2D Transformations

also today: how to read a research paper

CS 4300/5310

Computer Graphics

ANNOUNCEMENTS

Deadlines

- 2D Project Proposal: January 22nd
  - Submit one per group
- 2D Project main deadline: February 5th
- Reading Response: January 22nd
  - It's short! Don't worry!

Global Game Jam!

Go to www.northeastern.edu/games/ggj and register now!

Game Demo Day

- Submission deadline: March 29
- Event: April 19

- Campus-wide demo event for games developed during 2012-2013

- Industry judges!

- Great to add to resume

MATRIX MATH: QUICK REVIEW

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

m x n (3 x 3) (rows x columns)

Matrix

- Square matrix

- Diagonal Matrix

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

- Zero Matrix

Matrix

- Square matrix
  - m=n
- Diagonal Matrix

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

- Zero Matrix

Matrix

- Square matrix
  - m=n
- Diagonal Matrix
  - $a_{ij} = 0$ if $i \neq j$
- Zero Matrix

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

Matrix

- Square matrix
  - m=n
- Diagonal Matrix
  - $a_{ij} = 0$ if $i \neq j$
- Zero Matrix
  - all $a_{ij}=0$

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}$$

Matrix addition

$$A + B = C$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 5 \\ 7 & 7 \end{pmatrix} + \begin{pmatrix} 4 & 6 \\ 9 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 11 \\ 16 & 12 \end{pmatrix}$$

Constraint: $m_A = m_B$ and $n_A = n_B$

Matrix Subtraction

$$A - B = C$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 5 \\ 7 & 7 \end{pmatrix} - \begin{pmatrix} 4 & 6 \\ 9 & 5 \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ -2 & 2 \end{pmatrix}$$

Constraint: $m_A = m_B$ and $n_A = n_B$

Matrix Scalar Multiplication

$$bA = C$$

$$b * \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} b * a_{11} & b * a_{12} \\ b * a_{21} & b * a_{22} \end{pmatrix}$$

$$2 * \begin{pmatrix} 2 & 5 \\ 7 & 7 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 14 & 14 \end{pmatrix}$$

Matrix Multiplication

$$AB = C, c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} (m_A \times n_B)$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} * \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{pmatrix} (3x2matrix)$$

$$\begin{pmatrix} 3 & 0 & 2 \\ 2 & 1 & 1 \\ 2 & 0 & 2 \end{pmatrix} * \begin{pmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 5 & 13 \\ 4 & 8 \end{pmatrix}$$

Constraint: $n_A = m_B$

Matrix Multiplication

- Distributive: A(B+C) = AB + AC
- Associative: (AB)C = A(BC)
- Not commutative:

