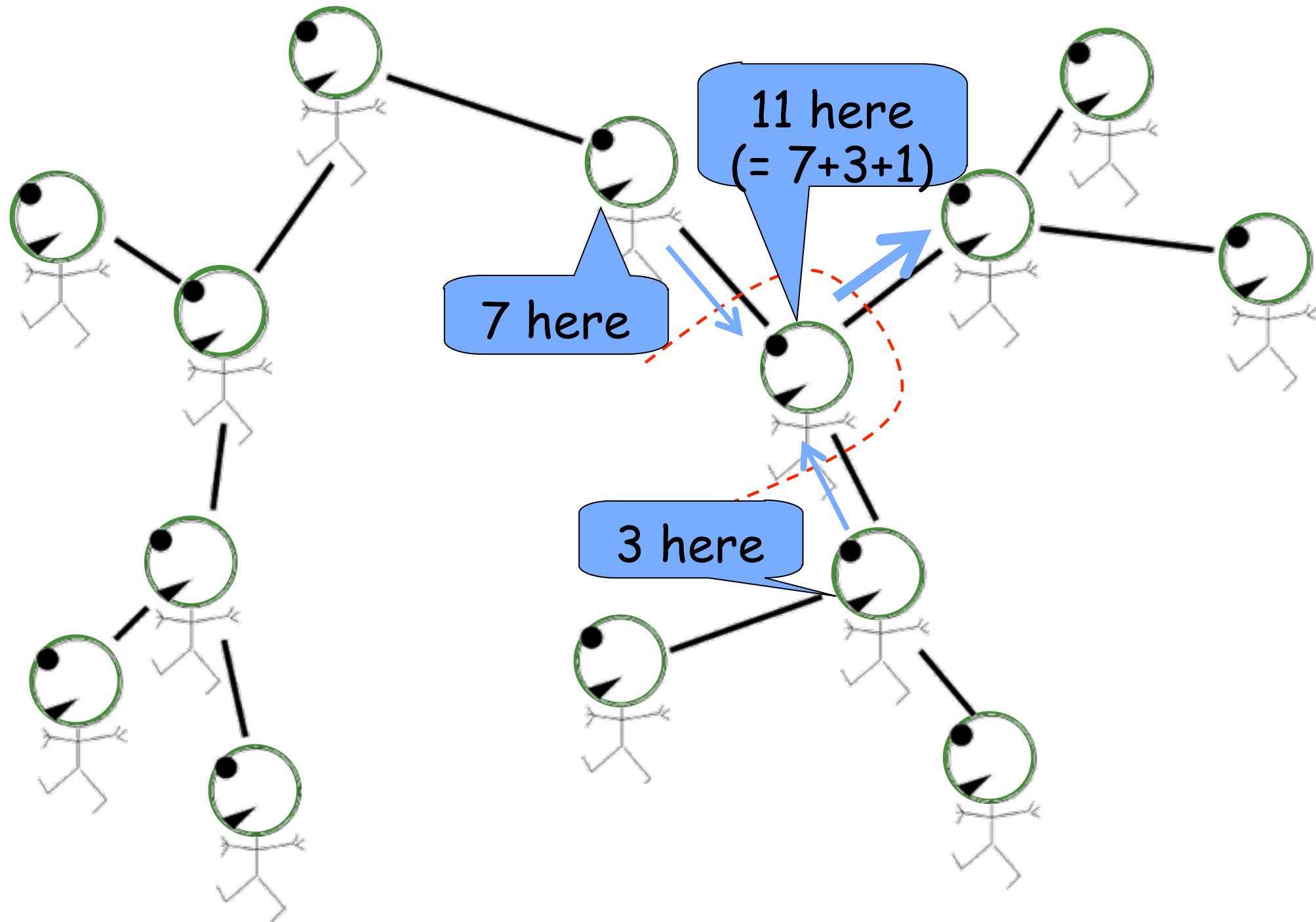
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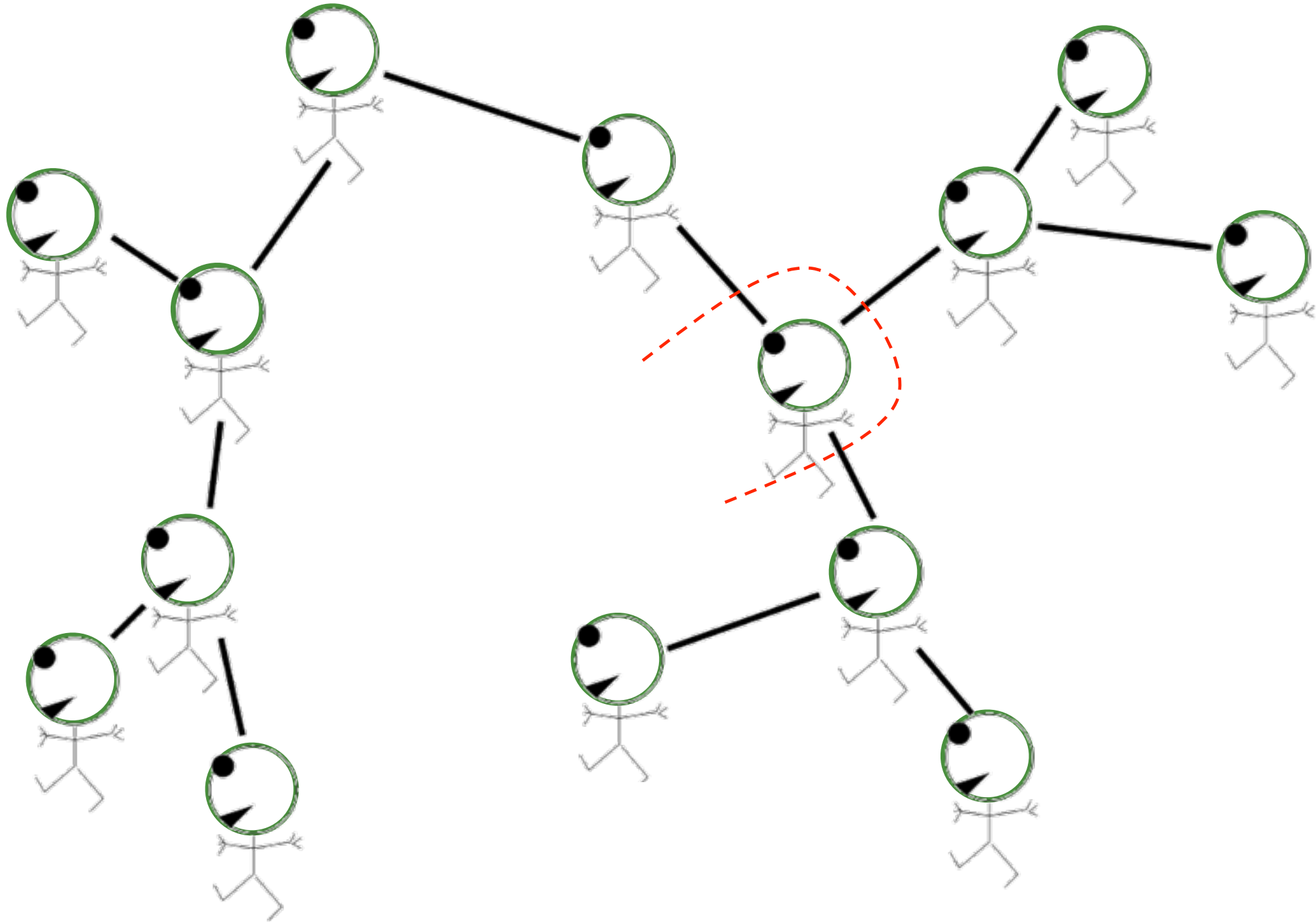
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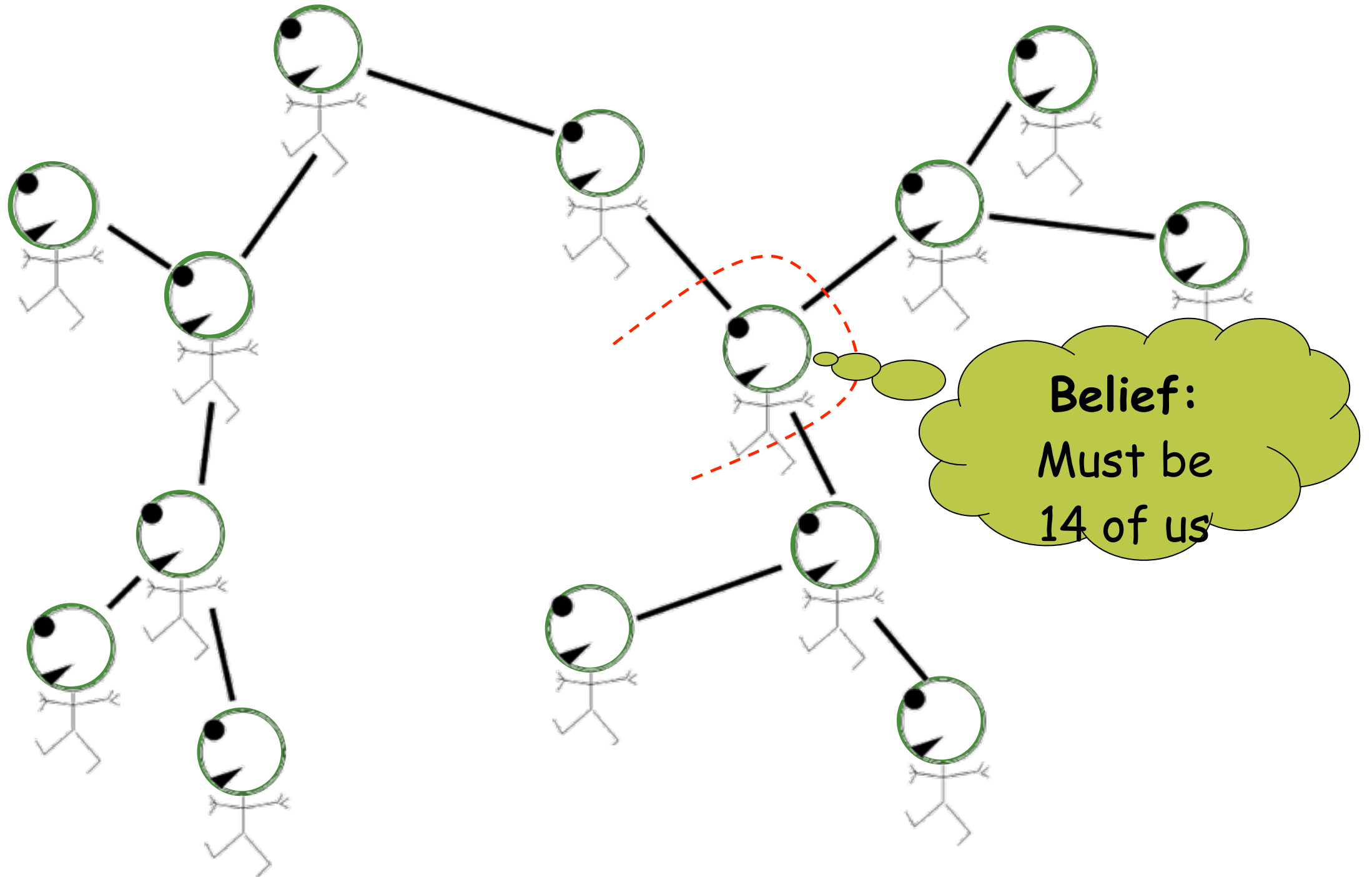
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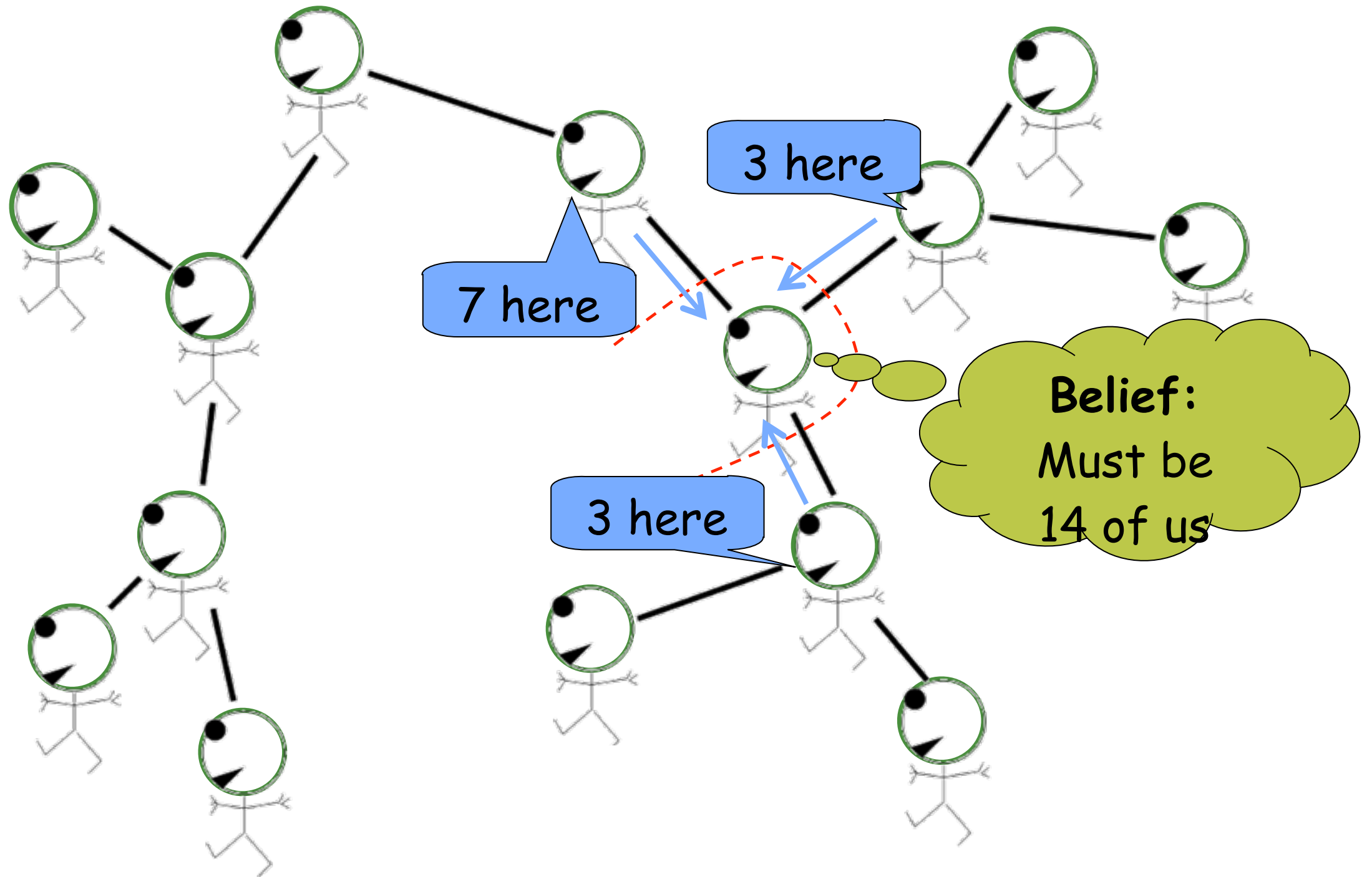
