

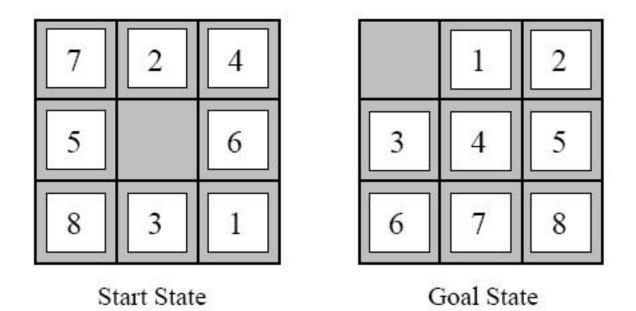
# Search

CS 4100/5100 Foundations of Al

#### Framework for Search Problems

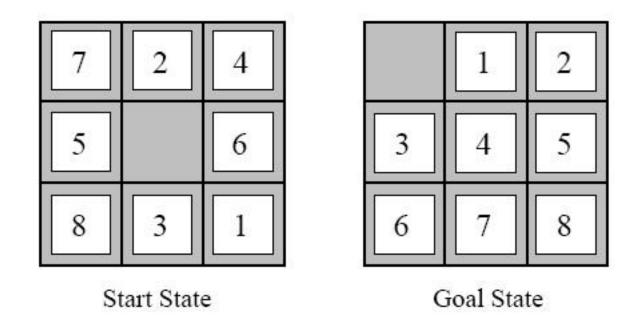
- Problem formulation
  - A set of discrete states (usually finite, usually huge)
  - Start state
  - Subset of states designated as goals (usually)
  - A set of actions the agent can perform
- Objective
  - Find a sequence of operators that lead to the goal state
    - Optimal or least-cost would be nice!

## **Example: 8 Puzzle**



States
Actions
Goal test
Path cost

## **Example: 8 Puzzle**



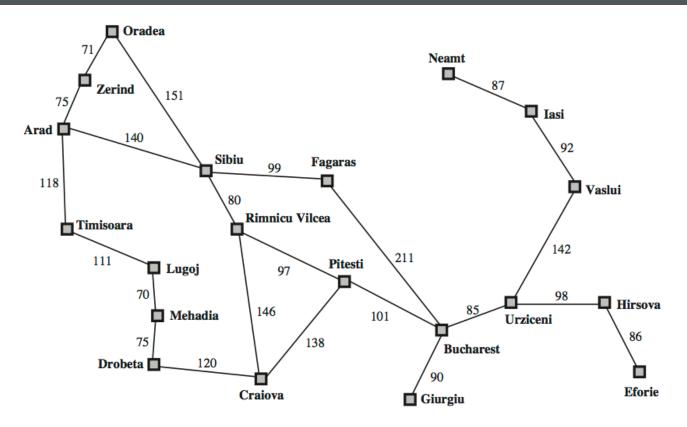
**States** locations of tiles + blanks

Actions move blank left, right, up, down

Goal test = goal state (given)

Path cost 1 per move

## **Example: Pathfinding**



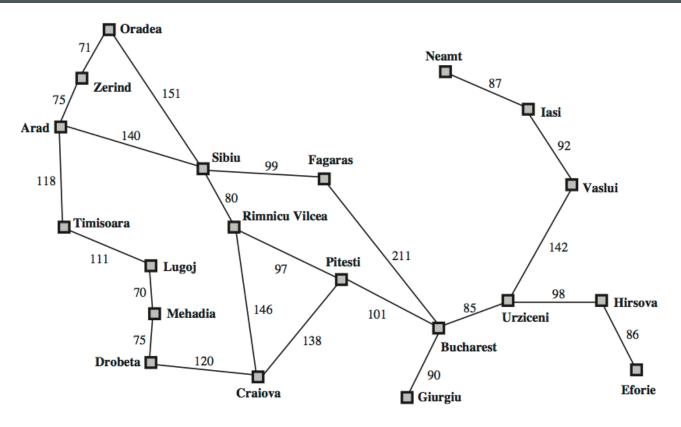
**States** 

**Actions** 

**Goal test** 

**Path cost** 

## **Example: Pathfinding**



**States** position of agent

**Actions** moving along path

Goal test at specified goal state

Path cost length of path between cities

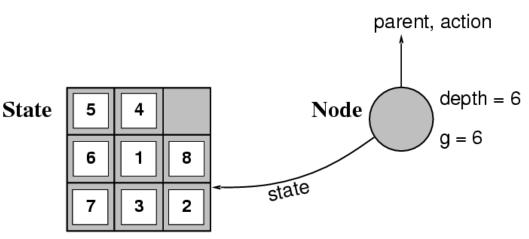
#### **Search Tree**

- State space forms a tree structure
  - Root = start state
  - Each node represents a state
  - Actions are branches, children are all possible next-states

 Search involves expanding a frontier of potential next states

### Implementation: States vs. Nodes

- A state is a configuration of the world
- A node is a data structure constituting the search tree, including:
  - State
  - Parent node
  - Action taken (edge)
  - Path cost
  - Depth
- More than one node can correspond to a single world state!



## **Evaluating a Search Strategy**

- Search strategy is defined by picking the order of node expansion
- Evaluation criteria
  - Completeness
  - Time complexity
  - Space Complexity
  - Optimality
- Time and space complexity measured in
  - *b* − maximum branching factor for the search tree
  - d − depth of least-cost solution
  - *m* − maximum depth of the state space