  AB is not equal to BA

Matrix Transpose

- A^T, a_{ij} becomes a_{ji}

$$A = \begin{pmatrix} 0 & 1 \\ 4 & 2 \\ 6 & 1 \end{pmatrix} (3 \times 2)$$

$$A^T = \begin{pmatrix} 0 & 4 & 6 \\ 1 & 2 & 1 \end{pmatrix} (2 \times 3)$$

2D TRANSFORMATIONS

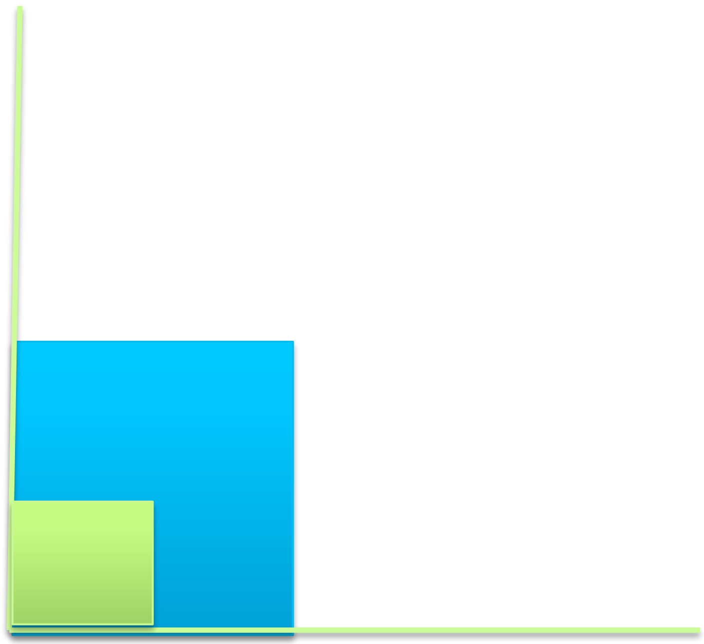
Transforms

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Transformation Matrix

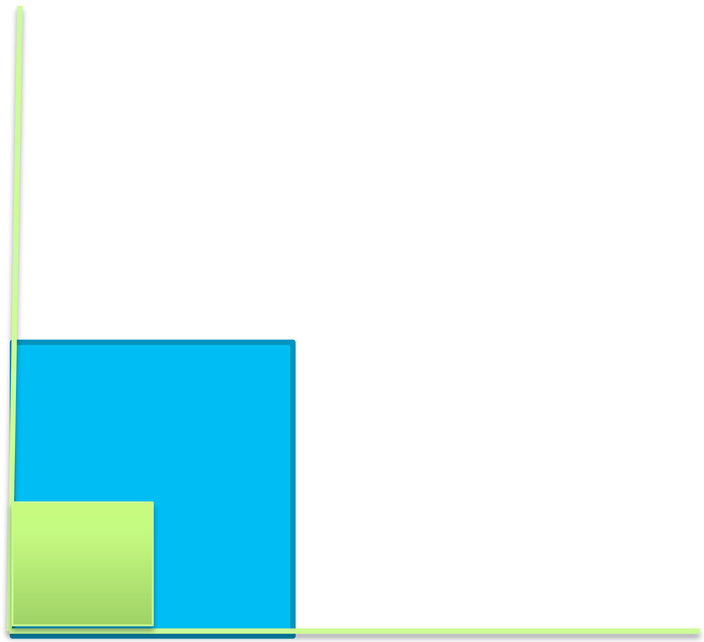
Transformation: Scaling

$$\begin{pmatrix} \quad ???? \quad \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Transformation: Scaling

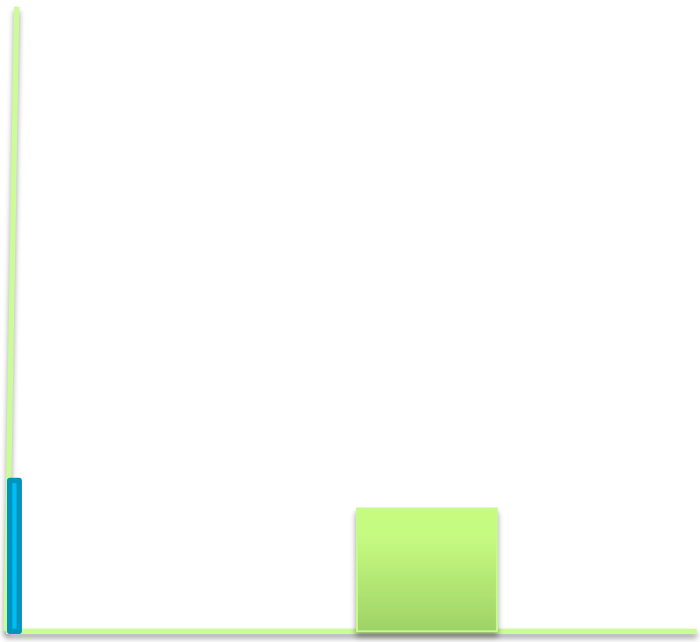
$$\begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Transformation: Projection