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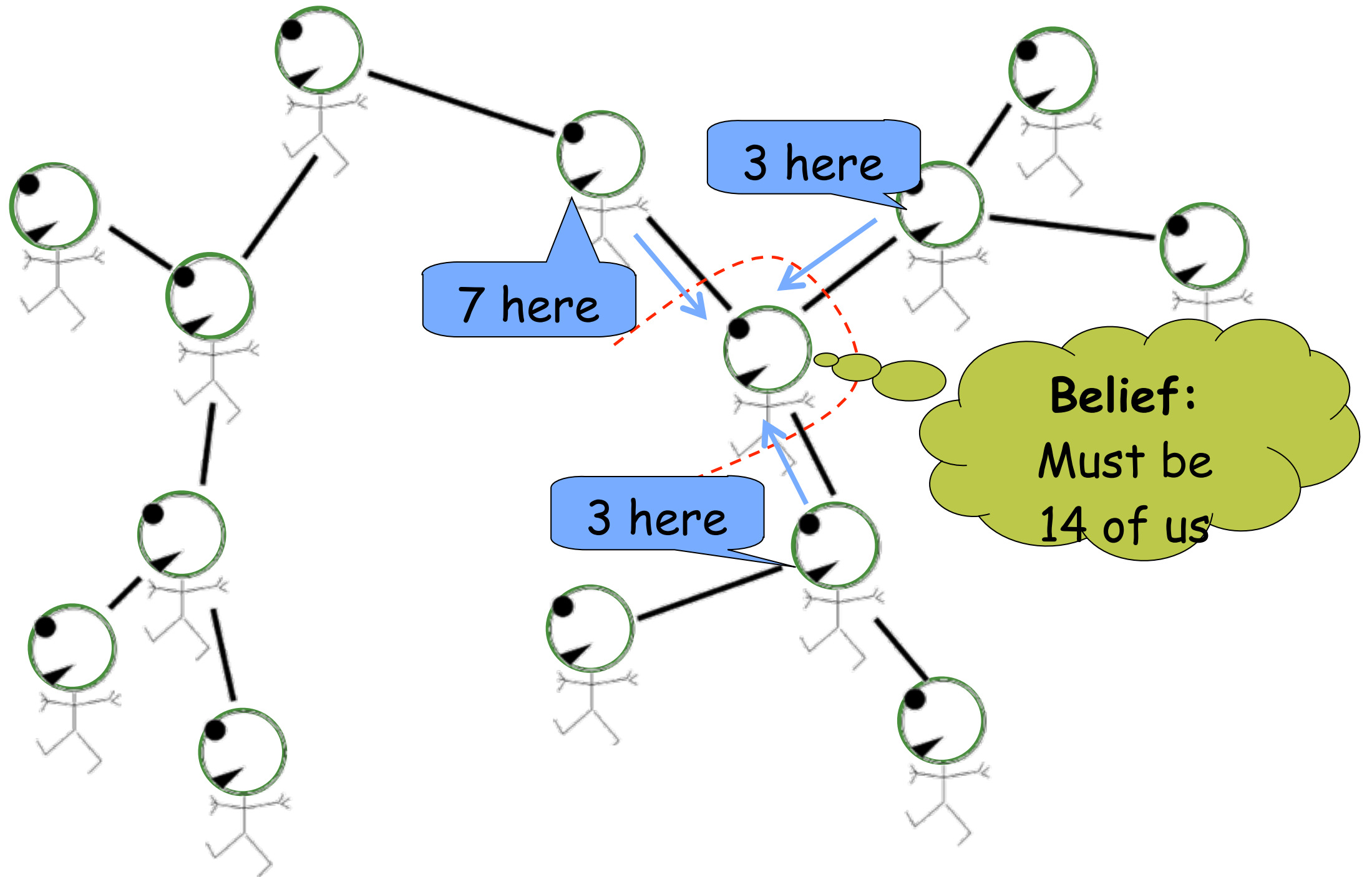
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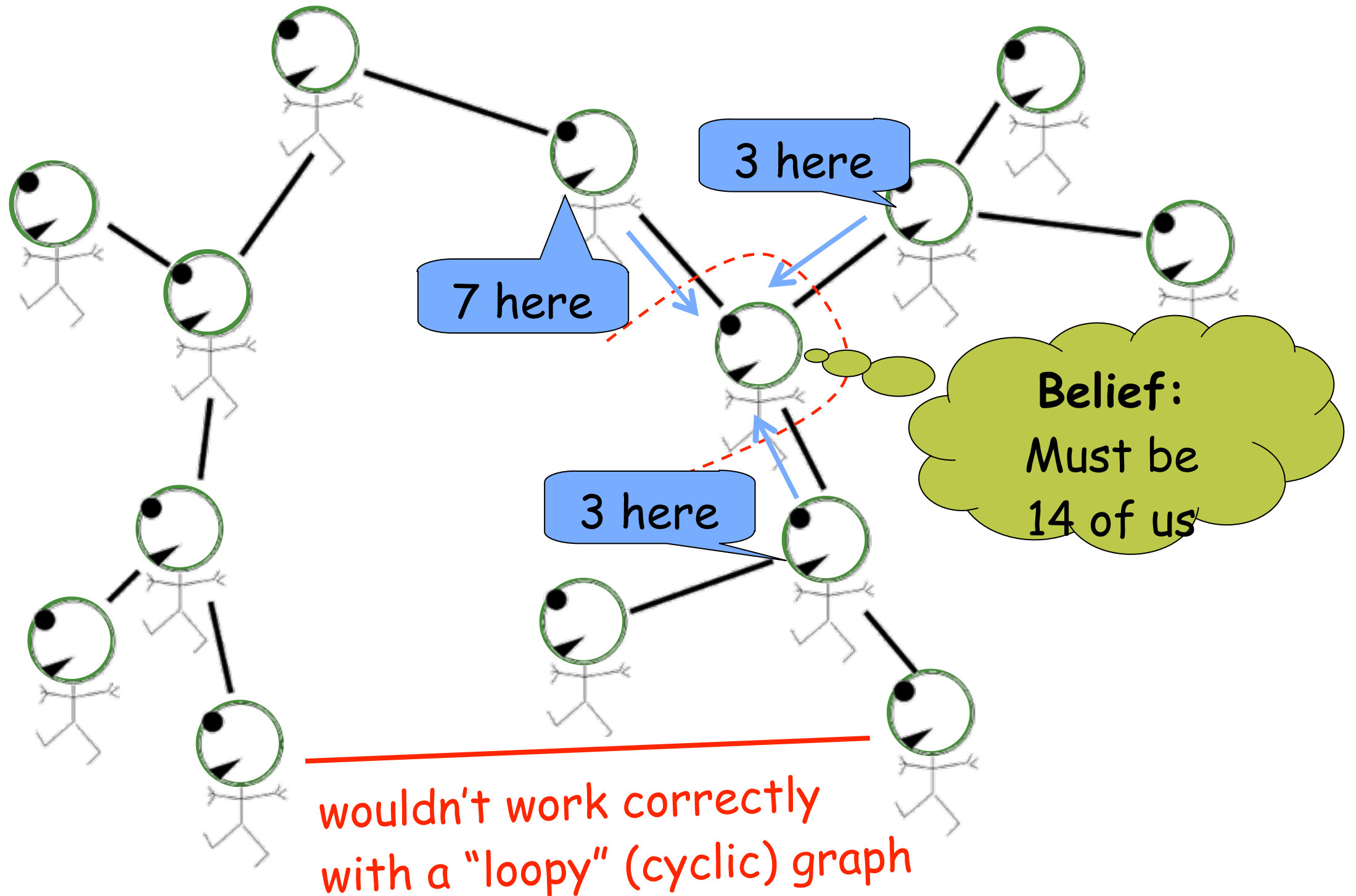
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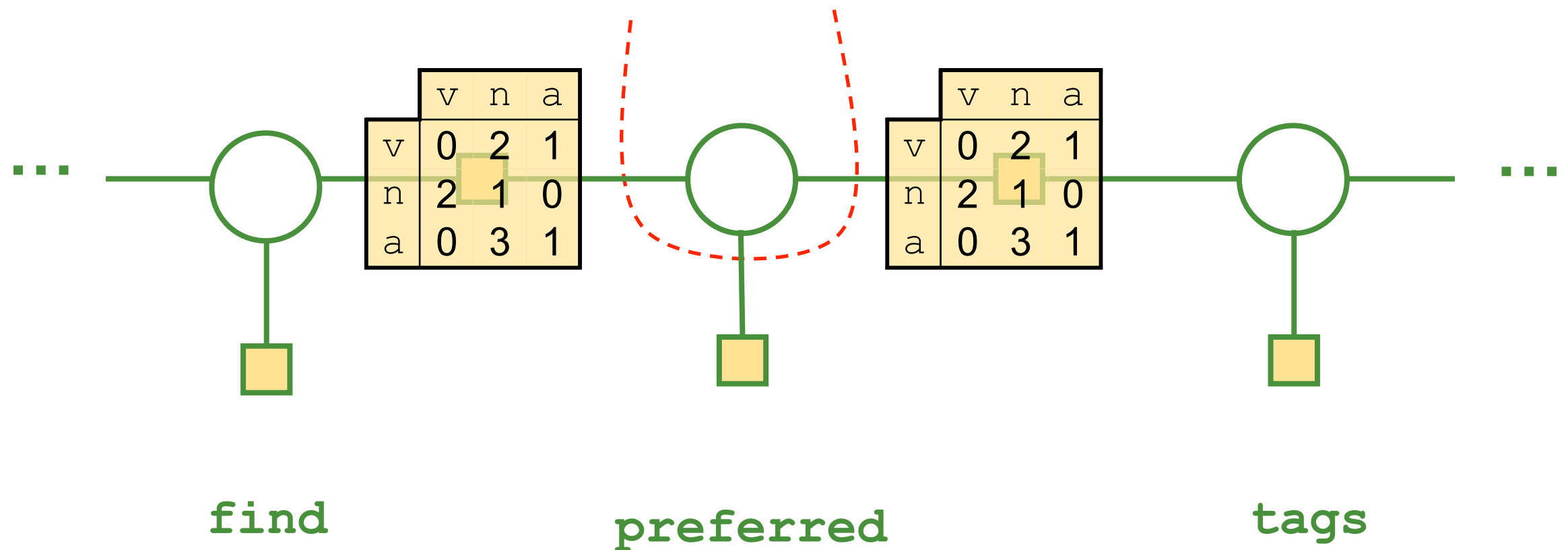
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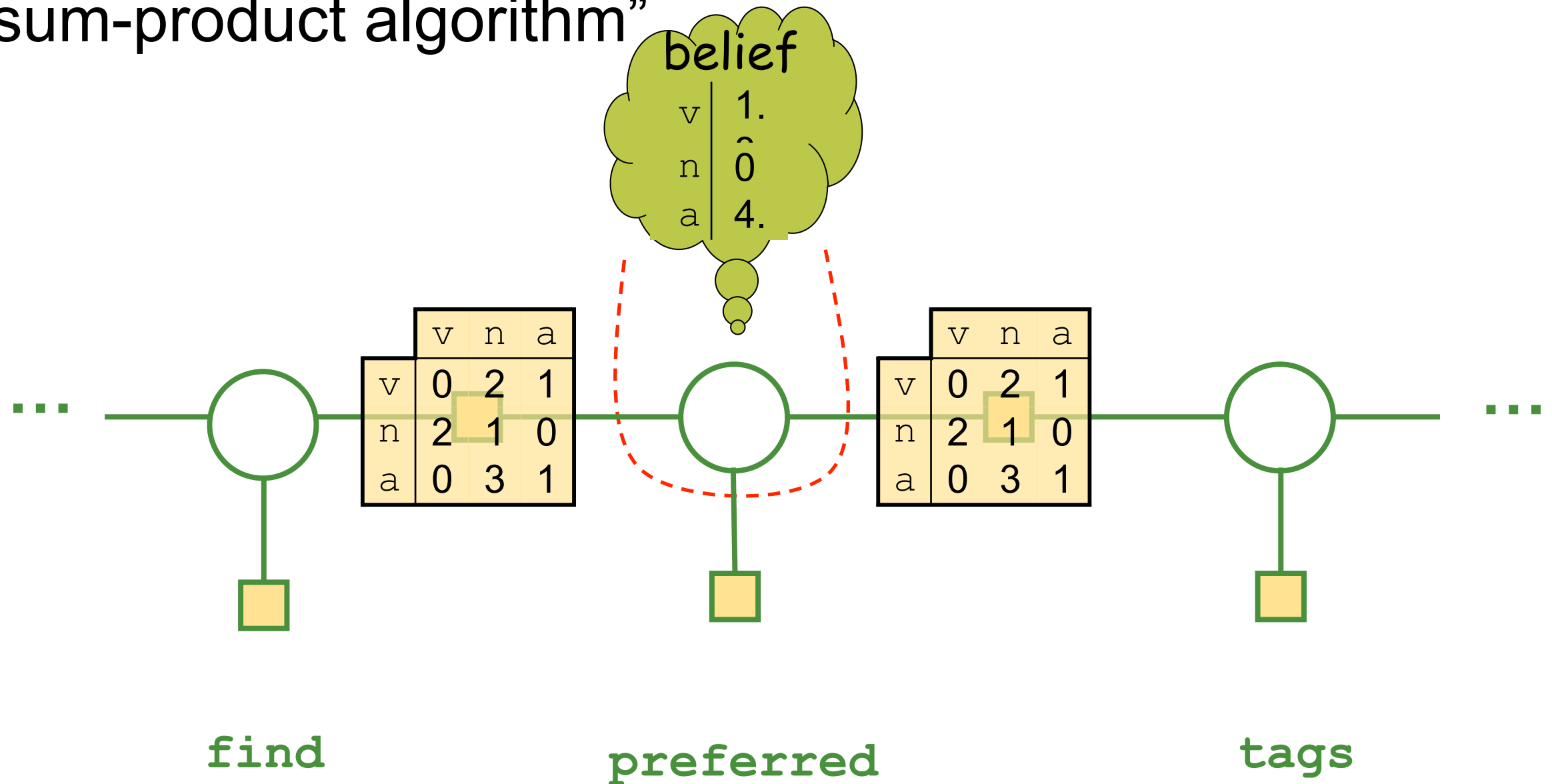
Great ideas in ML: Forward-Backward

