

# Log-Linear Models

a.k.a. Logistic Regression,  
Maximum Entropy Models

Natural Language Processing  
CS 6120—Spring 2013  
Northeastern University

David Smith  
(some slides from Jason Eisner and Dan Klein)

*summary of half of the course (statistics)*

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  - Bayesian smoothing:  **$\max p(\theta | \text{data}) = \max p(\theta, \text{data}) = p(\theta)p(\text{data} | \theta)$**

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- **$p(\dots)$  has to capture our intuitions about the ling. data**

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- Old AI hacking technique:
  - Possible parses (or whatever) have scores.
  - Pick the one with the best score.
  - How do you define the score?
    - Completely ad hoc!
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- “Learn
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Exposé at 9

## **Probabilistic Revolution Not Really a Revolution, Critics Say**

Log-probabilities no more  
than scores in disguise

“We’re just adding stuff up  
like the old corrupt regime  
did,” admits spokesperson



uses and  
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- *Note: Today we'll use +logprob not -logprob: i.e., bigger weights are better.*

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  - Can regard any linguistic object as a collection of features (here, tree = a collection of context-free rules)
  - Weight of the object = total weight of features
  - Our weights have always been conditional log-probs ( $\leq 0$ )
    - but that is going to change in a few minutes!
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- $p(\text{semantics}) * p(\text{syntax} \mid \text{semantics}) * p(\text{morphology} \mid \text{syntax}) * p(\text{phonology} \mid \text{morphology}) * p(\text{sounds} \mid \text{phonology})$

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- Consider e.g. Naïve Bayes for text categorization:
  - Buy this supercalifragilistic Ginsu knife set for only \$39 today ...



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- Ad-hoc approach does have one advantage
- Consider e.g. Naïve Bayes for text categorization:
  - Buy this supercalifragilistic Ginsu knife set for only \$39 today ...
- Some useful features:
  - Contains Buy
  - Contains supercalifragilistic
  - Contains a dollar amount under \$100
  - Contains an imperative sentence
  - Reading level = 8<sup>th</sup> grade
  - Mentions money (use word classes and/or regexp to detect this)

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ling  
.5 | .02  
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- |      |      |   |
|------|------|---|
| spam | ling |   |
| .5   | .02  | 50% of spam has this – 25x more likely than in ling |
|      |      | ■ Contains a dollar amount under \$100              |
|      |      | 90% of spam has this – 9x more likely than in ling  |
| .9   | .1   | ■ Mentions money                                    |

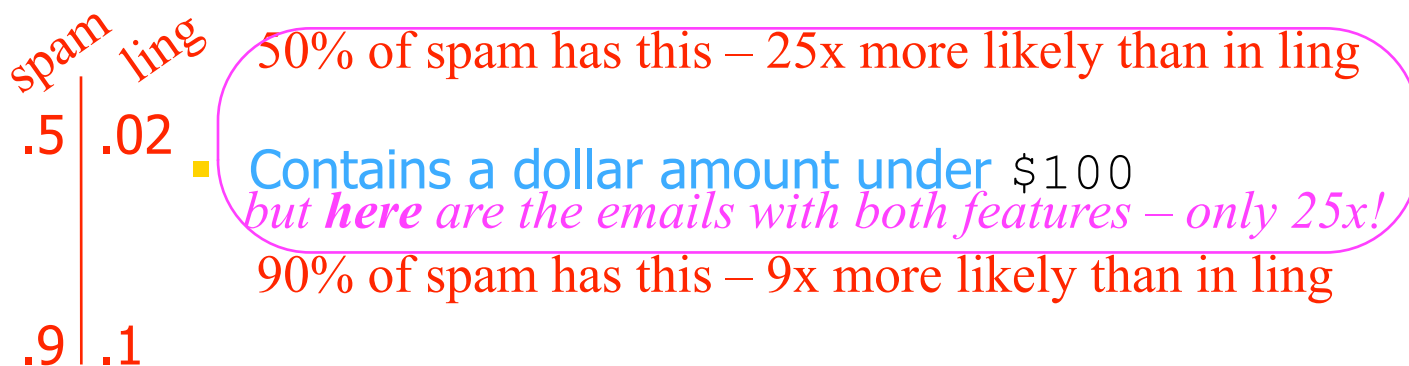
Naïve Bayes claims  $.5 * .9 = 45\%$  of spam has **both** features –  $25 * 9 = 225x$  more likely than in ling.