#### **Kinds of Search**

Uninformed

Informed

Optimization

Adversarial

## **UNINFORMED SEARCH**

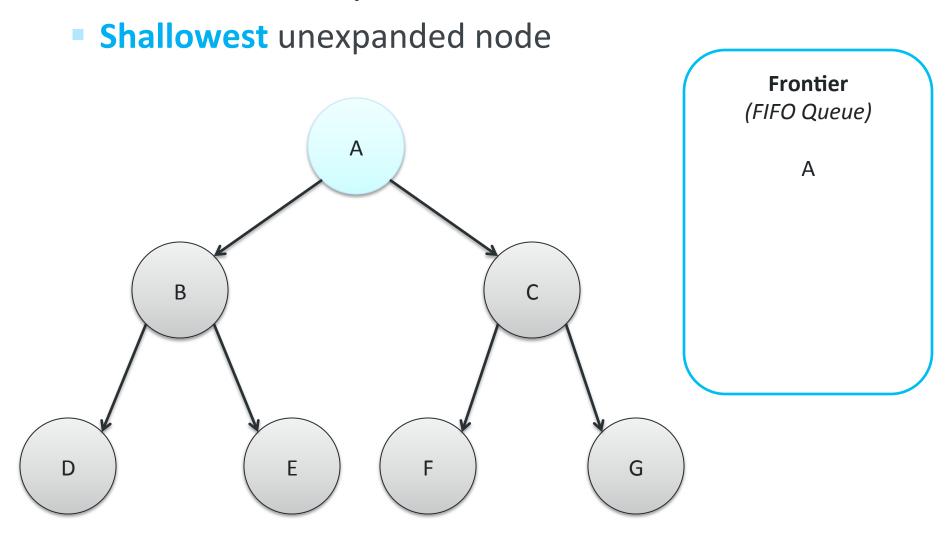
## **Uninformed Search Strategies**

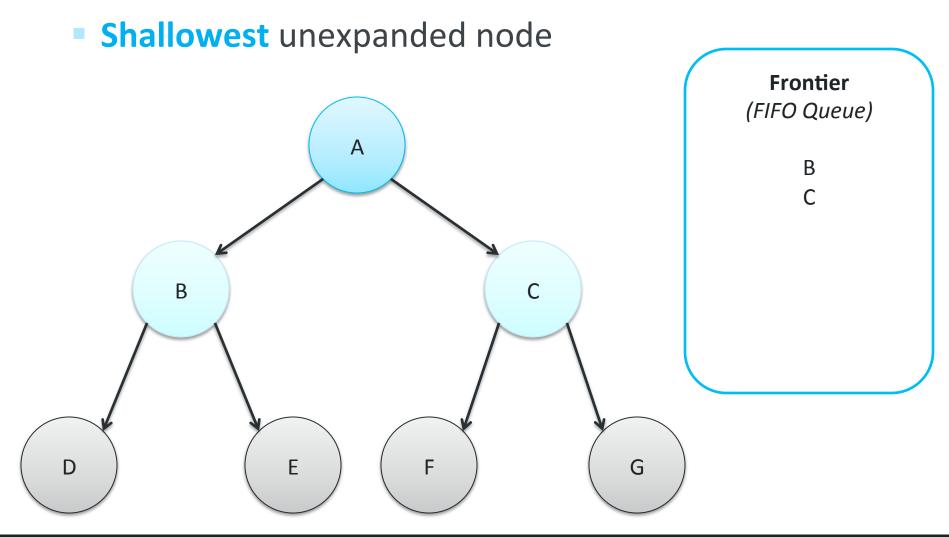
Breadth-First Search

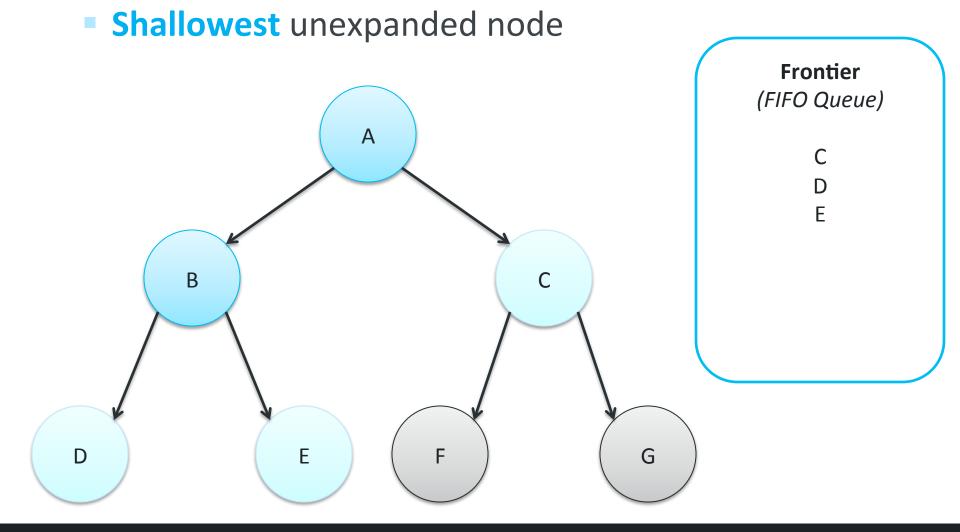
Depth-First Search

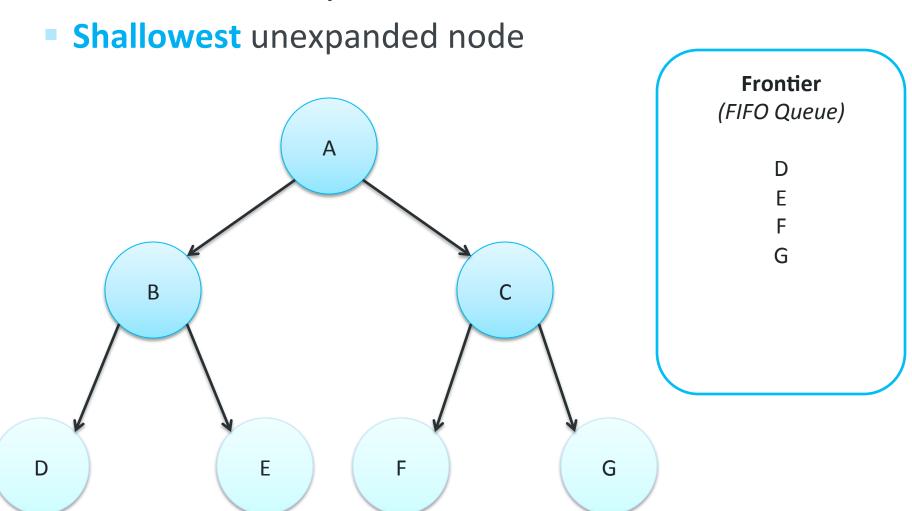
Limited Depth Search

Iterative Deepening









- Complete?
- Optimal?
- Time cost?
- Space cost?