- In the CRF, message passing = forward-backward = “sum-product algorithm”



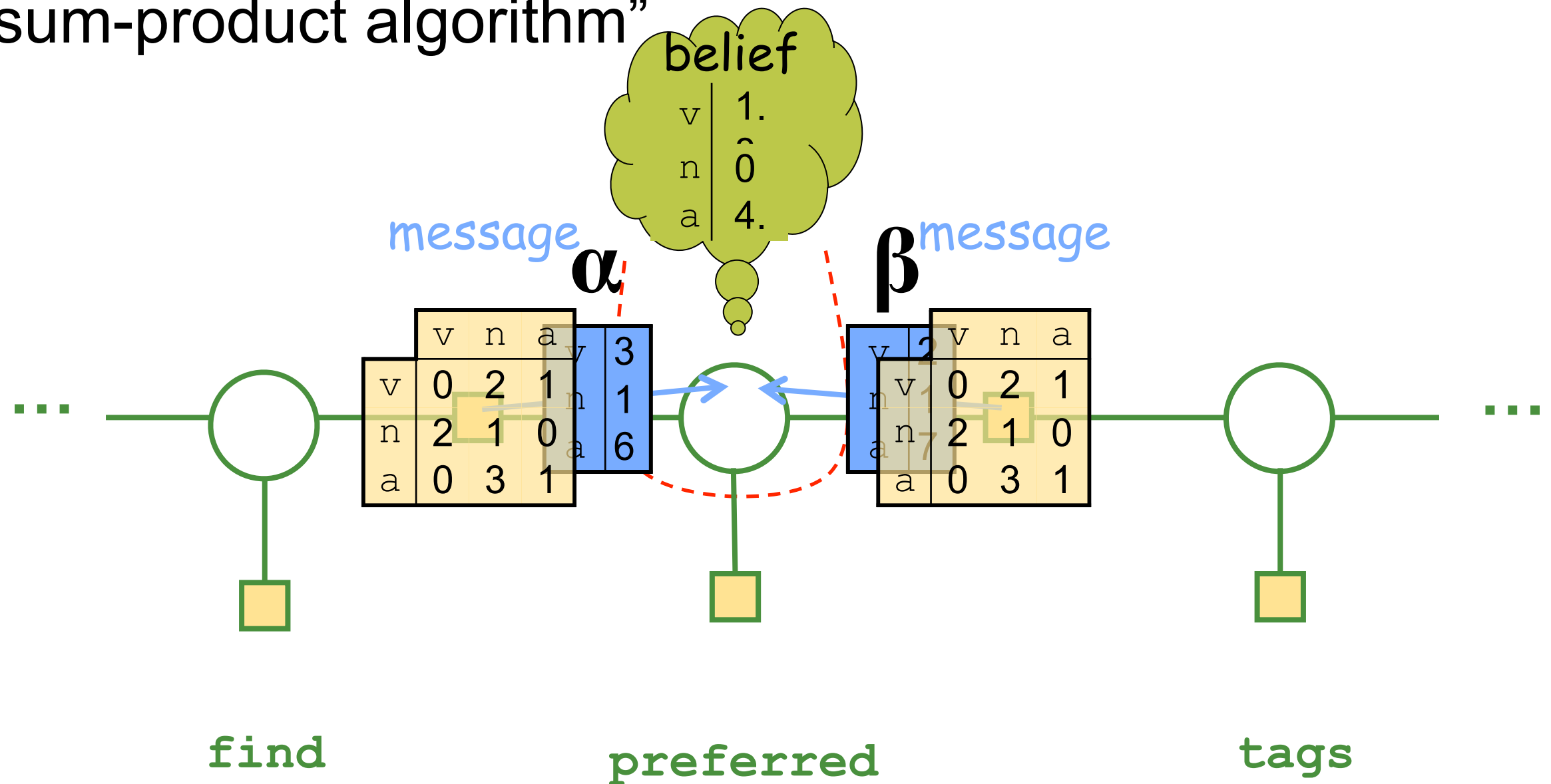
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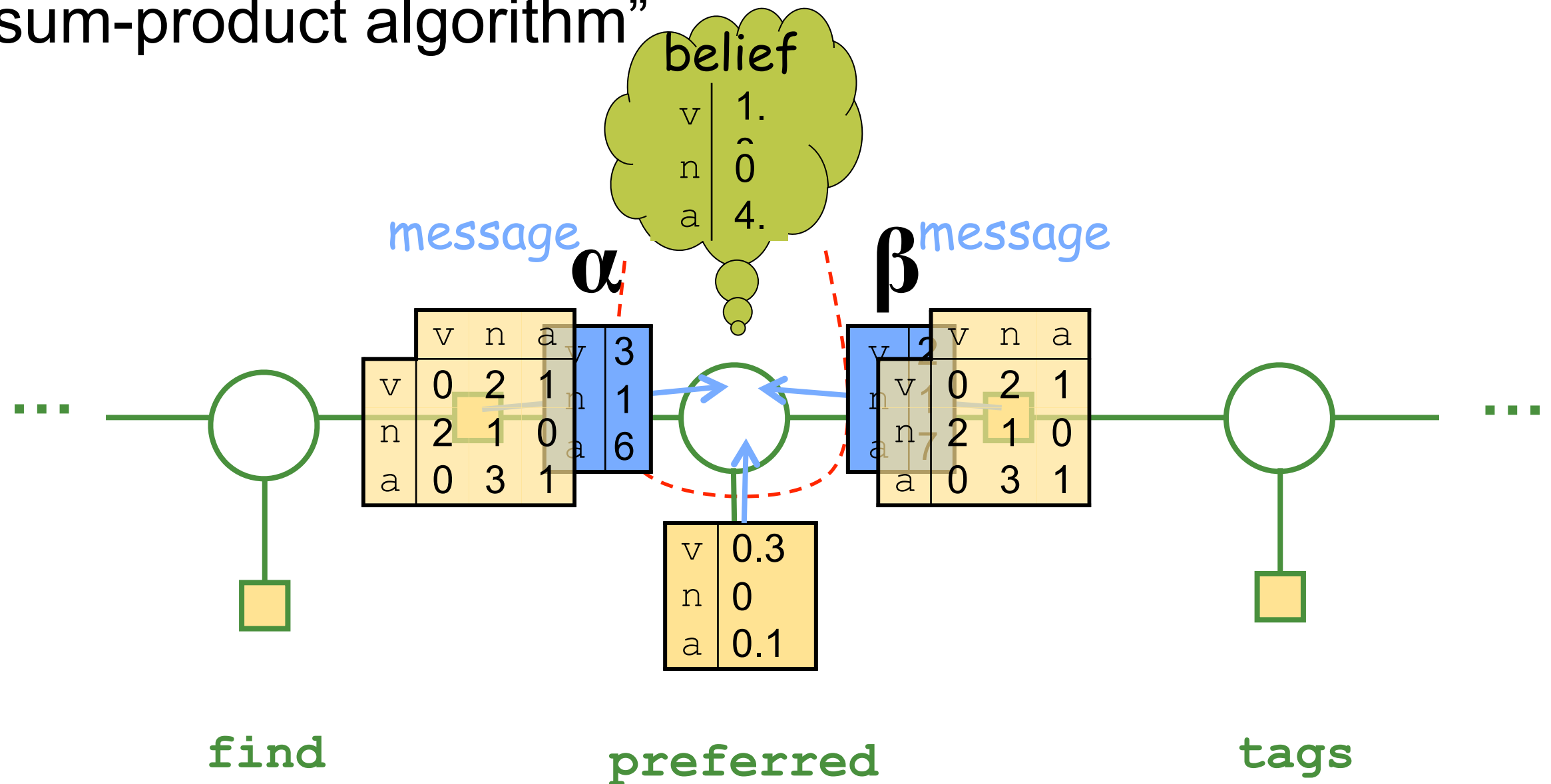
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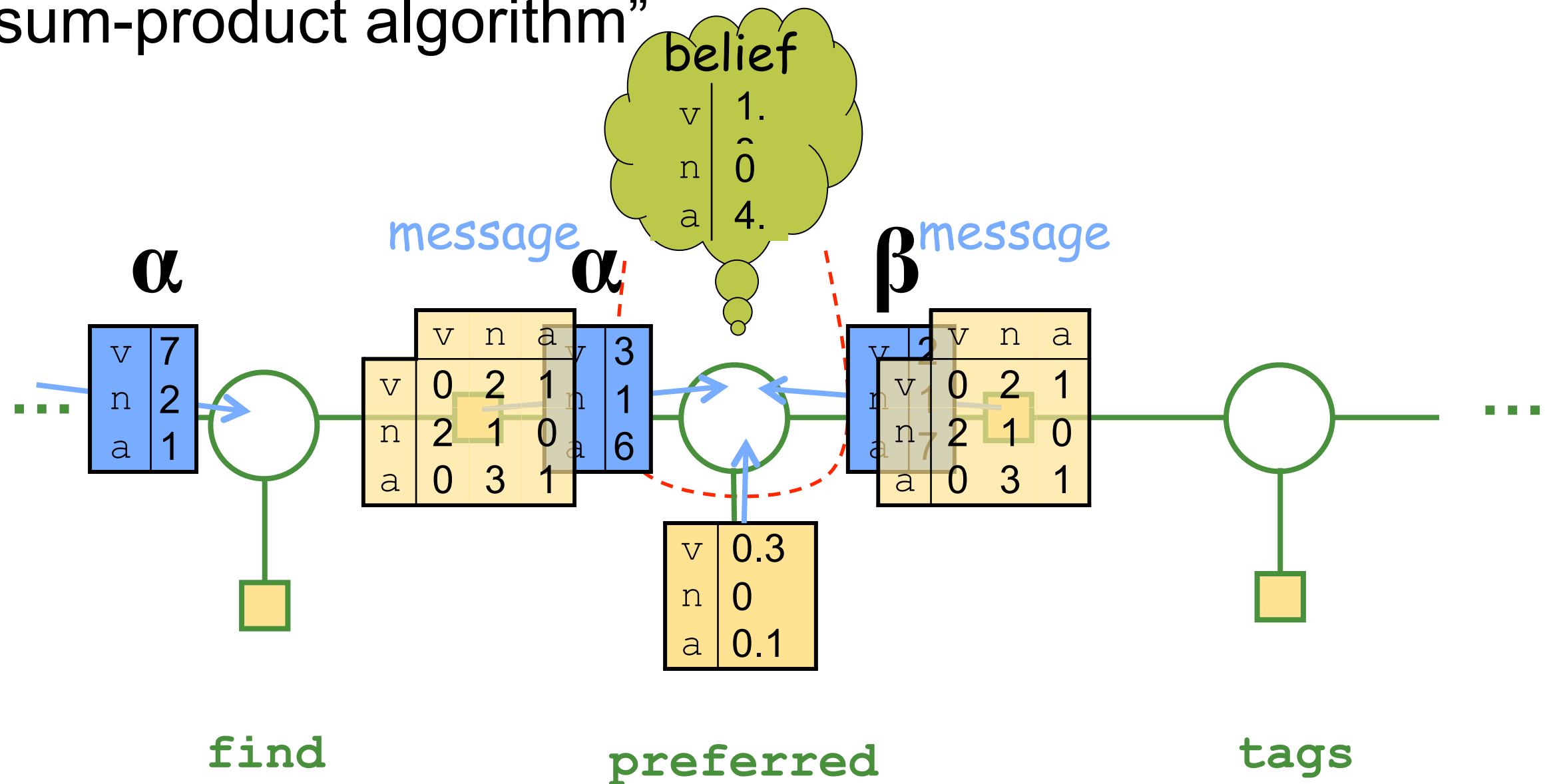
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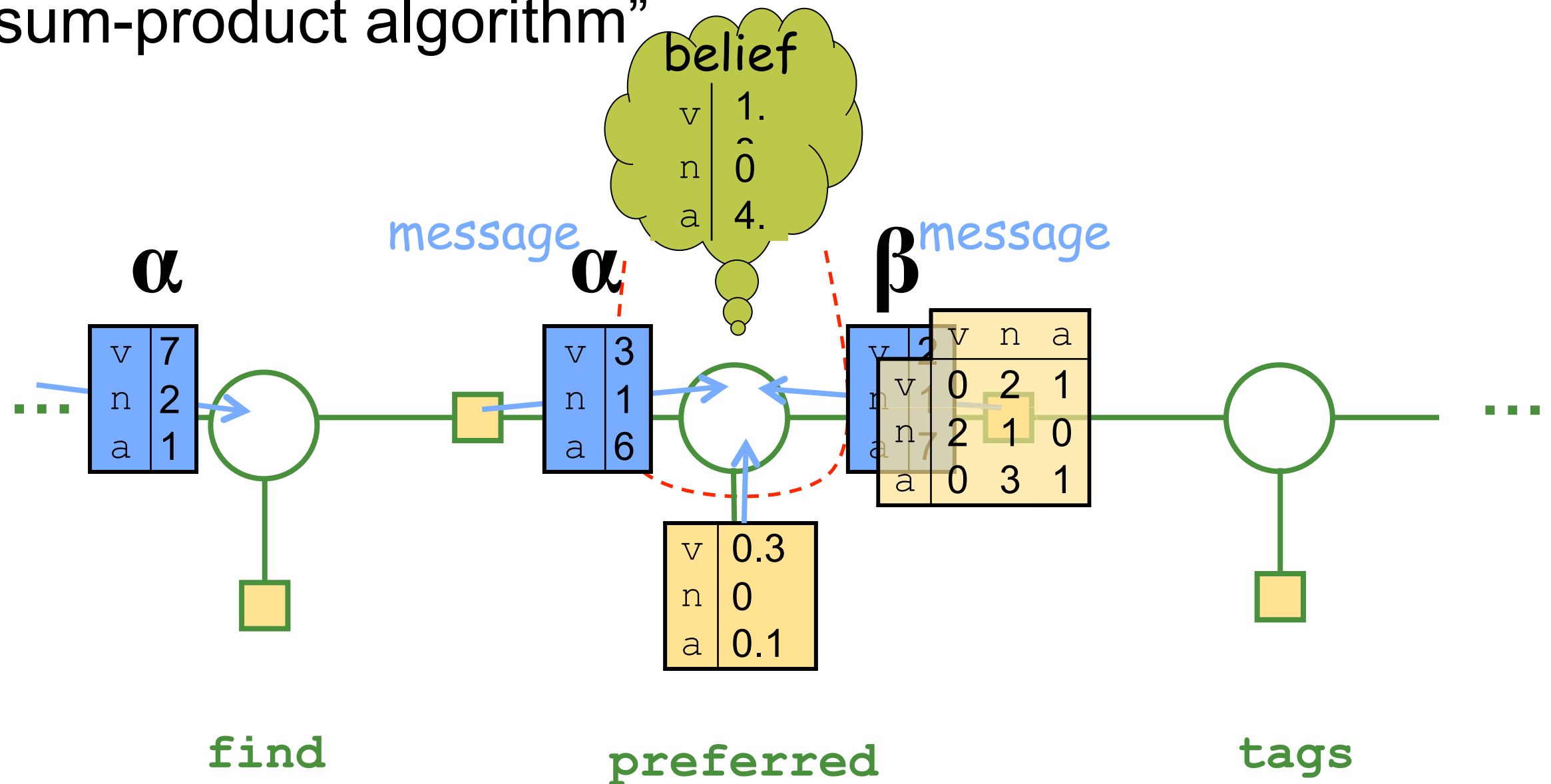