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Naïve Bayes claims  $.5 * .9 = 45\%$  of spam has **both** features –  $25 * 9 = 225x$  more likely than in ling.

- Contains a dollar amount under \$100  
*but here are the emails with both features – only 25x!*
- 90% of spam has this – 9x more likely than in ling
- Mentions money
- Naïve Bayes: pick C maximizing  $p(C) * p(\text{feat 1} | C) * \dots$
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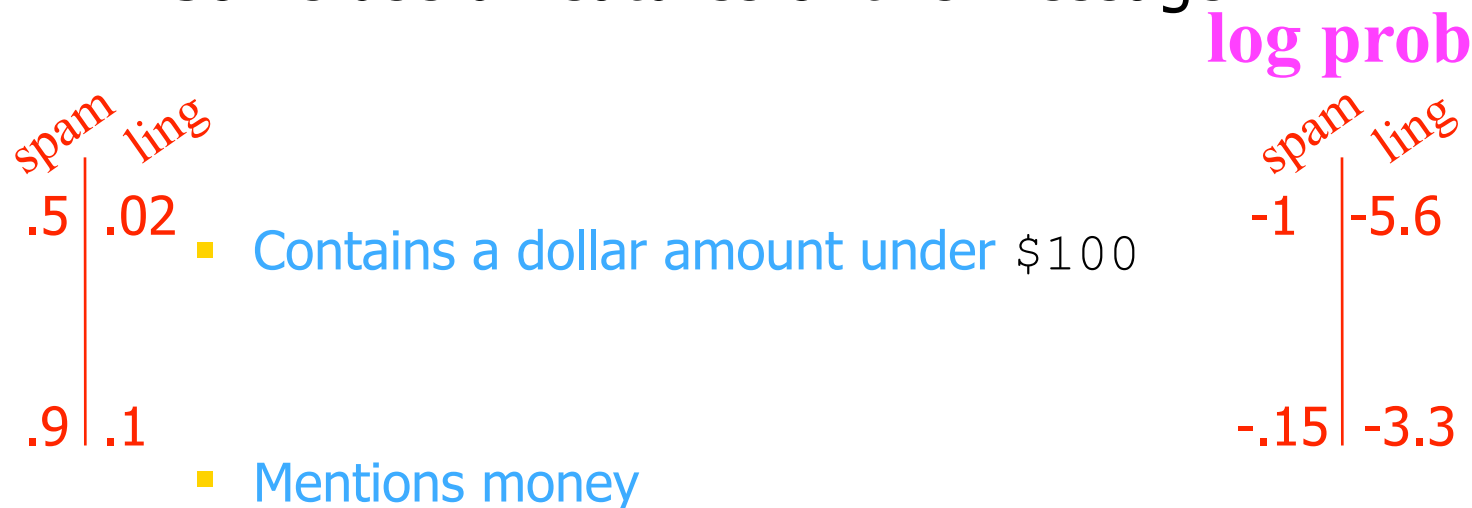
- But ad-hoc approach does have one advantage
  - **Can adjust scores to compensate for feature overlap ...**
- Some useful features of this message:

spam  
ling  
.5  
.02  
.9  
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- Contains a dollar amount under \$100
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		log prob	adjusted
spam	ling	spam	ling
.5	.02	-1	-.85
.9	.1	-.15	-.15
	Contains a dollar amount under \$100		
	Mentions money		

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# **Revolution Corrupted by Bourgeois Values**

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- But not clear how to restructure these features like that:
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  - **Contains** `supercalifragilistic`
  - **Contains a dollar amount under** `$100`
  - **Contains an imperative sentence**
  - **Reading level =** `7th grade`
  - **Mentions money** (use word classes and/or regexp to detect this)
  - ...