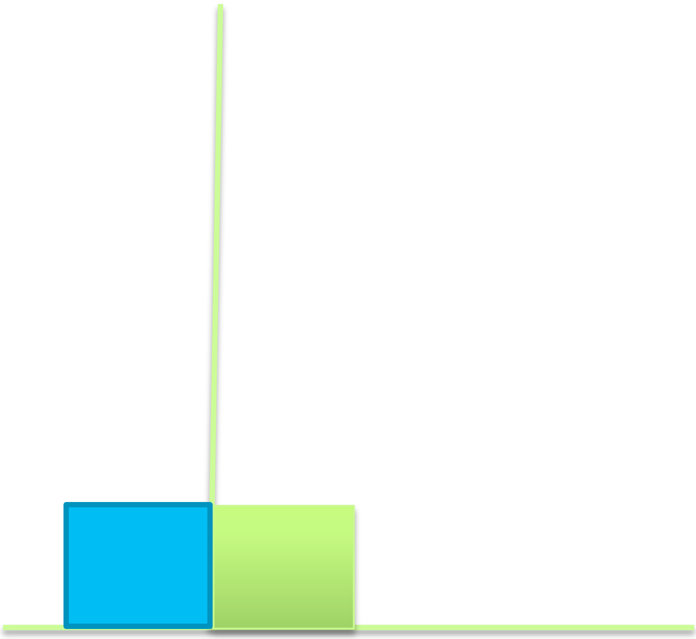
$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$



Transformation: Reflection

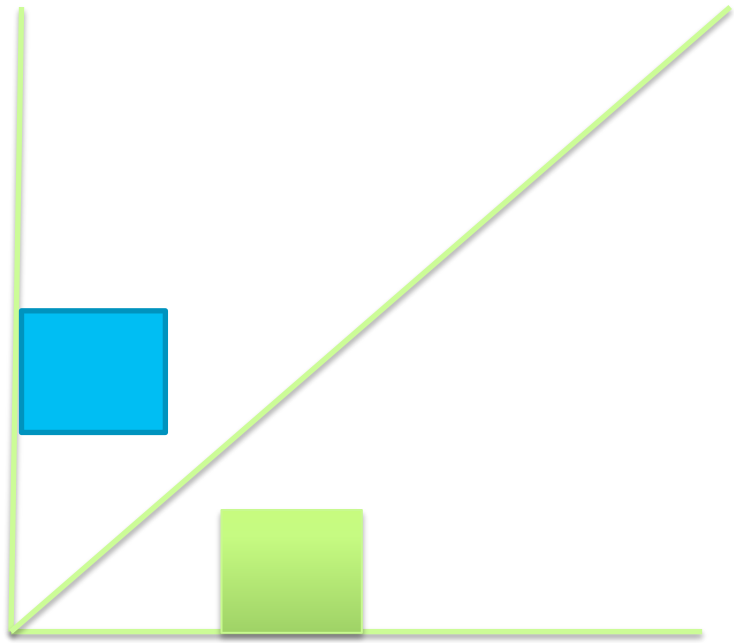
$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$



Transformation: Reflection

- Reflection over y=x line

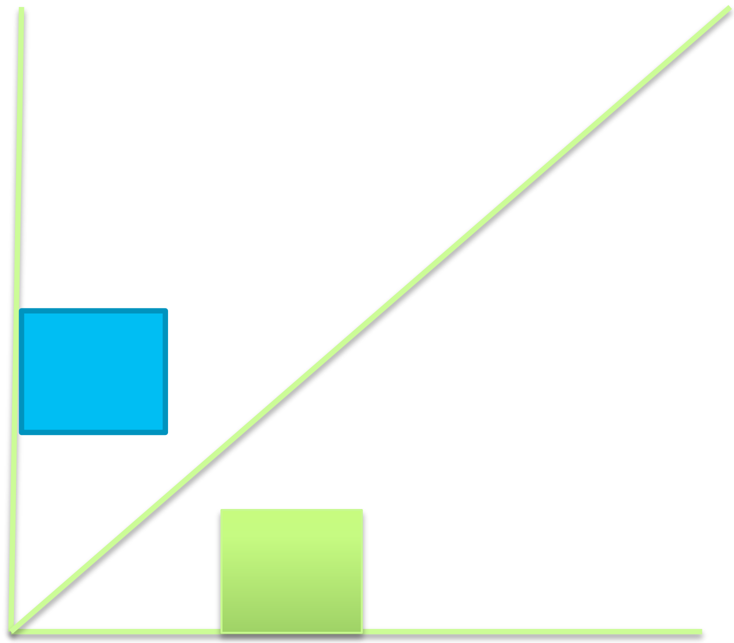
$$\begin{pmatrix} ???? \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$