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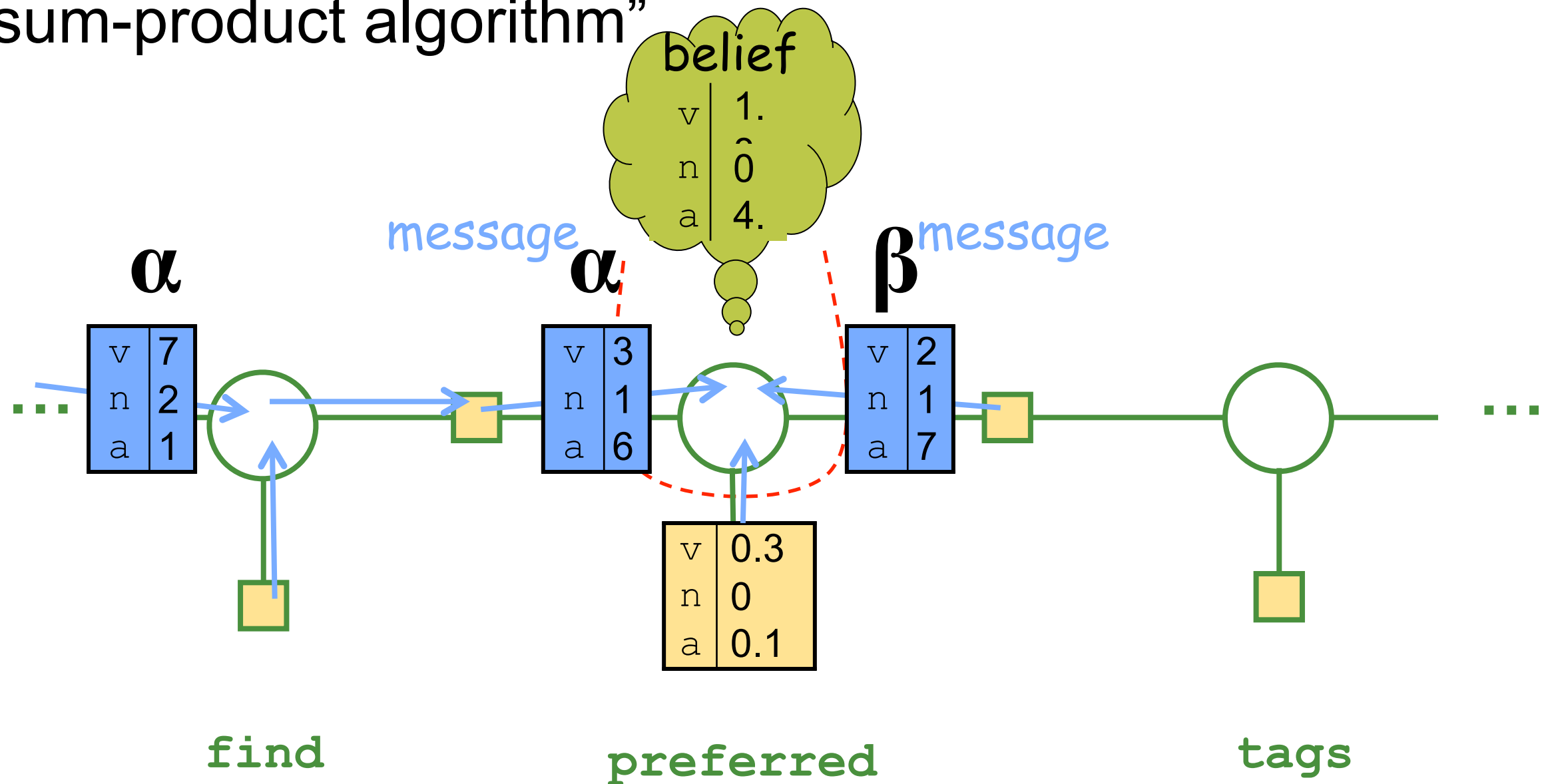
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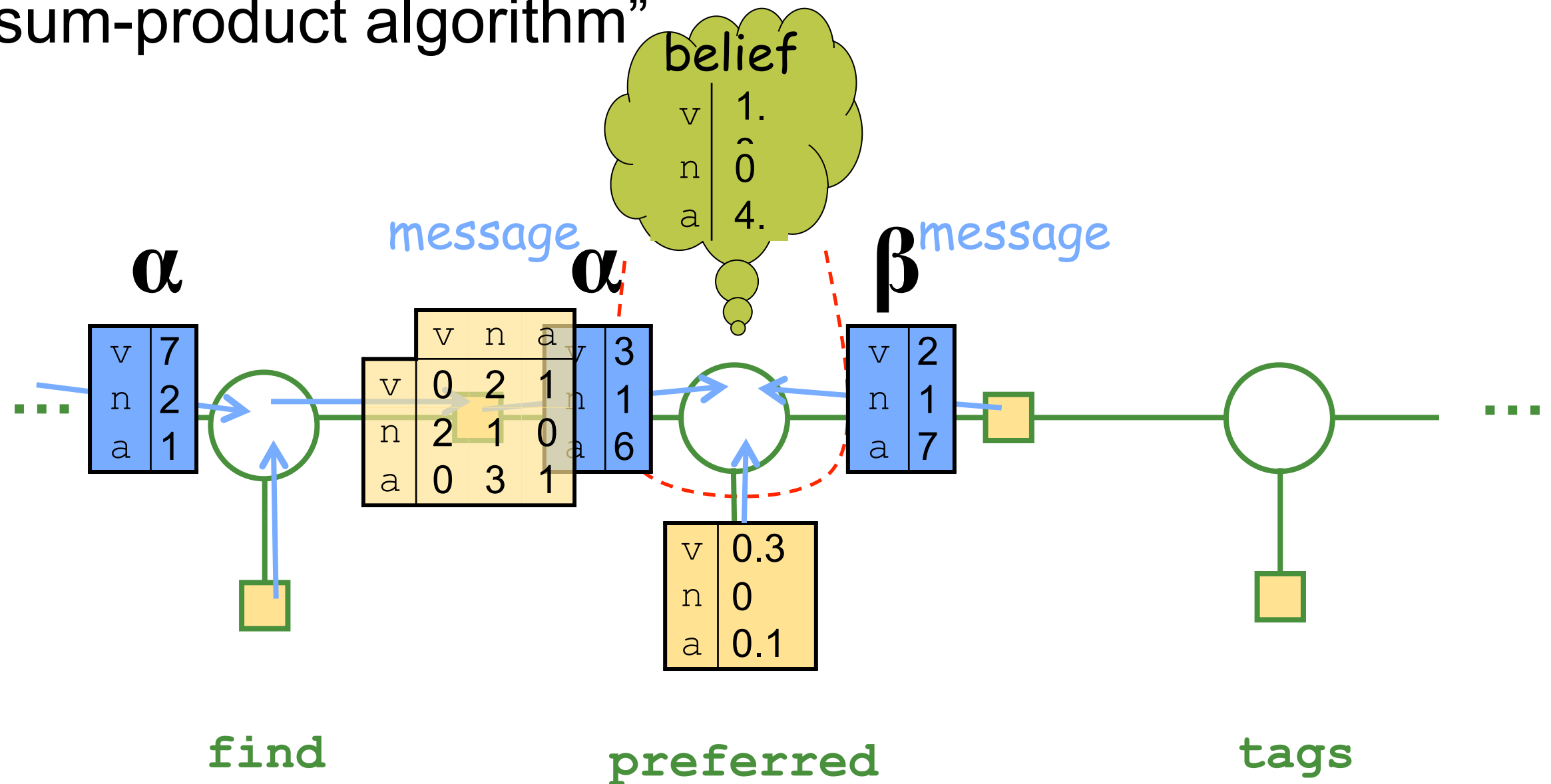
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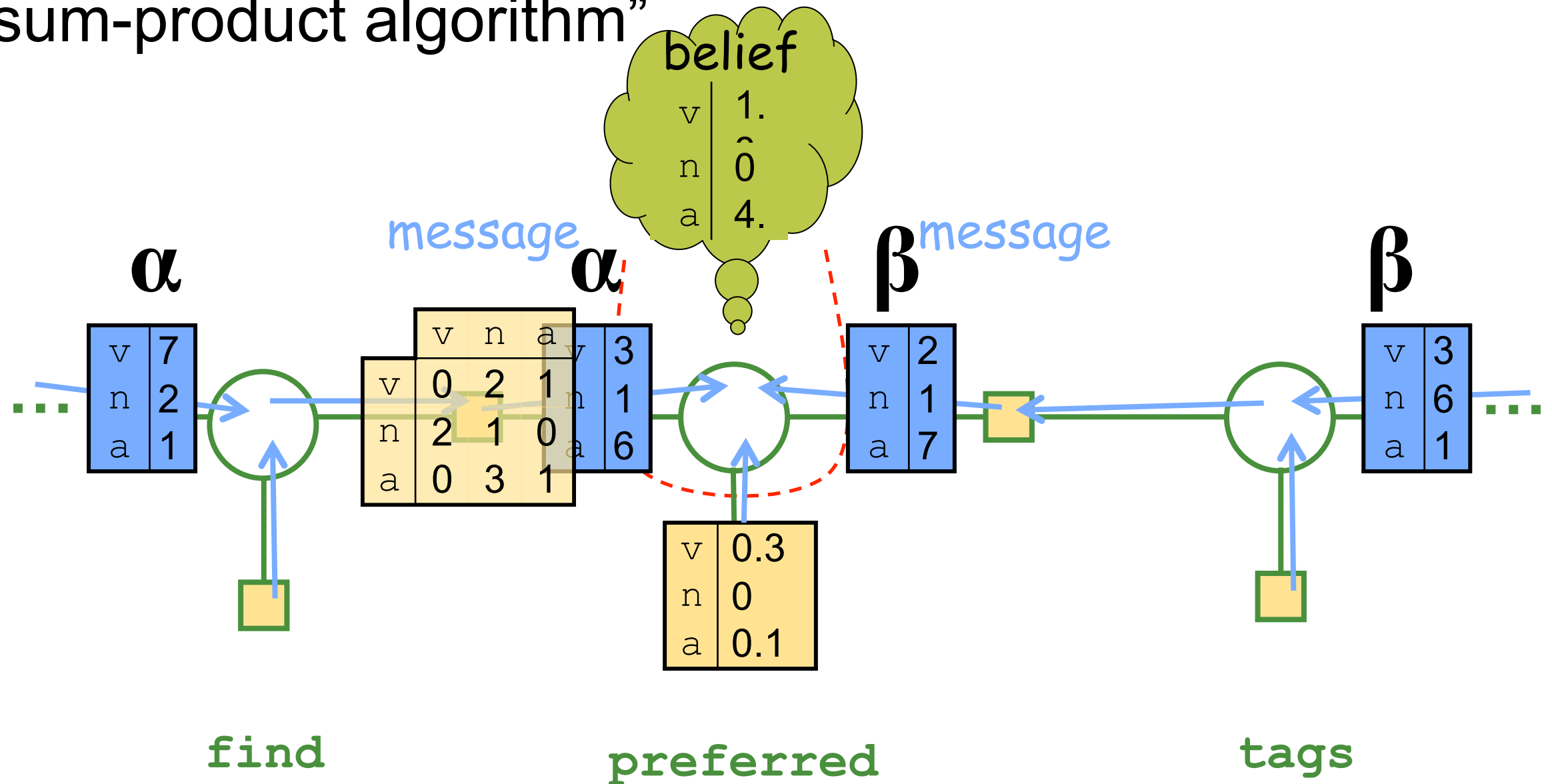
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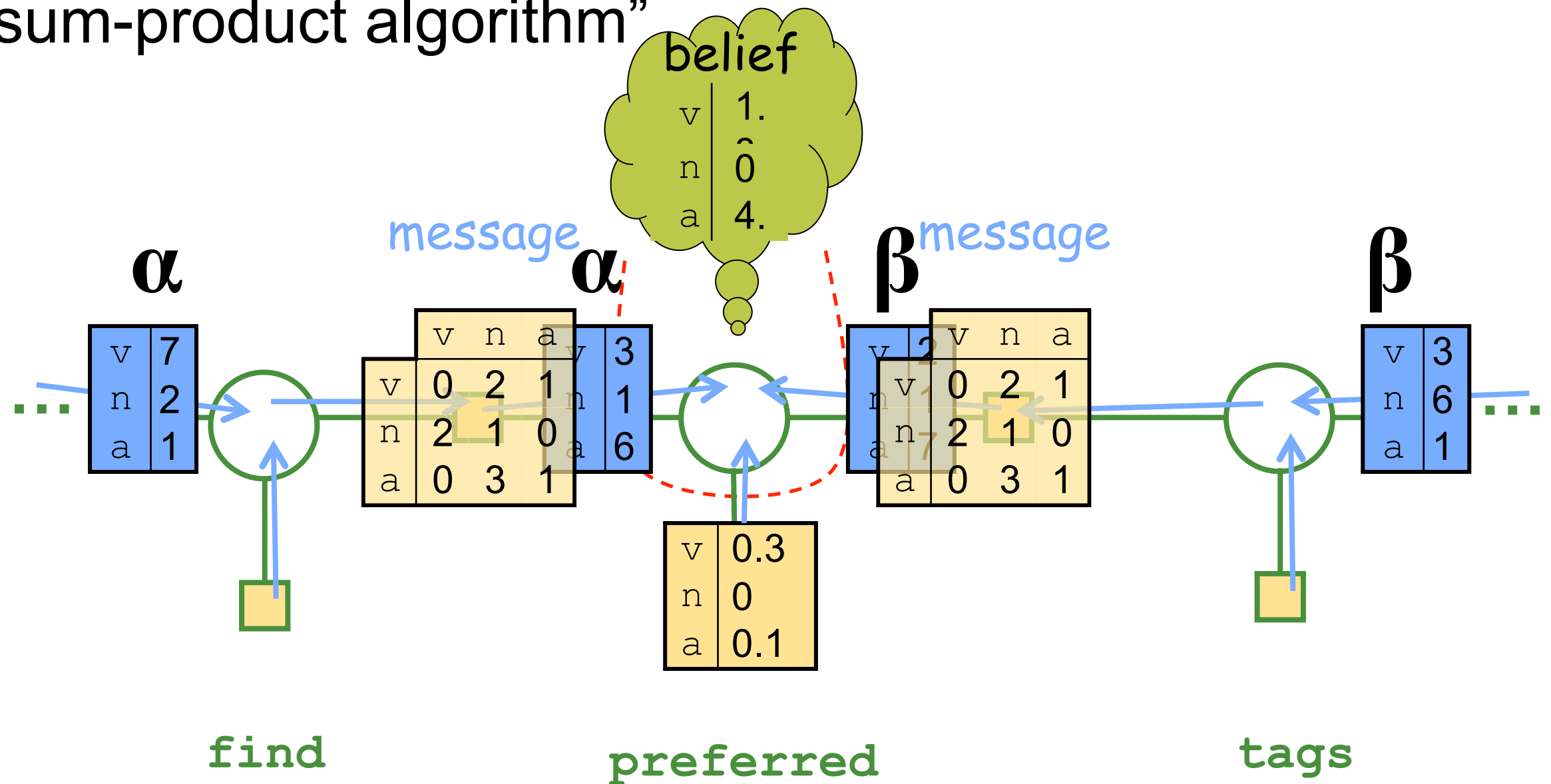
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Sum-Product Equations