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- Boy, we'd like to be able to throw all that useful stuff in without worrying about feature overlap/independence.

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- 3 ■ Reading level = 7<sup>th</sup> grade
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total: 5.77

- Boy, we'd like to be able to throw all that useful stuff in without worrying about feature overlap/independence.
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total: 5.77

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- Oops, then  **$p(\text{feats} \mid \text{spam}) = \exp 5.77 = 320.5$**

# Renormalize by $1/Z$ to get a Log-Linear Model

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everything  $< 1$   
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- $p(\text{feats} \mid \text{spam}) = \exp 5.77 = 320.5$

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- $p(m \mid \text{spam}) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m)$  where

$m$  is the email message

$\lambda_i$  is weight of feature  $i$

$f_i(m) \in \{0,1\}$  according to whether  $m$  has feature  $i$

More generally, allow  $f_i(m) = \text{count or strength of feature.}$

$1/Z(\lambda)$  is a normalizing factor making  $\sum_m p(m \mid \text{spam}) = 1$

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- Why is it called "log-linear"?

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- **But:**  $p(m_j | c_j)$  for a given  $\lambda$  requires  $Z(\lambda)$ : hard!

# Attempt to Cancel out Z

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- Alas, doesn't cancel out because Z differs for the spam and ling models
- But we can fix this ...

# **So: Modify Setup a Bit**

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- Instead of having separate models

$$p(m|\text{spam}) * p(\text{spam}) \quad \text{vs.} \quad p(m|\text{ling}) * p(\text{ling})$$



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- Instead of having separate models  
 $p(m|\text{spam}) * p(\text{spam})$  vs.  $p(m|\text{ling}) * p(\text{ling})$
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- Equivalent to changing feature set to:
  - spam
  - spam and Contains Buy
  - spam and Contains supercalifragilistic
  - ...
  - ling
  - ling and Contains Buy
  - ling and Contains supercalifragilistic

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- No real change, but 2 categories now share single feature set and single value of  $Z(\lambda)$

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- spam and Contains Buy ←old spam model's weight for "contains Buy"

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← weight of this feature is  $\log p(\text{spam}) + \text{a constant}$

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  - Easy to compute now ...
  - $\prod_j p(c_j | m_j)$  is still convex, so easy to maximize too

# Generative vs. Conditional

- What is the most likely label for a given input?
- How likely is a given label for a given input?
- What is the most likely input value?
- How likely is a given input value?
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- **OUCH!**

# Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045

- Column A sums to 0.55 (“55% of all messages are in class A”)

# Maximum Entropy

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- Column A sums to 0.55
- Row `Buy` sums to 0.1 (“10% of all messages contain `Buy`”)

# Maximum Entropy

	A	B	C	D	E	F	G	H	I	J
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	0.003	0.003
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- Row Buy sums to 0.1
- (Buy, A) and (Buy, C) cells sum to 0.08 (“80% of the 10%”)

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$$\text{Entropy} = -.051 \log .051 - .0025 \log .0025 - .029 \log .029 - \dots$$

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Entropy =  $-.051 \log .051 - .0025 \log .0025 - .029 \log .029 - \dots$

Largest if probabilities are evenly distributed

# Maximum Entropy

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- $(\text{Buy}, A)$  and  $(\text{Buy}, C)$  cells sum to 0.08 (“80% of the 10%”)
- Given these constraints, fill in cells “as equally as possible”: maximize the entropy
- Now  $p(\text{Buy}, C) = .029$  and  $p(C | \text{Buy}) = .29$
- We got a compromise:  $p(C | \text{Buy}) < p(A | \text{Buy}) < .55$

# Generalizing to More Features

	A	B	C	D	E	F	G	H	...
Buy	0.051	0.003	0.029	0.003	0.003	0.003	0.003	0.003	
Other	0.499	0.045	0.045	0.045	0.045	0.045	0.045	0.045	

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$$p(m,c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m,c)$$
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- Gives another motivation for our log-linear approach.