- Complete?
  - Yes! As long as b is finite
- Optimal?
- Time cost?
- Space cost?

- Complete?
  - Yes! As long as b is finite
- Optimal?
  - Yes! Assuming?
- Time cost?
- Space cost?

- Complete?
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- Optimal?
  - Yes! Assuming?
- Time cost?
  - O(b<sup>d</sup>)
- Space cost?

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  - Yes! Assuming?
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- Space cost?
  - O(b<sup>d</sup>)

Suppose b=10, a node uses 1000 bytes, 1m nodes/sec can be generated

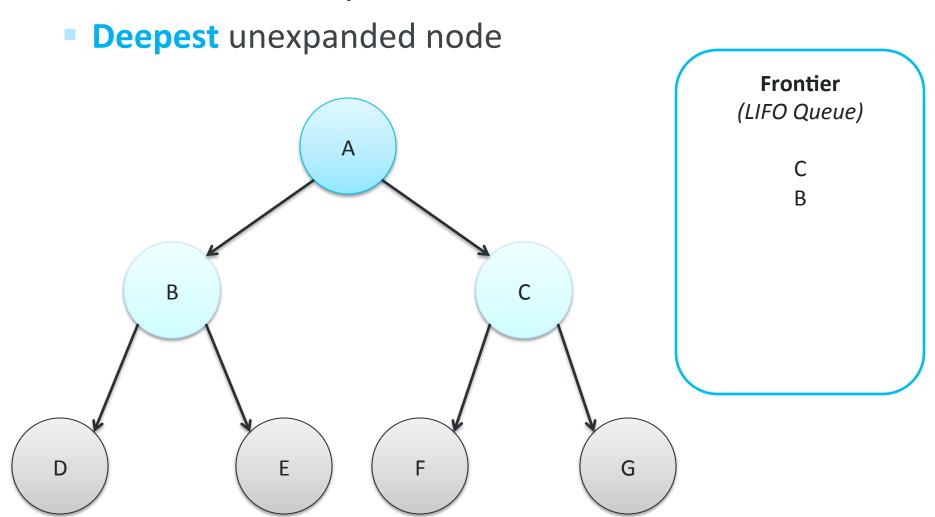
Depth 10: ~3 hours, ~10 terabytes

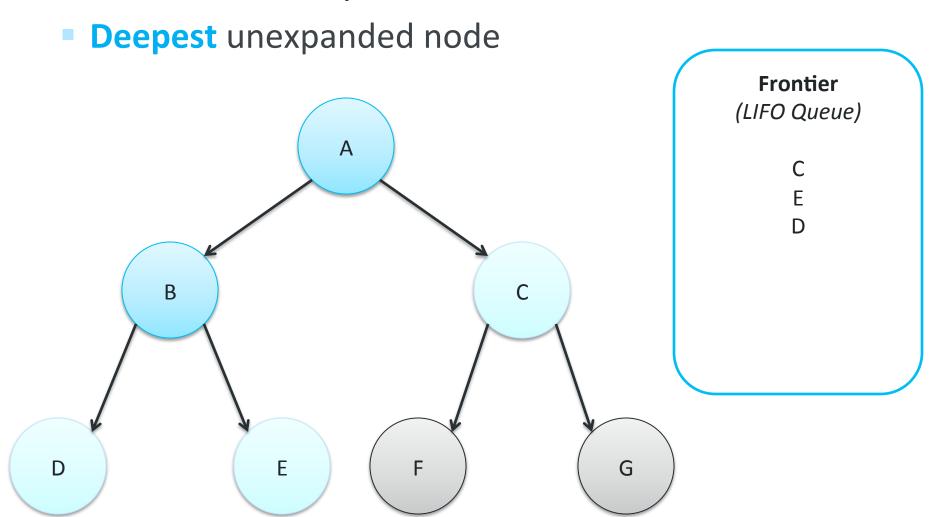
Depth 12: ~13 days, ~1 petabyte

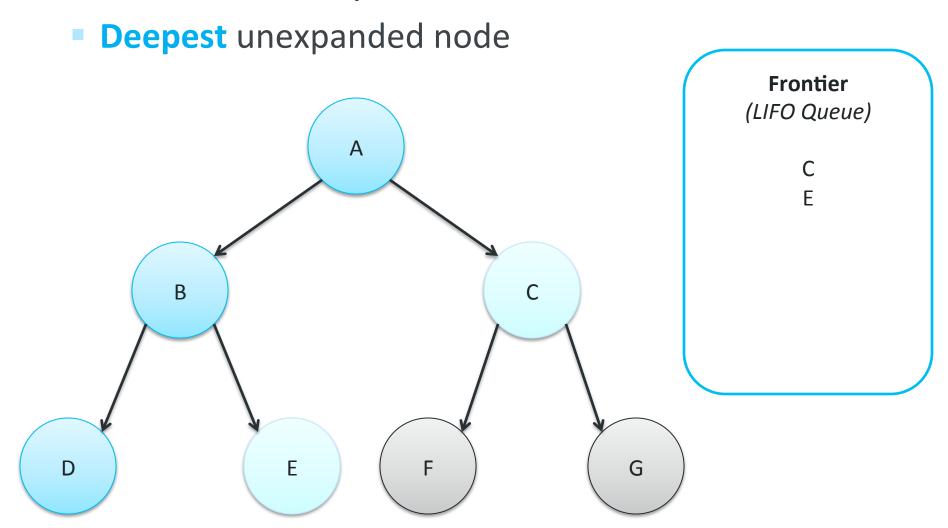
Depth 14: ~3.5 years, ~99 petabytes

Order of node expansion?