Transformation: Reflection

- Reflection over y=x line

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$



Transformation: Shearing

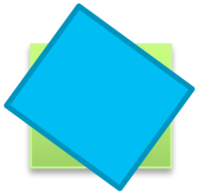
$$x - axis \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ y \end{pmatrix}$$

$$y - axis \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x + y \end{pmatrix}$$

Transformation: Rotation

$$clockwise\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$counter-clockwise\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$



Transformation: Composition

- Order is very important! Read right-left.

- What does this do?

$$\begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

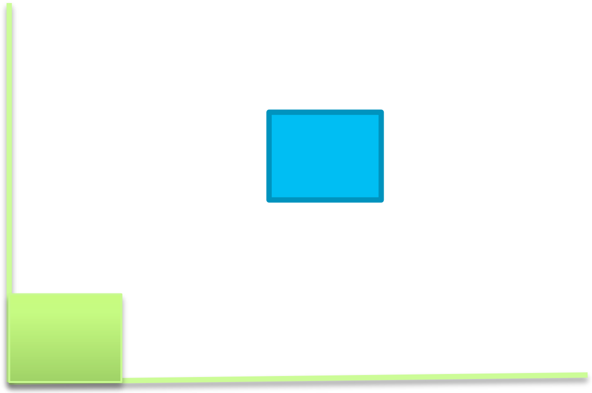
Translation

- Rotation, scaling, shearing, etc. are **linear** transformations

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix}$$

- But we want:

$$\begin{pmatrix} x + x_t \\ y + y_t \end{pmatrix}$$



Changing our representation...

- Represent the point x, y by vector

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogenous Coordinates

- Thus:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + x_t \\ y + y_t \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

- Vectors vs. Points:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$ Points

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$ Vectors

- Thus:

$$\begin{pmatrix} x' \\ y' \\ 0 \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & x_t \\ a_{21} & a_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \\ 0 \end{bmatrix}$$