# Log-linear form derivation

- Say we are given some **constraints** in the form of feature expectations:

$$\sum_x p(x) f_i(x) = \alpha_i$$

- In general, there may be many distributions  $p(x)$  that satisfy the constraints. Which one to pick?
- The one with maximum entropy (making fewest possible additional assumptions---Occum's Razor)
- This yields an optimization problem

$$\max H(p(x)) = - \sum_x p(x) \log p(x)$$

$$\text{Subject to } \sum_x p(x) f_i(x) = \alpha_i, \forall i \text{ and } \sum_x p(x) = 1$$

# Log-linear form derivation

- To solve the maxent problem, we use Lagrange multipliers:

$$L = - \sum_{\mathbf{x}} p(\mathbf{x}) \log p(\mathbf{x}) - \sum_i \theta_i \left( \sum_{\mathbf{x}} p(\mathbf{x}) f_i(\mathbf{x}) - \alpha_i \right) - \mu \left( \sum_{\mathbf{x}} p(\mathbf{x}) - 1 \right)$$

$$\frac{\partial L}{\partial p(\mathbf{x})} = 1 + \log p(\mathbf{x}) - \sum_i \theta_i f_i(\mathbf{x}) - \mu$$

$$p^*(\mathbf{x}) = e^{\mu-1} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

$$Z(\theta) = e^{1-\mu} = \sum_{\mathbf{x}} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

$$p(\mathbf{x}|\theta) = \frac{1}{Z(\theta)} \exp \left\{ \sum_i \theta_i f_i(\mathbf{x}) \right\}$$

- So feature constraints + maxent implies exponential family.
- Problem is convex, so solution is unique.

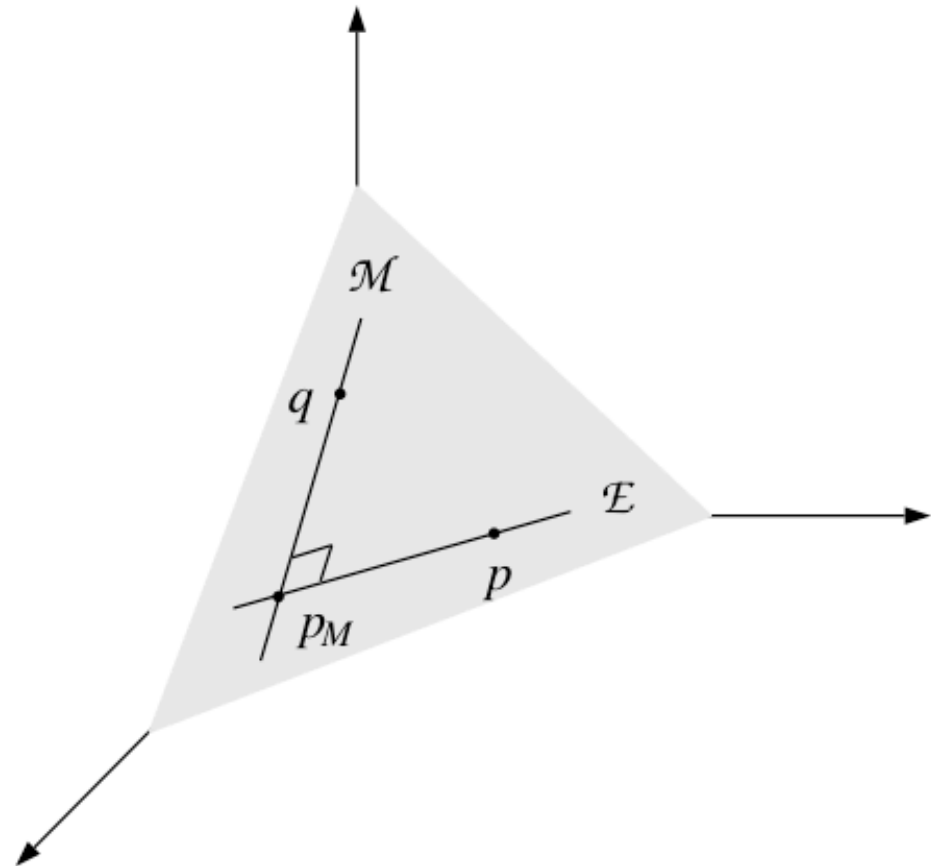
# MaxEnt = Max Likelihood

Define two submanifolds on the probability simplex  $p(\mathbf{x})$ .