**Deepest** unexpanded node **Frontier** (LIFO Queue) Α В G





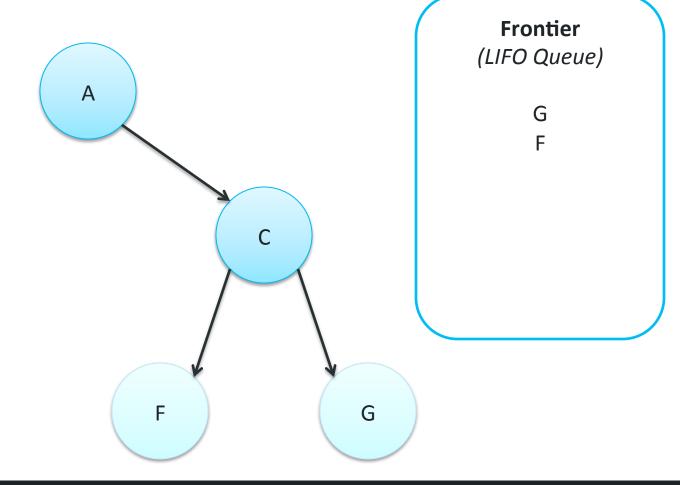


Order of node expansion?

**Deepest** unexpanded node **Frontier** (LIFO Queue) Α В Ε G

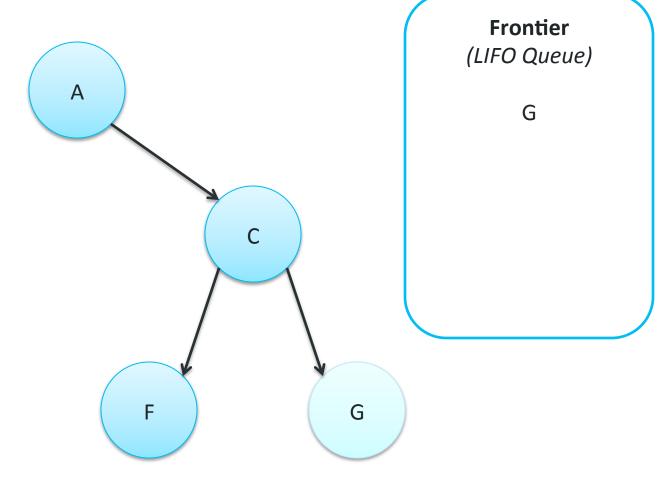
Order of node expansion?

Deepest unexpanded node



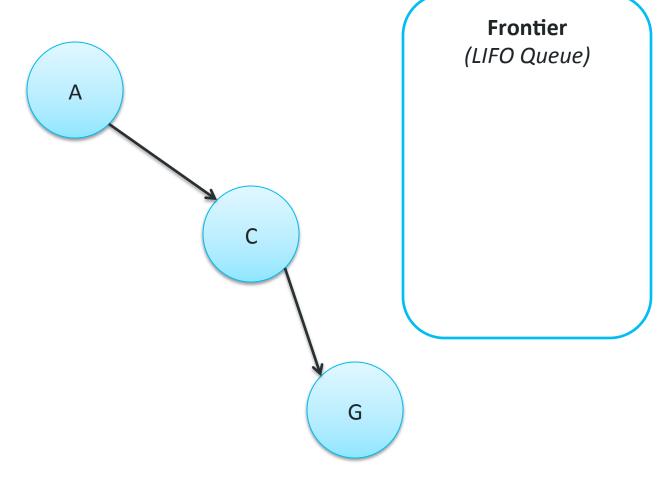
Order of node expansion?

Deepest unexpanded node



Order of node expansion?

Deepest unexpanded node



- Complete?
- Optimal?
- Time cost?
- Space cost?

- Complete?
  - No! Why? Unless?
- Optimal?
- Time cost?
- Space cost?

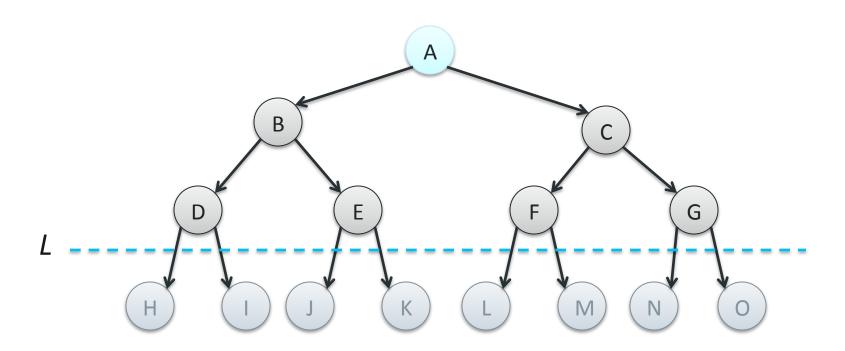
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- Optimal?
  - No! Why?
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  - O(b<sup>m</sup>)
- Space cost?
  - O(bm)