- Message from variable v to factor f

$$m_{v \rightarrow f}(x) = \prod_{f' \in N(v) \setminus \{f\}} m_{f' \rightarrow v}(x)$$

- Message from factor f to variable v

Recipe for Conditional Training of $p(y | x)$

1. Gather constraints/features from training data

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero

3. Classify training data with current parameters; calculate expectations

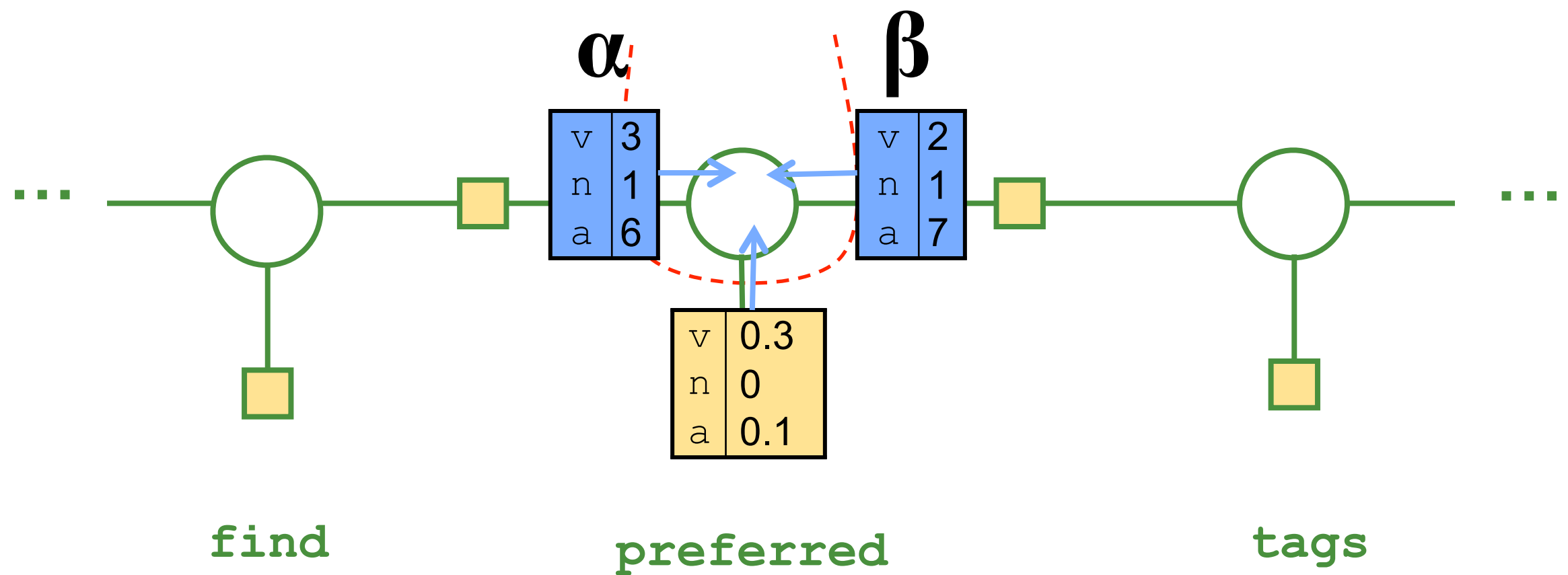
$$E_{\Theta}[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_{\Theta}(y' | x_j) f_{iy}(x_j, y')$$

4. Gradient is $\tilde{E}[f_{iy}] - E_{\Theta}[f_{iy}]$

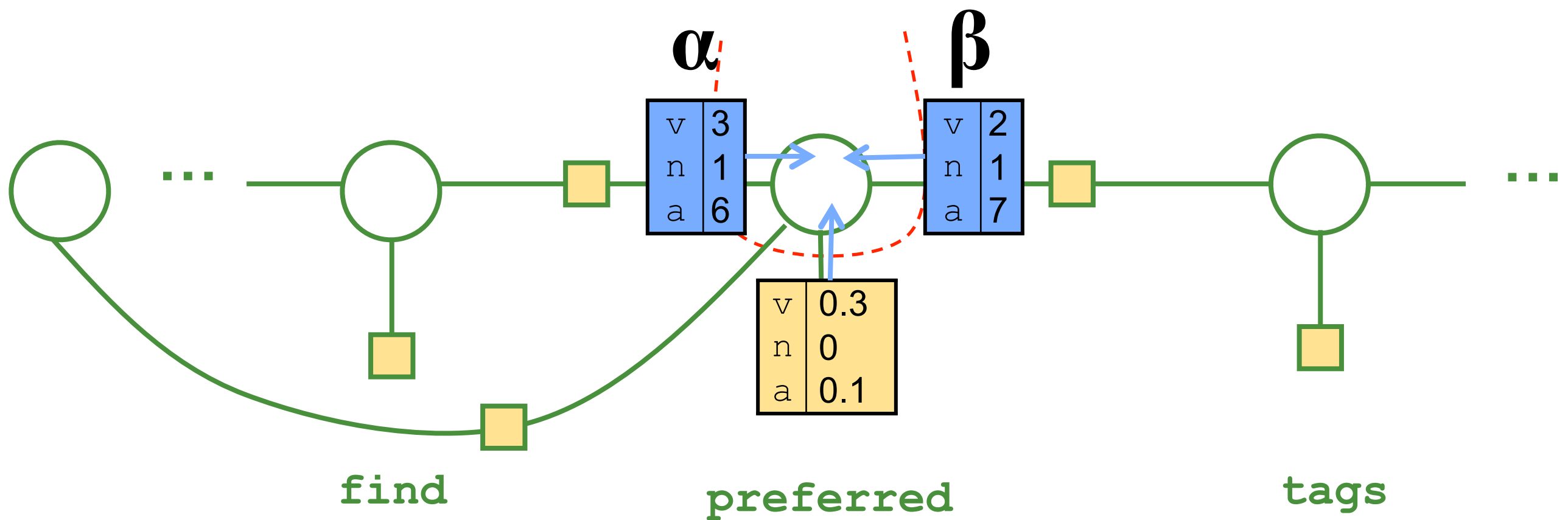
5. Take a step in the direction of the gradient