The first is  $\mathcal{E}$ , the set of all exponential family distributions based on a particular set of features  $f_i(\mathbf{x})$ .

The second is  $\mathcal{M}$ , the set of all distributions that satisfy the feature expectation constraints.

They intersect at a single distribution  $p_M$ , the maxent, maximum likelihood





# Exponential Model Likelihood

- Maximum Likelihood (Conditional) Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.
- Exponential model form, for a data set (C,D):

$$\log P(C | D, \lambda) = \sum_{(c,d) \in (C,D)} \log P(c | d, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c, d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c', d)}$$





# Building a Maxent Model

- Define features (indicator functions) over data points.
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
  - Usually features are added incrementally to “target” errors.
- For any given feature weights, we want to be able to calculate:
  - Data (conditional) likelihood
  - Derivative of the likelihood wrt each feature weight
    - Use expectations of each feature according to the model
- Find the optimum feature weights (next part).



# The Likelihood Value

- The (log) conditional likelihood is a function of the iid data  $(C,D)$  and the parameters  $\lambda$ :

$$\log P(C | D, \lambda) = \log \prod_{(c,d) \in (C,D)} P(c | d, \lambda) = \sum_{(c,d) \in (C,D)} \log P(c | d, \lambda)$$

- If there aren't many values of  $c$ , it's easy to calculate:

$$\log P(C | D, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c, d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c', d)}$$

- We can separate this into two components:

$$\log P(C | D, \lambda) = \sum_{(c,d) \in (C,D)} \log \exp \sum_i \lambda_i f_i(c, d) - \sum_{(c,d) \in (C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c', d)$$

$$\log P(C | D, \lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component



# The Derivative I: Numerator

$$\begin{aligned}\frac{\partial N(\lambda)}{\partial \lambda_i} &= \frac{\partial \sum_{(c,d) \in (C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d) \in (C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} f_i(c,d)\end{aligned}$$

Derivative of the numerator is: the empirical count( $f_i, c$ )



# The Derivative II: Denominator

$$\begin{aligned}\frac{\partial M(\lambda)}{\partial \lambda_i} &= \frac{\partial \sum_{(c,d) \in (C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'', d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'', d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c', d)}{1} \frac{\partial \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c', d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'', d)} \frac{\partial \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i} \\ &= \sum_{(c,d) \in (C,D)} \sum_{c'} P(c' | d, \lambda) f_i(c', d) = \text{predicted count}(f_i, \lambda)\end{aligned}$$



# The Derivative III

$$\frac{\partial \log P(C | D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature's **predicted expectation** equals its **empirical expectation**. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if features counts are from actual data).
- Features can have high model expectations (predicted counts) either because they have large weights or because they occur with other features which have large weights.



# Summary

- We have a function to optimize:

$$\log P(C | D, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

- We know the function's derivatives:

$$\partial \log P(C | D, \lambda) / \partial \lambda_i = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

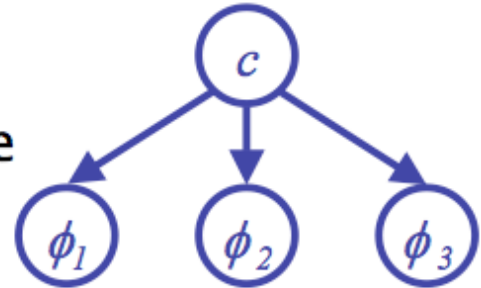
- Perfect situation for general optimization (Part II)

By gradient ascent or conjugate gradient.