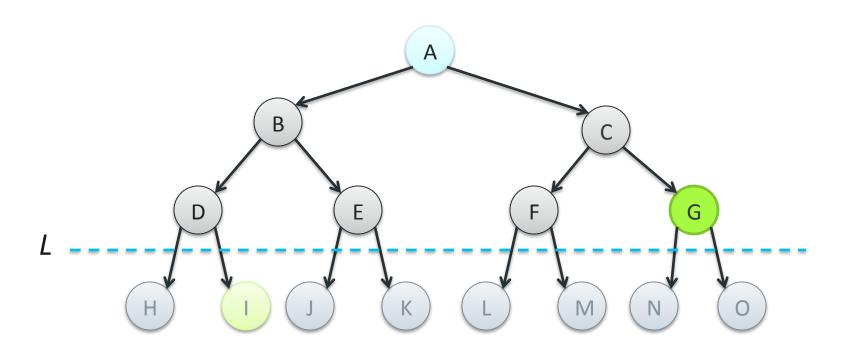
## **Depth-Limited Search**

 Depth first search, but don't go past a pre-set limit (L) while searching



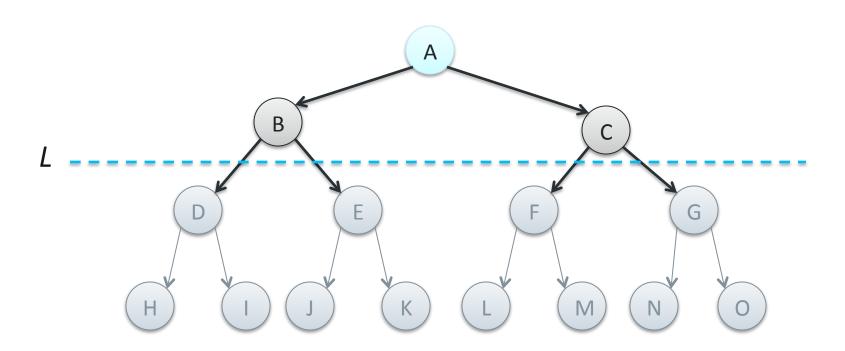
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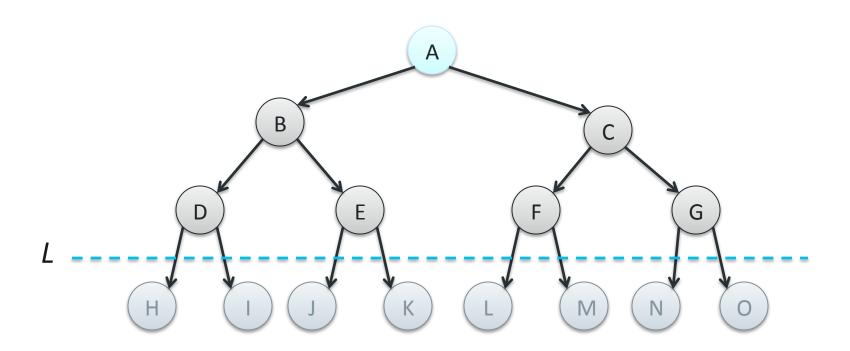
### **Iterative Deepening**

 Limited-depth search, progressively lowering the limit



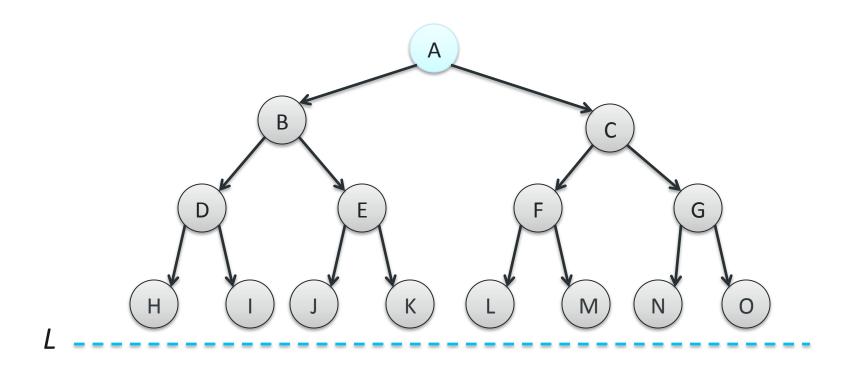
### **Iterative Deepening**

 Limited-depth search, progressively lowering the limit



# **Iterative Deepening**

 Limited-depth search, progressively lowering the limit



# **Limited Depth Search**

Combines benefits of depth- and breadth-first

- Number of nodes generated:
  - $(d+1)b^0 + (d)b^1 + (d-1)b^2 + ... + (2)b^{d-1} + (1)b^d = O(b^d)$

### Limited Depth Search

Combines benefits of depth- and breadth-first

- Number of nodes generated:
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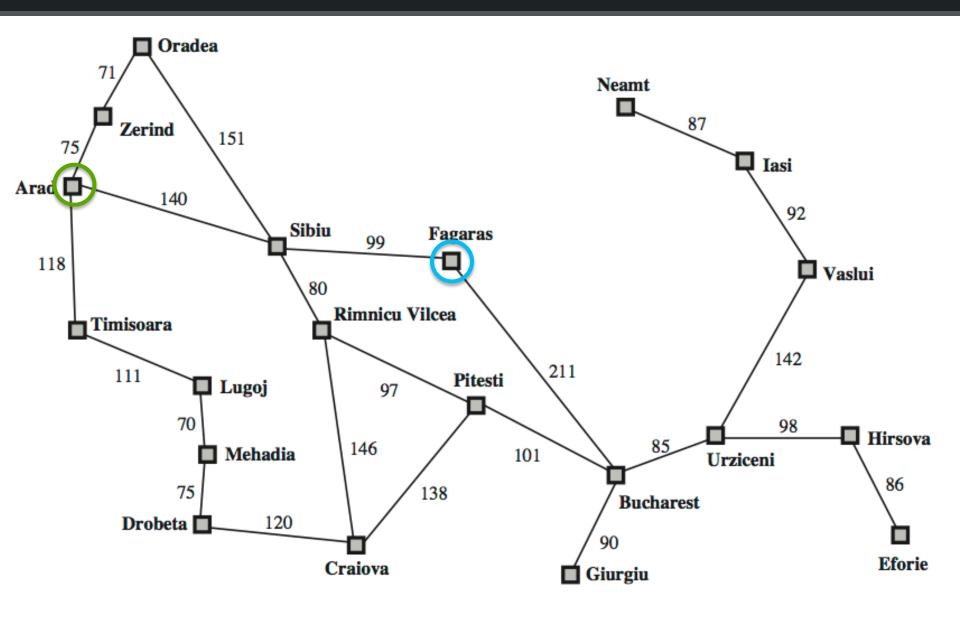
#### **Iterative Deepening Search**

- Combines benefits of depth- and breadth-first
- Number of nodes generated:
  - $(d+1)b^0 + (d)b^1 + (d-1)b^2 + ... + (2)b^{d-1} + (1)b^d = O(b^d)$
- Spatial complexity: O(bd)
- Comparison to breadth-first search
  - Assume b = 10, d = 5
  - N(IDS) = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456
  - N(BFS) = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111
  - Overhead: (123,456 111,111)/111,111 = 11%

# **Summary**

Search Type	Pros	Cons
Breadth-first search	Good for problems with expected shallow solutions	Takes up too much space
Depth-first search	Efficient space usage Good when lots of solutions	Gets stuck in loops (w/o modification) Can go too deep
Iterative deepening	Efficient space usage Does not get stuck on a bad path	Overhead of additional searching