6. Repeat from 3 until convergence

Great ideas in ML: Forward-Backward

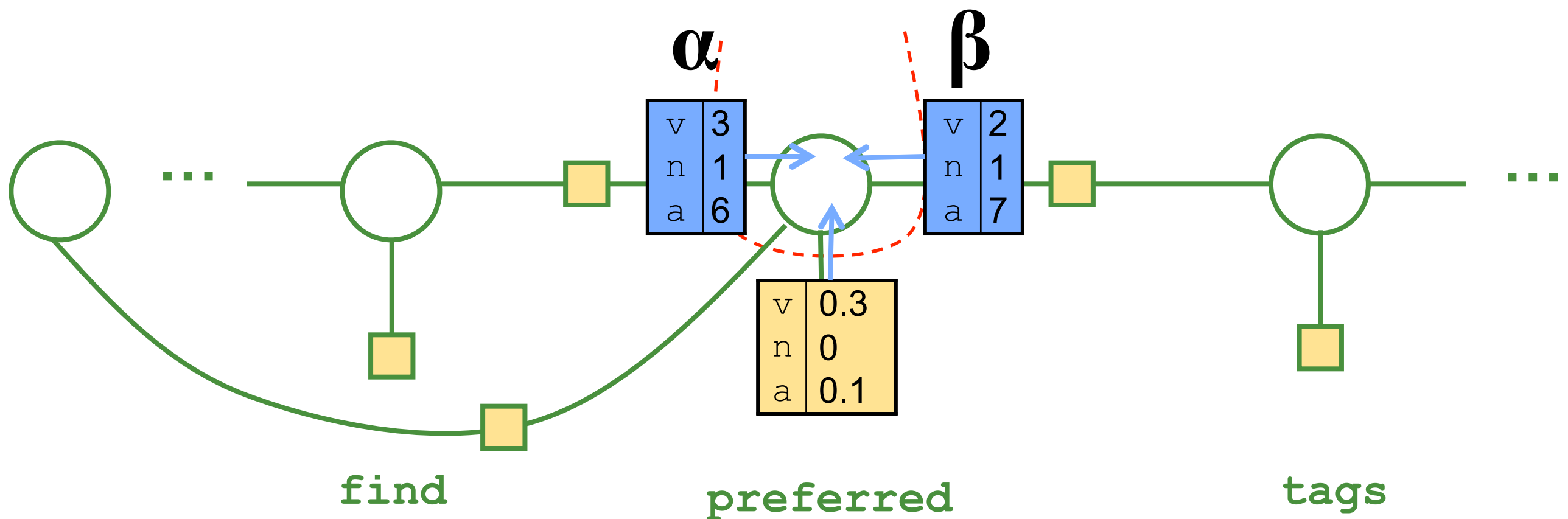


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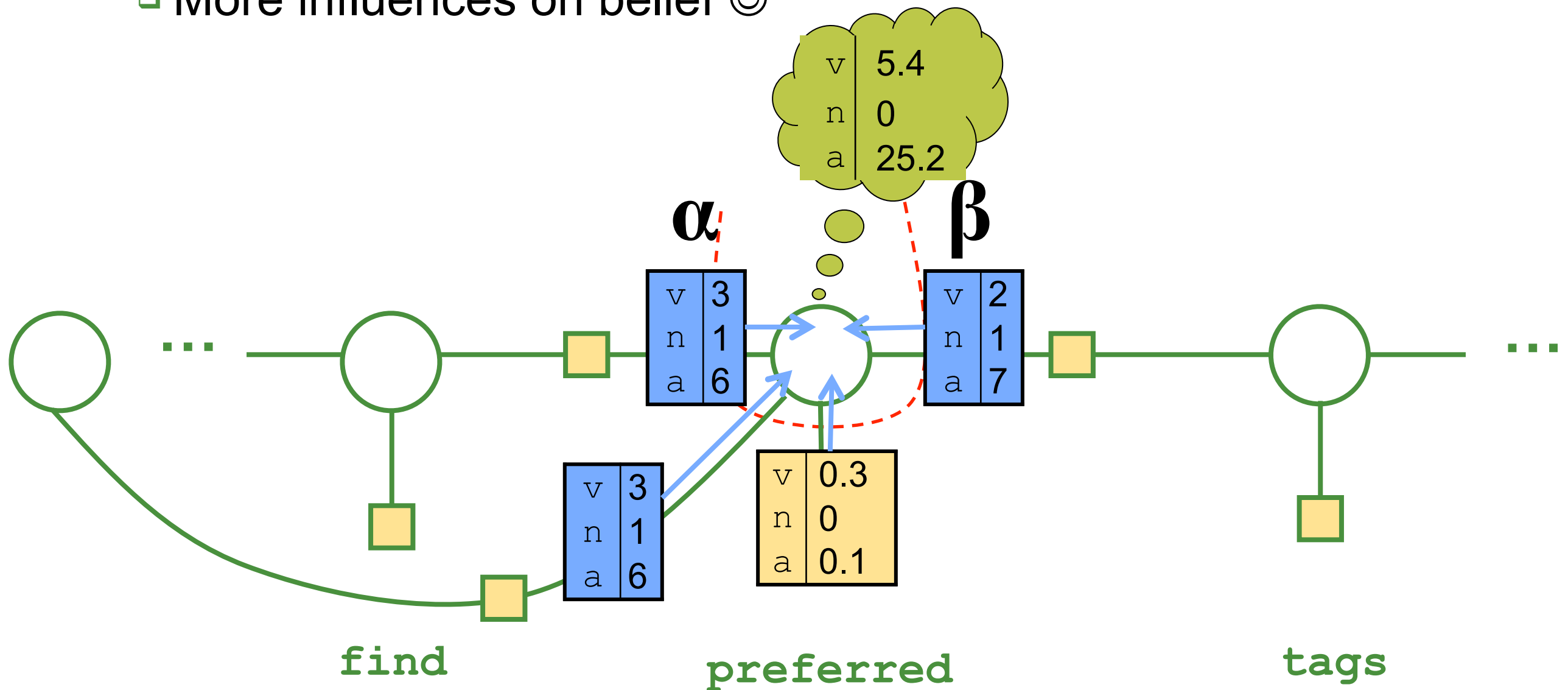
Great ideas in ML: Forward-Backward

- Extend CRF to “skip chain” to capture non-local factor



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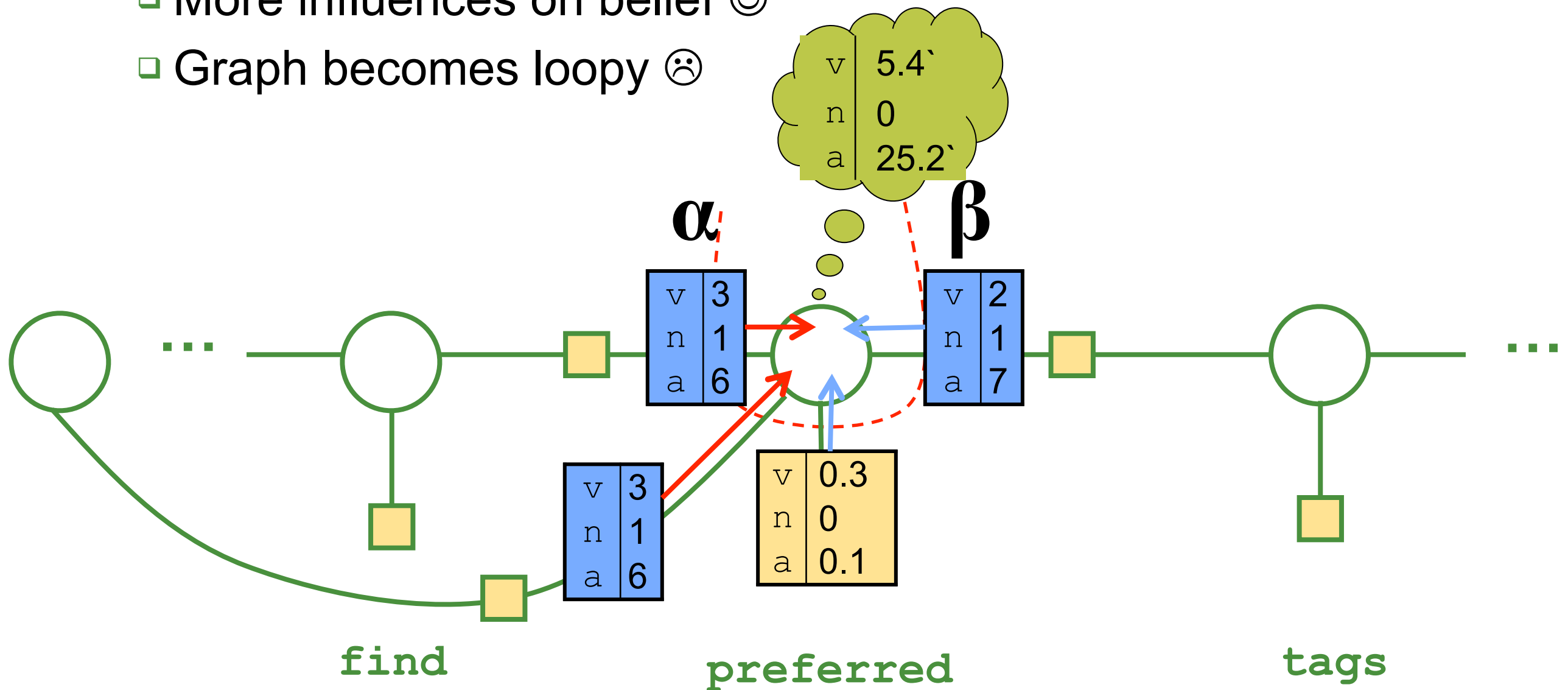
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 - More influences on belief 😊



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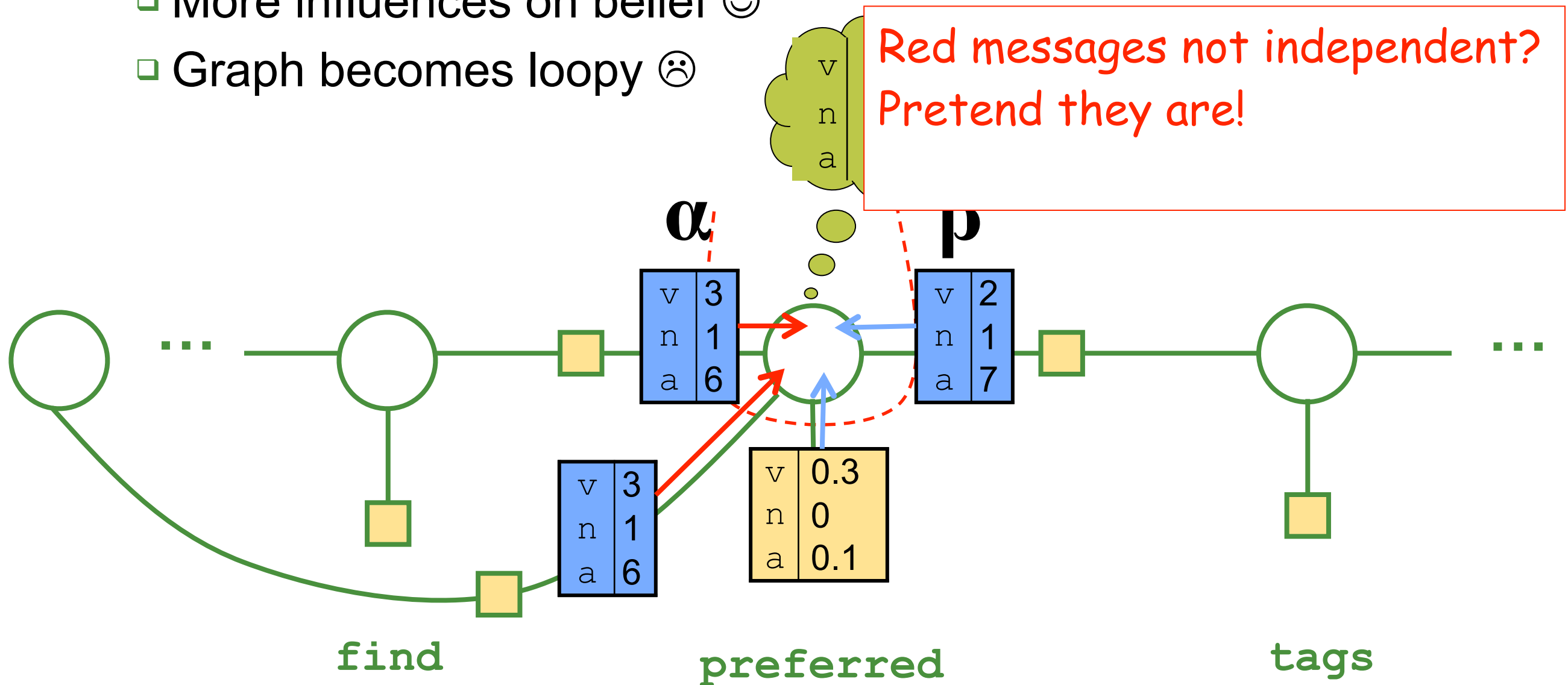
- More influences on belief 😊
- Graph becomes loopy 😞



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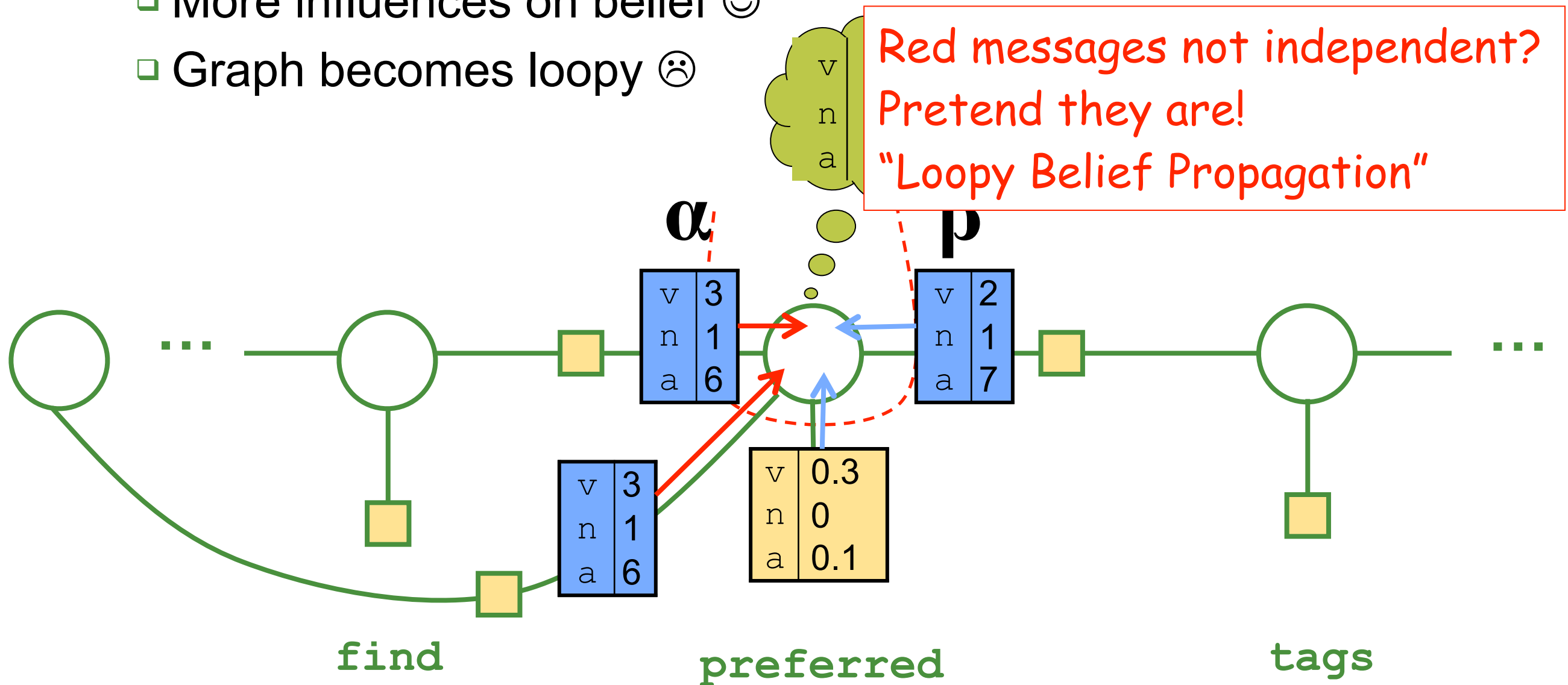
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Terminological Clarification

`propagation`

Terminological Clarification


belief propagation

Terminological Clarification



Terminological Clarification



Terminological Clarification

loopy **belief** **propagation**



Terminological Clarification

loopy **belief** **propagation**

A diagram with three green text elements: 'loopy', 'belief', and 'propagation'. 'loopy' and 'belief' are enclosed in a single pair of green brackets. A curved black arrow points from 'loopy' to 'belief'. Another curved black arrow points from 'belief' to 'propagation'.