# Comparison to Naïve-Bayes

- Naïve-Bayes is another tool for classification:
  - We have a bunch of random variables (data features) which we would like to use to predict another variable (the class):
  - The Naïve-Bayes likelihood over classes is:



$$P(c | d, \lambda) = \frac{P(c) \prod_i P(\phi_i | c)}{\sum_{c'} P(c') \prod_i P(\phi_i | c')} \Rightarrow \frac{\exp\left[\log P(c) + \sum_i \log P(\phi_i | c)\right]}{\sum_{c'} \exp\left[\log P(c') + \sum_i \log P(\phi_i | c')\right]}$$

Naïve-Bayes is just an exponential model.

$$\Rightarrow \frac{\exp\left[\sum_i \lambda_{ic} f_{ic}(d, c)\right]}{\sum_{c'} \exp\left[\sum_i \lambda_{ic'} f_{ic'}(d, c')\right]}$$





# Comparison to Naïve-Bayes

- The primary differences between Naïve-Bayes and maxent models are:

## Naïve-Bayes

Trained to maximize joint likelihood of data and classes.

Features assumed to supply independent evidence.

Feature weights can be set independently.

Features must be of the conjunctive  $\Phi(d) \wedge c = c_i$  form.

## Maxent

Trained to maximize the conditional likelihood of classes.

Features weights take feature dependence into account.

Feature weights must be mutually estimated.

Features need not be of the conjunctive form (but usually are).



# Overfitting

- If we have too many features, we can choose weights to model the training data perfectly.
- If we have a feature that only appears in spam training, not ling training, it will get weight  $\infty$  to maximize  $p(\text{spam} \mid \text{feature})$  at 1.
- These behaviors overfit the training data.
- Will probably do poorly on test data.

# Solutions to Overfitting

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  - Add one at a time, always greedily picking the one that most improves performance on held-out data.
3. Smooth the observed feature counts.
4. Smooth the weights by using a prior.
  - $\max p(\lambda|\text{data}) = \max p(\lambda, \text{data}) = p(\lambda)p(\text{data}|\lambda)$
  - decree  $p(\lambda)$  to be high when most weights close to 0



# Smoothing: Priors (MAP)

- What if we had a prior expectation that parameter values wouldn't be very large?
- We could then balance evidence suggesting large parameters (or infinite) against our prior.
- The evidence would never totally defeat the prior, and parameters would be smoothed (and kept finite!).
- We can do this explicitly by changing the optimization objective to maximum posterior likelihood:

$$\log P(C, \lambda | D) = \log P(\lambda) + \log P(C | D, \lambda)$$

Posterior

Prior

Evidence

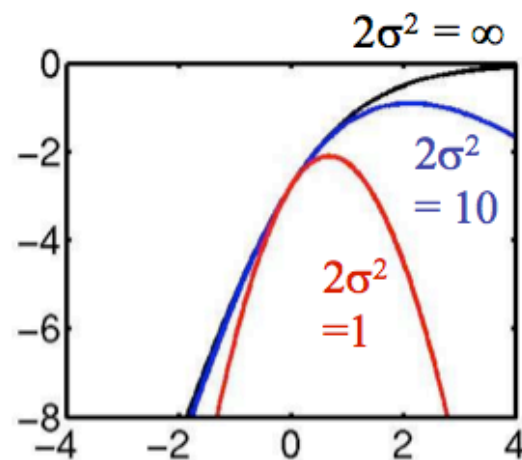


# Smoothing: Priors

- Gaussian, or quadratic, priors:
  - Intuition: parameters shouldn't be large.
  - Formalization: prior expectation that each parameter will be distributed according to a gaussian with mean  $\mu$  and variance  $\sigma^2$ .

$$P(\lambda_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(\lambda_i - \mu_i)^2}{2\sigma_i^2}\right)$$

- Penalizes parameters for drifting to far from their mean prior value (usually  $\mu=0$ ).
- $2\sigma^2=1$  works surprisingly well.



They don't even capitalize my name anymore!





# Recipe for a Conditional MaxEnt Classifier

1. Gather *constraints* from training data:

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero.
3. Classify training data with current parameters. Calculate *expectations*.

$$E_{\Theta}[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_{\Theta}(y' | x_j) f_{iy}(x_j, y')$$

4. Gradient is  $\tilde{E}[f_{iy}] - E_{\Theta}[f_{iy}]$
5. Take a step in the direction of the gradient
6. Until convergence, return to step 3.