#### **In-Class Exercise**



# **INFORMED SEARCH**

#### **Informed Search**

 Use problem-specific knowledge beyond the problem definition to find solutions more efficiently

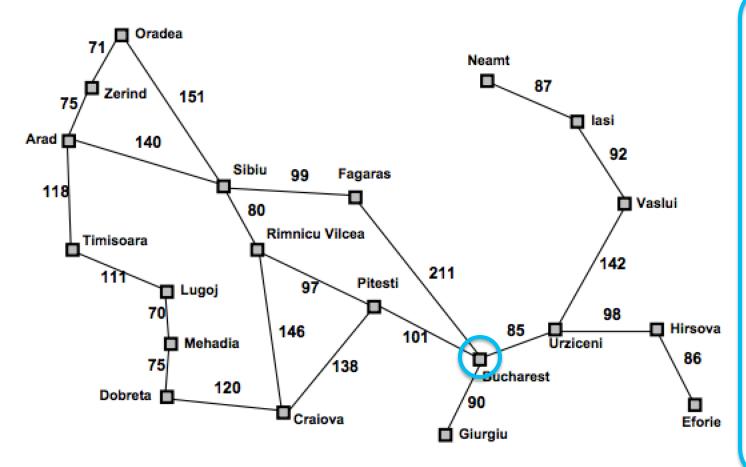
Heuristic: which node should be expanded next?

#### **Heuristic Search**

- Use an evaluation function f(n) for each node
  - Estimate of "desirability"
  - Includes a heuristic function, h(n)

Expand the most desirable unexpanded node

#### **Example: Heuristic for Path-Finding**

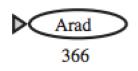


Straight-line distant to Bucharest	ce
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

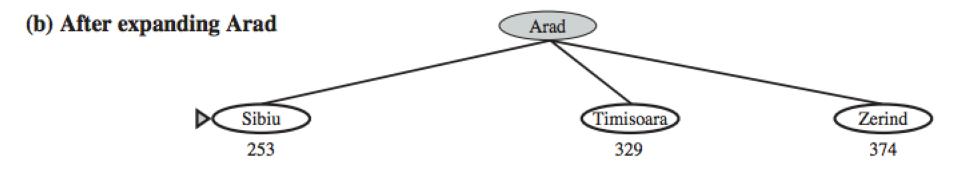
- Evaluation function f(n) = h(n)
  - E.g. h(n) = straight line distance from n to goal

- Search strategy
  - Expand the node that appears closest to the goal

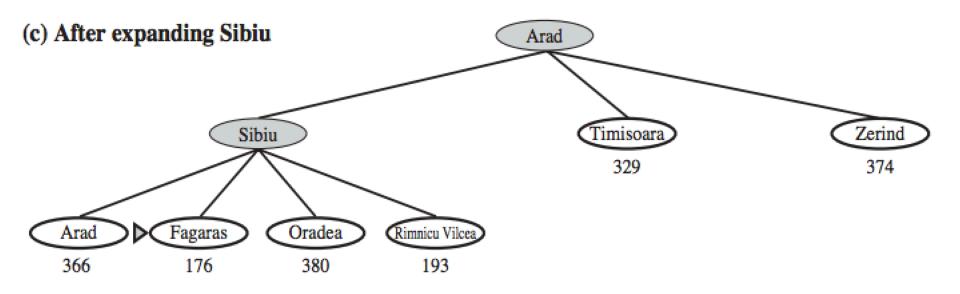
(a) The initial state



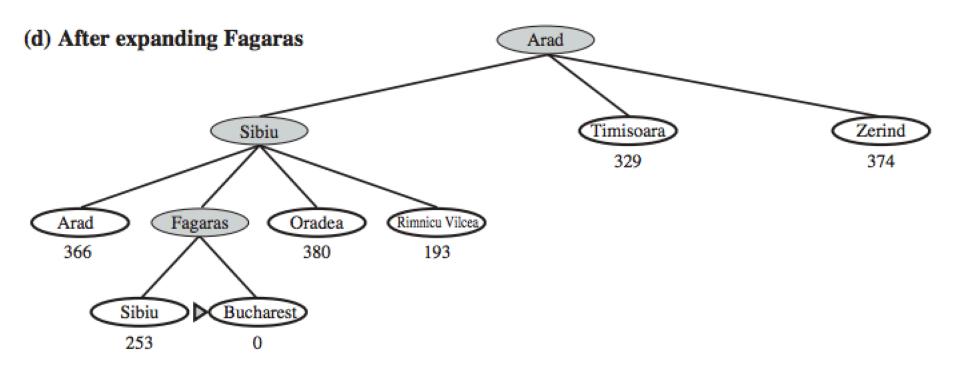
Arad	366	Fagaras	178	Mehadia	241	Sibiu	253
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- Complete?
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  - O(b<sup>m</sup>), but improves with great heuristic
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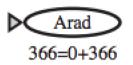
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- Space?
  - O(b<sup>m</sup>), keeps all nodes in memory

#### A\* Search

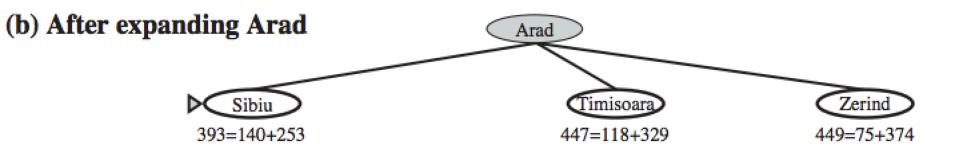
Avoid expanding paths that are already expensive

- Evaluation function: f(n) = g(n) + h(n)
  - g(n) = cost of path so far
  - h(n) = estimated cost from n to goal
  - f(n) = estimated total cost of path through n to goal