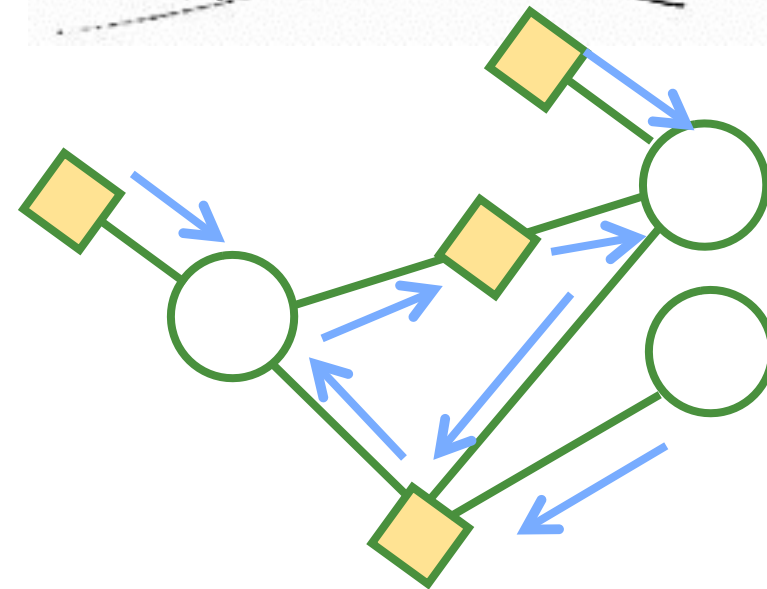
loopy **belief propagation**

A diagram with two green text elements: 'loopy' and 'belief propagation'. 'belief propagation' is enclosed in a single pair of green brackets. A curved black arrow points from 'loopy' to 'belief propagation'.

Terminological Clarification

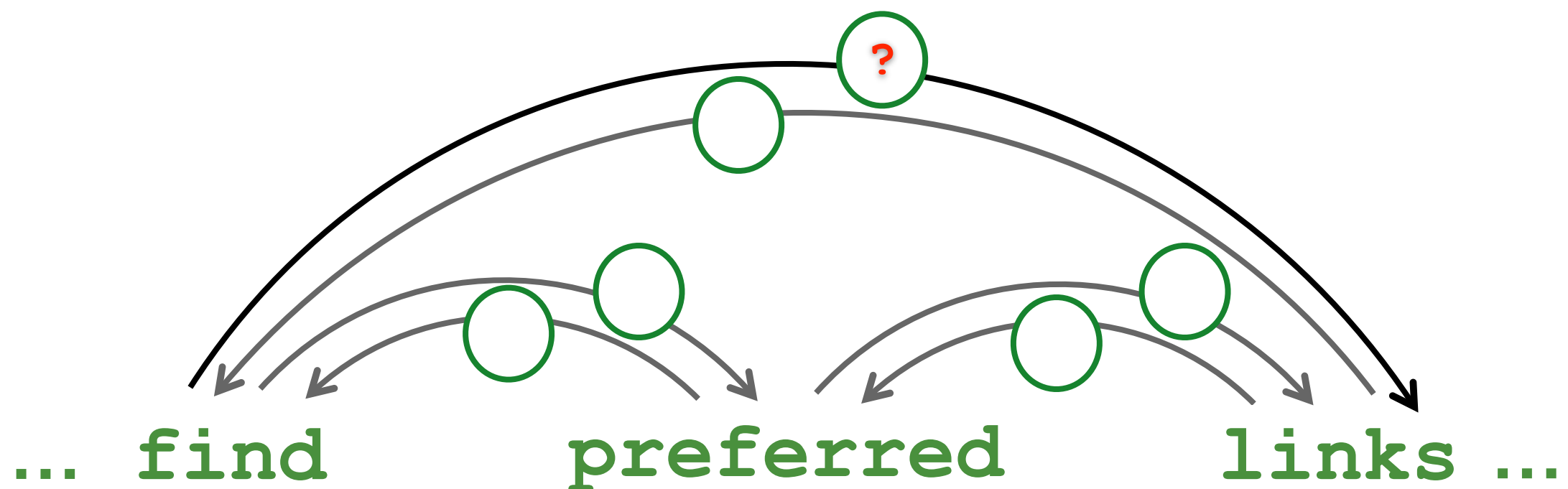
[loopy belief] propagation

loopy [belief propagation]



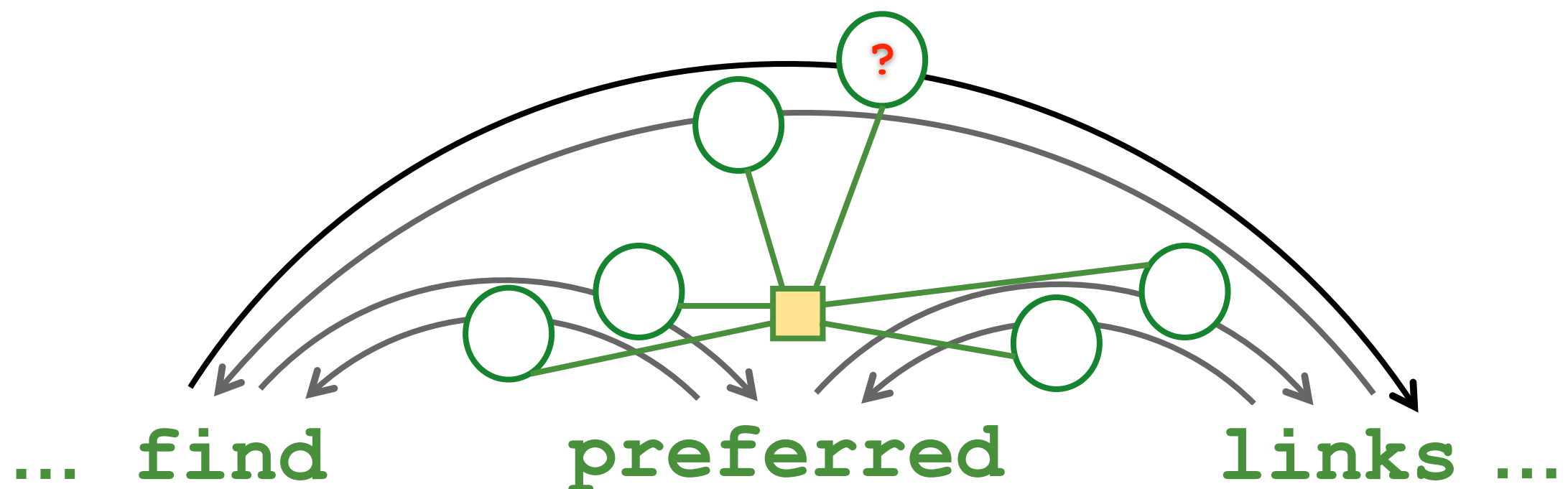
Propagating Global Factors

- Loopy belief propagation is easy for local factors
- How do combinatorial factors (like TREE) compute the message to the link in question?
 - ❖ “Does the TREE factor think the link is probably **t** given the messages it receives from *all* the other links?”



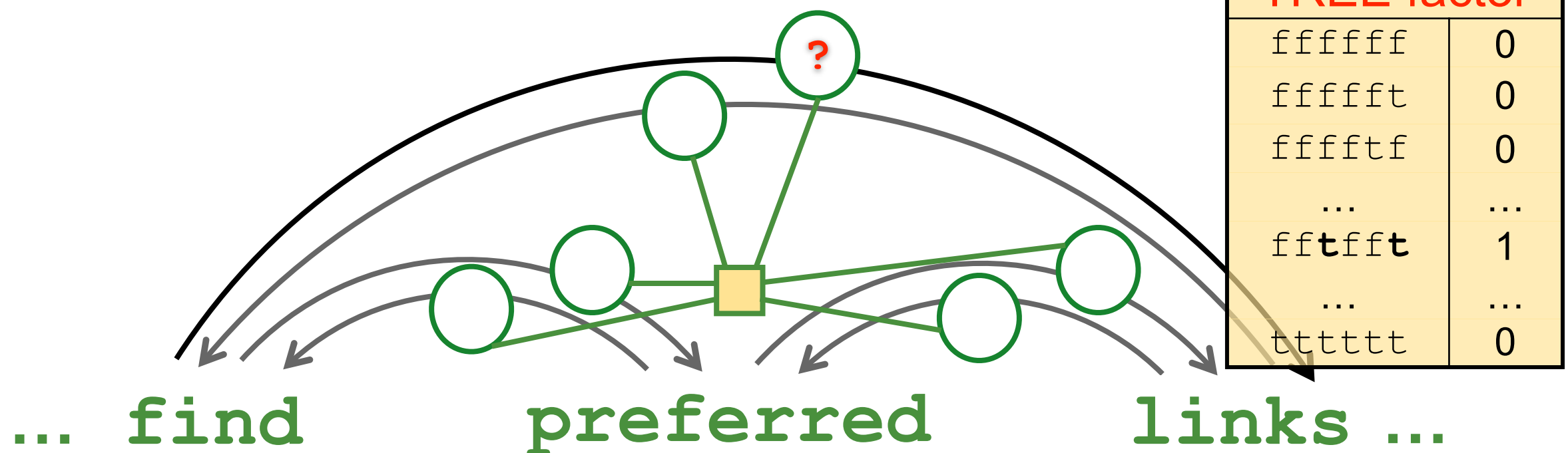
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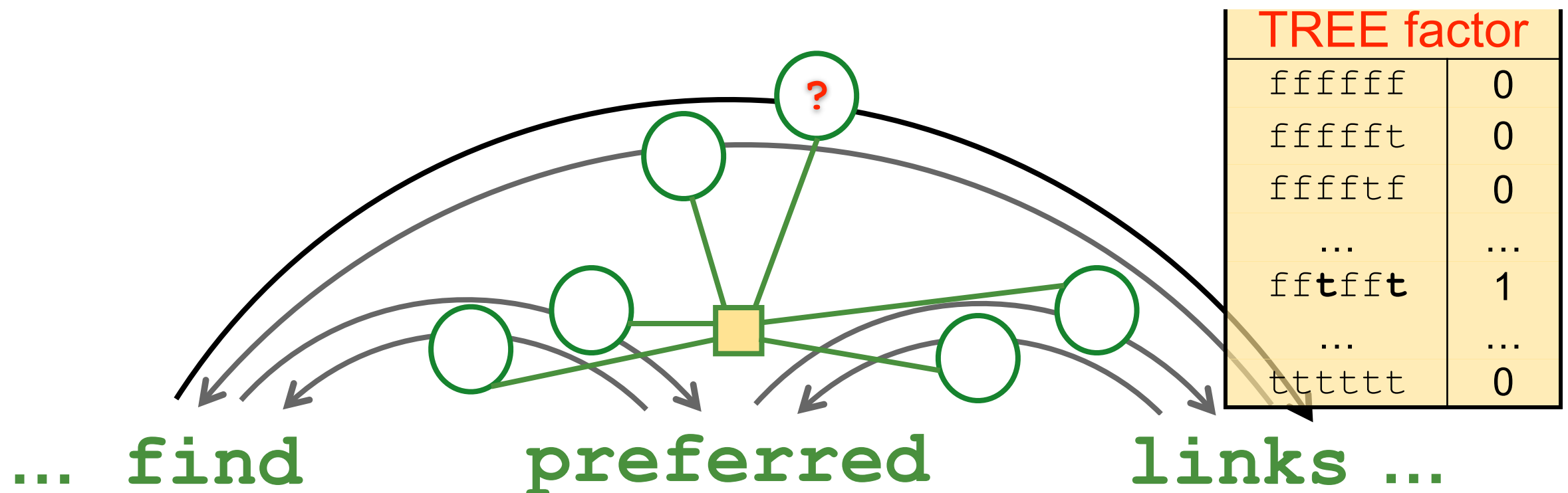
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Propagating Global Factors

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Old-school parsing to the rescue!

This is the outside probability of the link in an *edge-factored* parser!

∴ TREE factor computes all outgoing messages at once
(given all incoming messages)

Projective case: total $O(n^3)$ time by inside-outside

Non-projective: total $O(n^3)$ time by inverting Kirchhoff matrix

Graph Theory to the Rescue!

Tutte's Matrix-Tree Theorem (1948)

The **determinant** of the Kirchhoff (aka Laplacian) adjacency matrix of directed graph G without row and column r is equal to the **sum of scores of all directed spanning trees** of G rooted at node r .



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Exactly the Z we need!



Graph Theory to the Rescue!

$O(n^3)$ time!