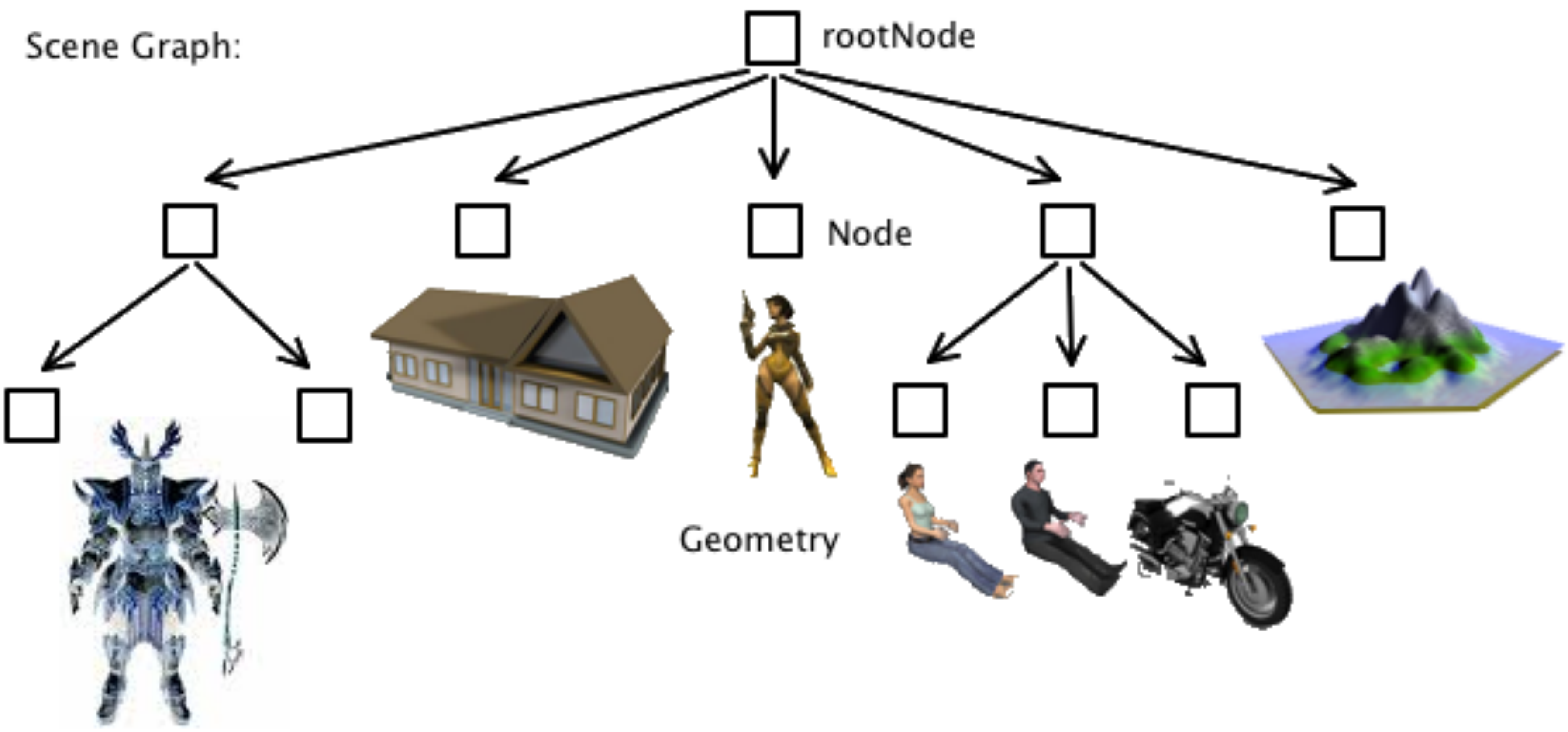
Transformation Matrix

- Use same matrix for scaling, rotation, etc.

- Example:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} a_{11} & 0 & x_t \\ 0 & a_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}x + x_t \\ a_{22}y + y_t \\ 1 \end{bmatrix}$$

Scene Graphs



HOW TO READ A RESEARCH PAPER

Step One: Authors are Human

- Authors are people like you!
- Read **critically** – don't assume they are correct
- Research papers are peer-reviewed

Step Two: Structure of a Paper

- Introduction
- Related Work
- Method/Approach
- Evaluation
- Conclusions
- References

Step Three: Reading Critically

- Introduction
  - What problem are they solving?
  - Does it make sense to solve it?
  - Is there a better problem?
  - Is the problem oversimplified?
  - Do you agree with their arguments?

Step Three: Reading Critically

- Related Work
  - Is the related work actually related?
  - Are they comparing appropriately?
  - Are they describing the other work fairly?

Step Three: Reading Critically

- Method/Approach
  - Is there enough detail?
  - Does this approach make sense?
  - Why are they making these decisions?
  - What assumptions are being made?
  - Could this be improved?

- But what if I really don’t understand?
  - Flag concepts you don’t understand
  - Go to the references
  - Ask questions on piazza/in class

Step Three: Reading Critically

- Evaluation
  - Are you convinced by the results?
  - Are they testing their approach appropriately?
  - What further information do you wish you had?

Step Three: Reading Critically

- Conclusion
  - Are the authors drawing the right conclusion?
  - Does the future work make sense?
  - Do the authors make their claims about what they’ve done clear?

Step Four: Reading *Creatively*

- What would I do differently?
- How would I extend the work presented?
- How would I evaluate it differently?
- What do I think the impact of this could be on other areas I'm interested in?
- If I were to start working on a project in this area, what's the first thing I would do next?

Reading Responses

- 1 page
- **Brief** summary of the paper
  - Aim for no more than 2-3 sentences
  - What problem were they solving? What did they learn?
- The rest is your opinion
  - What did/didn't you like in the paper?
  - What questions did you ask yourself when reading?