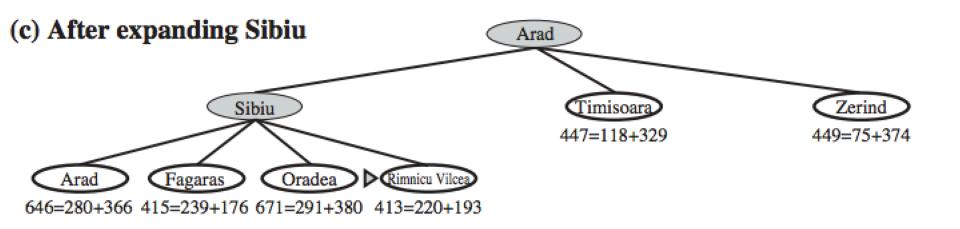
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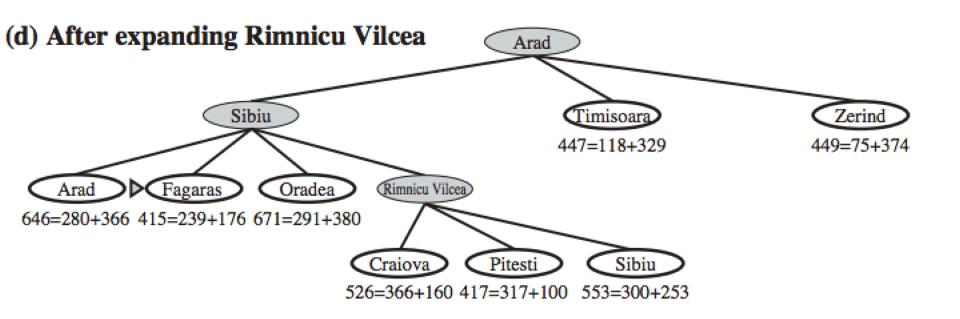
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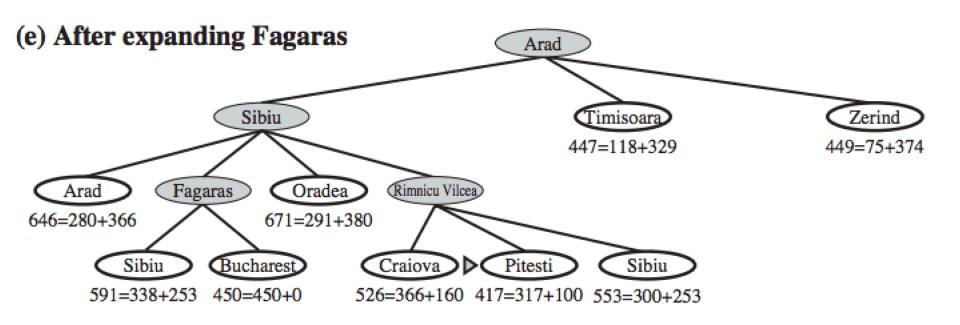
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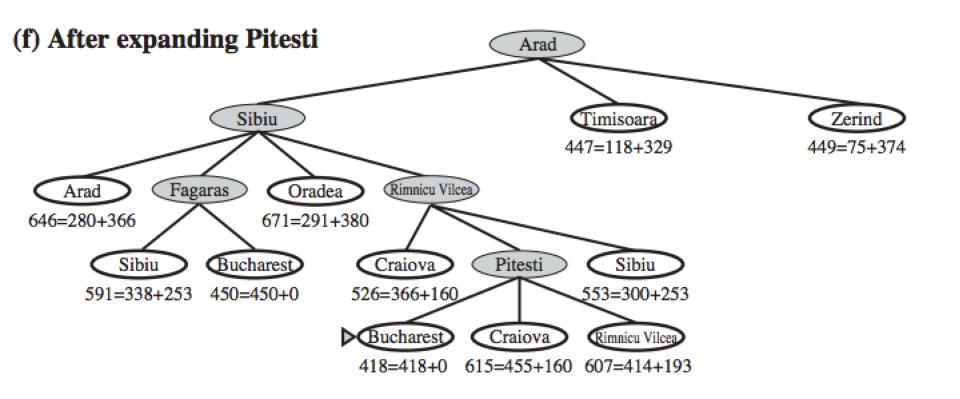
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#### A\* Search

- Complete?
  - Yes
- Optimal?
  - Yes!
- Time?
  - Exponential
- Space?
  - Keeps all nodes in memory

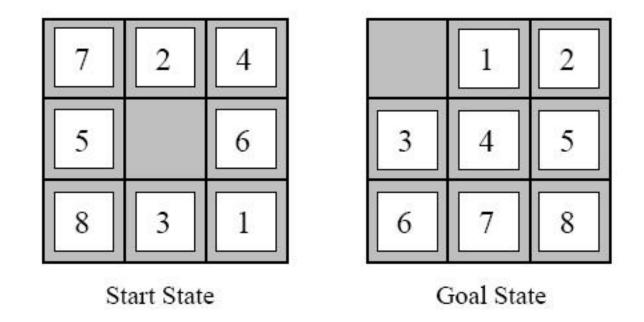
#### **Admissible Heuristics**

- A heuristic is admissible if, for every node n, h(n) ≤ h\*(n)
  - h\*(n): true cost to reach the goal from state n

 Example: path-finding heuristic never overestimates actual road distance

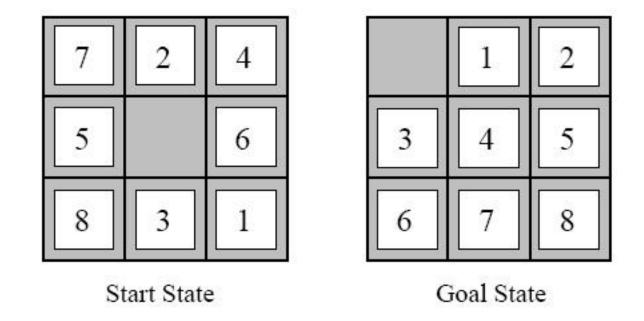
- Generate from a relaxed problem
  - Fewer restrictions than original problem

### **Class Activity**



What are some heuristics for the 8 puzzle?