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Kirchoff (Laplacian) Matrix



$$\begin{bmatrix} 0 & -s(1,0) & -s(2,0) & \cdots & -s(n,0) \\ 0 & 0 & -s(2,1) & \cdots & -s(n,1) \\ 0 & -s(1,2) & 0 & \cdots & -s(n,2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -s(1,n) & -s(2,n) & \cdots & 0 \end{bmatrix}$$

- Negate edge scores
- Sum columns (children)
- Strike root row/col.
- Take determinant



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Kirchoff (Laplacian) Matrix



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$$\begin{vmatrix} \sum_{j \neq 1} s(1,j) & -s(2,1) & \cdots & -s(n,1) \\ -s(1,2) & \sum_{j \neq 2} s(2,j) & \cdots & -s(n,2) \\ \vdots & \vdots & \ddots & \vdots \\ -s(1,n) & -s(2,n) & \cdots & \sum_{j \neq n} s(n,j) \end{vmatrix}$$

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- Negate edge scores
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N.B.: This allows multiple children of root, but see Koo et al. 2007.

Transition-Based Parsing

- Linear time
- Online
- Train a classifier to predict next action
- Deterministic or beam-search strategies
- But... generally less accurate

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Start state: $([], [1, \dots, n], \{ \})$

Final state: $(S, [], A)$

Shift: $(S, i|B, A) \Rightarrow (S|i, B, A)$

Reduce: $(S|i, B, A) \Rightarrow (S, B, A)$

Right-Arc: $(S|i, j|B, A) \Rightarrow (S|i|j, B, A \cup \{i \rightarrow j\})$

Left-Arc: $(S|i, j|B, A) \Rightarrow (S, j|B, A \cup \{i \leftarrow j\})$

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

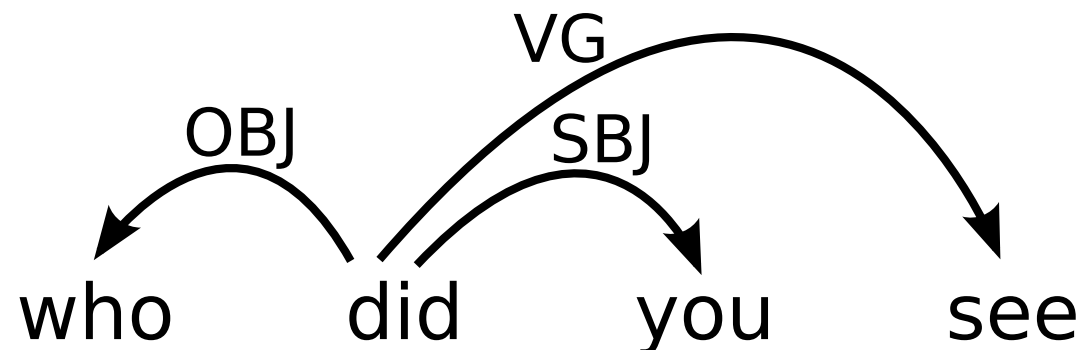
[]_S

Buffer

[who, did, you, see]_B

Arcs

{ }



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[who]_S

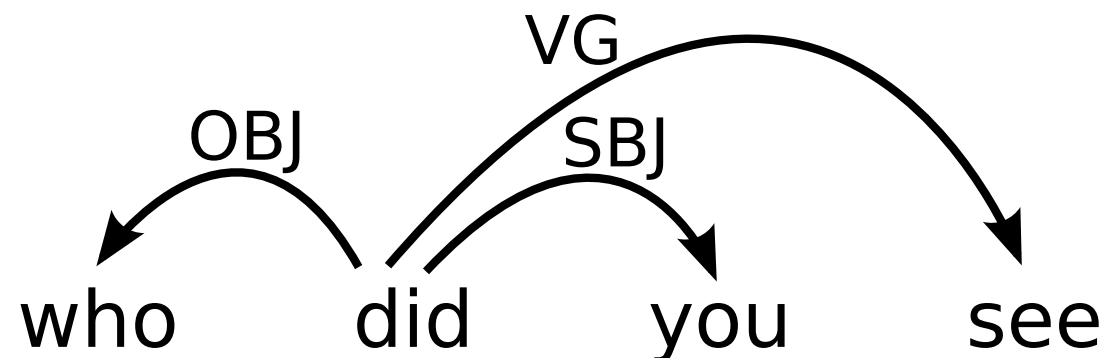
Buffer

[did, you, see]_B

Arcs

{ }

Shift



Transition-Based Parsing

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Stack

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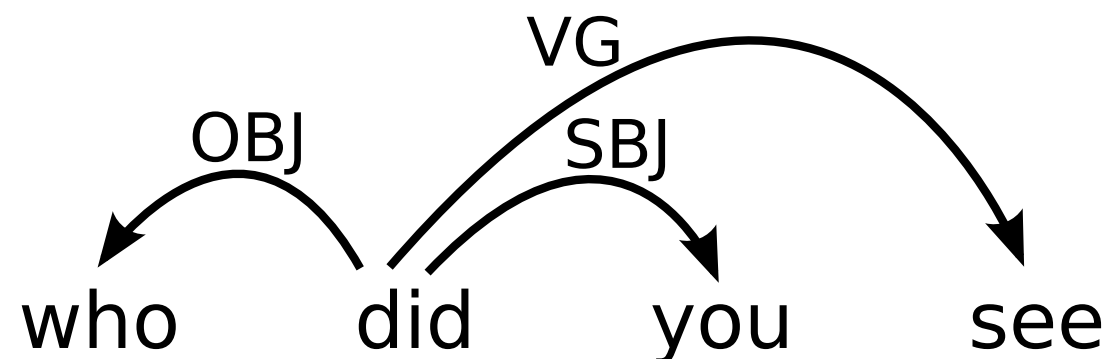
Buffer

[did, you, see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did }

Left-arc
OBJ



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did]_S

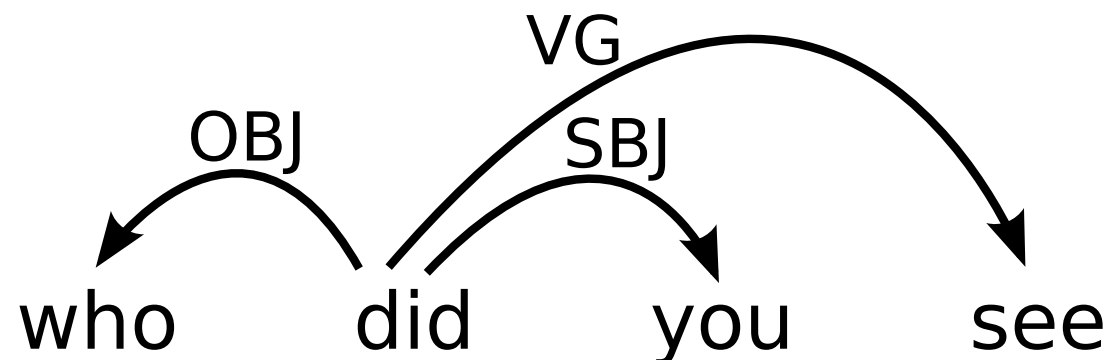
Buffer

[you, see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did }

Shift



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Stack

[did, you]_S

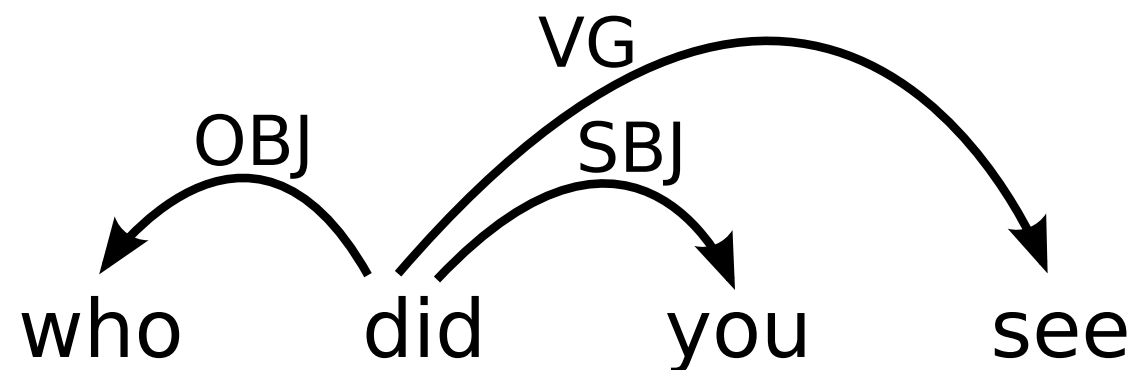
Buffer

[see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did,
did $\xrightarrow{\text{SBJ}}$ you }

Right-arc
SBJ



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did]_S

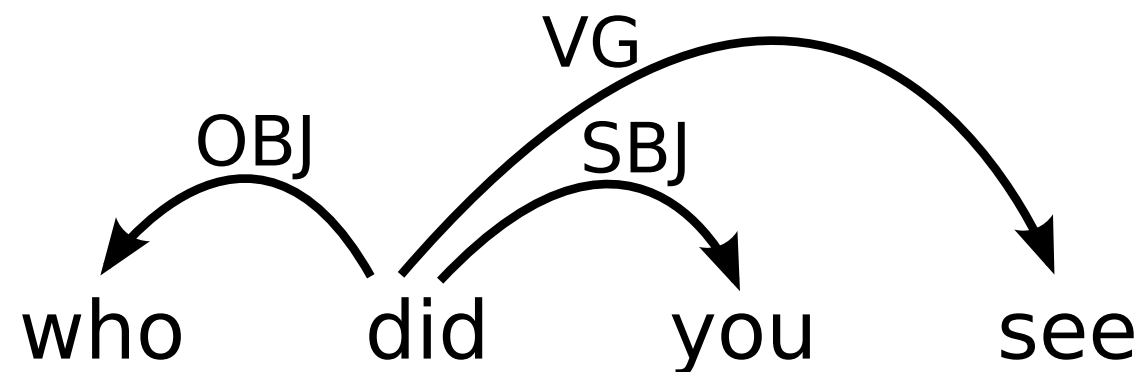
Buffer

[see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did,
did $\xrightarrow{\text{SBJ}}$ you }

Reduce



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did, see]_S

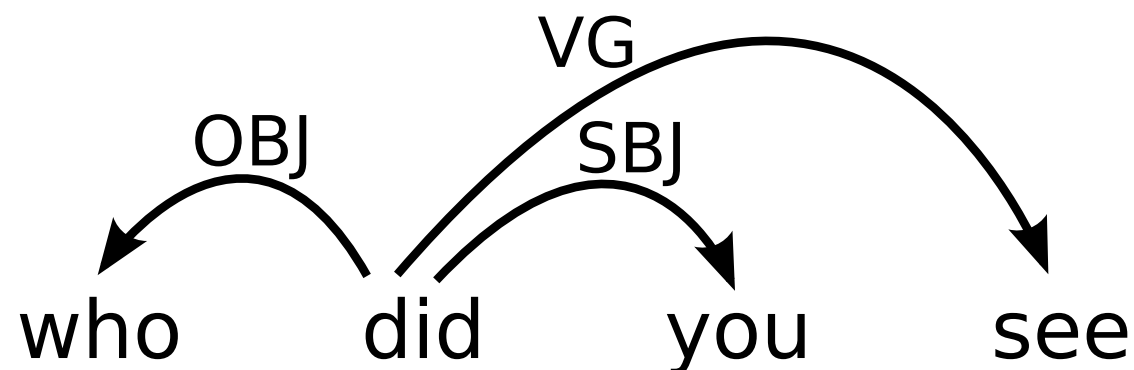
Buffer

[]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did,
did $\xrightarrow{\text{SBJ}}$ you,
did $\xrightarrow{\text{VG}}$ see }

Right-arc
VG



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did, you]_S

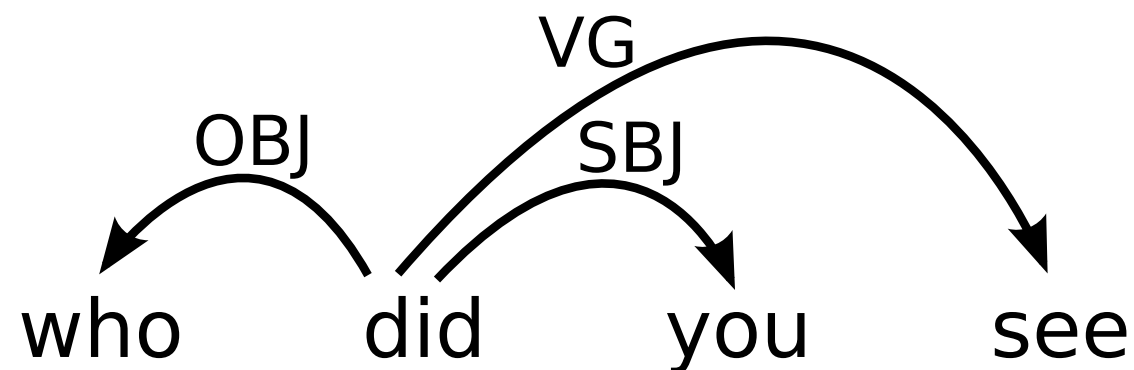
Buffer

[see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did,
did $\xrightarrow{\text{SBJ}}$ you }

Right-arc
SBJ



Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did, you]_S

Buffer

[see]_B

Arcs

{ who $\xleftarrow{\text{OBJ}}$ did,
did $\xrightarrow{\text{SBJ}}$ you }

Right-arc
SBJ

VG

Choose action w/best classifier score
100k - 1M features

Transition-Based Parsing

Arc-eager shift-reduce parsing (Nivre, 2003)

Stack

[did, you]_S

B

[s

Very fast linear-time performance
WSJ 23 (2k sentences) in 3 s

did → you }

Right-arc
SBJ

VG

Choose action w/best classifier score
100k - 1M features