### **Class Activity**



 Sketch out first few steps of a\* search given the total number of misplaced tiles heuristic

#### **OPTIMIZATION**

### **Optimization Problems**

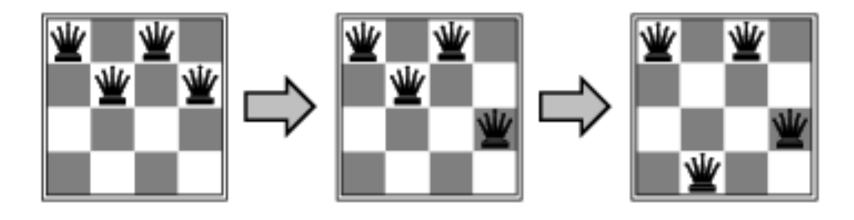
Path to goal state is irrelevant

Don't know what the goal state is exactly, trying to find it!

- Local search algorithms
  - Keep single "current" state, hunt for solution

#### Example: *n*-queens

 Put n queens on an n x n board with no two queens on the same row, column, or diagonal

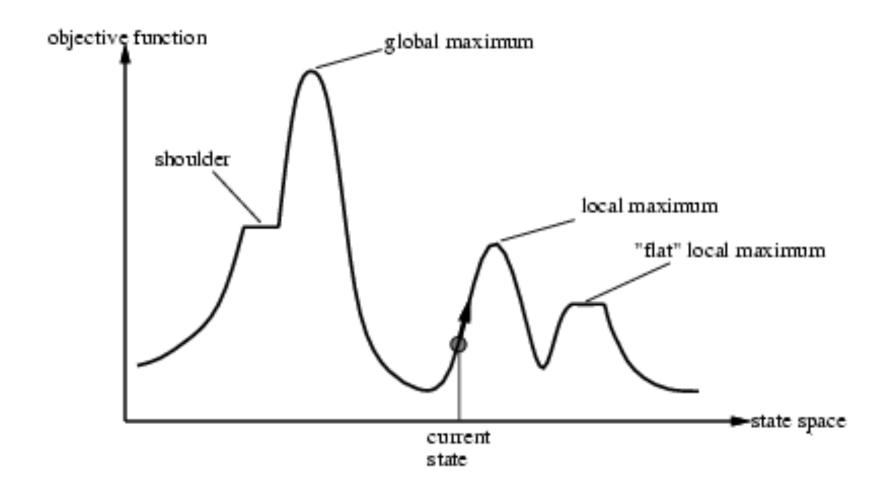


### Hill-Climbing

"Like climbing Mount Everest in fog with amnesia"

- Set current state = initial state
- Loop
  - Pick the best neighboring state
  - If the neighboring state is better than the current state
    - Current state = neighboring state
  - Else
    - Current state is the best fit

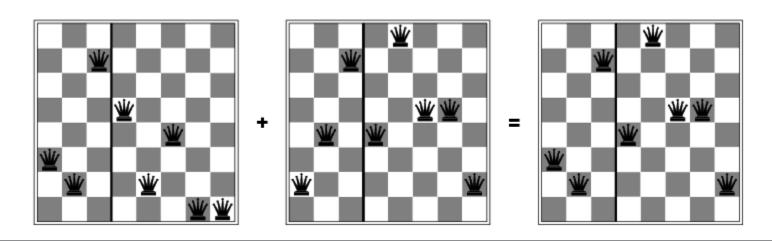
## Hill-Climbing



#### **Genetic Algorithms**

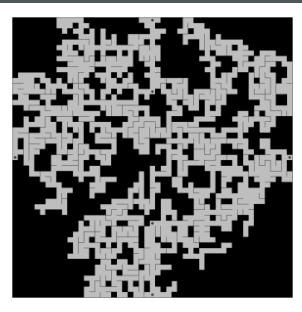
Instead of keeping 1 state, keep a population

- Evolve population based on fitness function
  - Keep the best x% with some probability
  - Crossover the best y%
  - Mutate the remainder

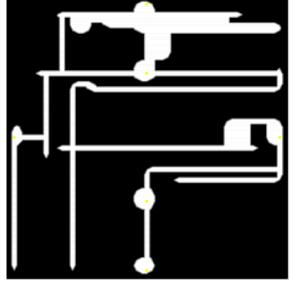


## **Genetic Algorithms: Representation**











#### **Humans as Fitness Functions**

<u>Picbreeder</u>

**Galactic Arms Race** 

#### **Evolving Virtual Creatures**

**Evolved Virtual Creatures Video** 

#### **ADVERSARIAL SEARCH**

	Perfect Information	Imperfect Information
Deterministic		
Stochastic		

	Perfect Information	Imperfect Information
Deterministic	Chess Checkers Go Othello	
Stochastic	Backgammon Monopoly Snakes and Ladders	

	Perfect Information	Imperfect Information
Deterministic	Chess Checkers Go Othello	Battleship Mastermind
Stochastic	Backgammon Monopoly Snakes and Ladders	

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	Perfect Information	Imperfect Information
Deterministic	Chess Checkers Go Othello	Battleship Mastermind
Stochastic	Backgammon Monopoly Snakes and Ladders	Poker Bridge Scrabble Civilization

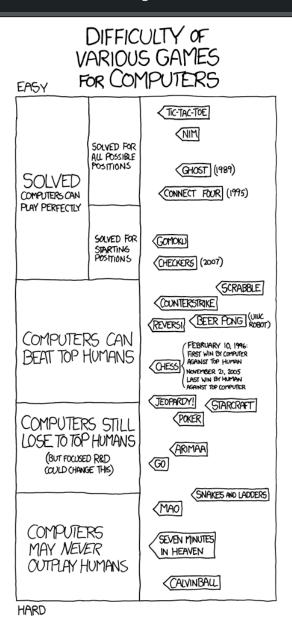
### What are Games? (in the context of AI)

Multi-agent environment

Adversarial

Planning a strategy

High complexity + time limits



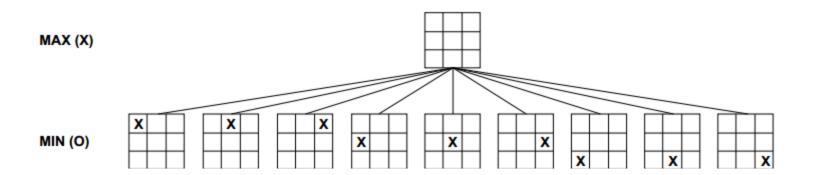
### Minimax Algorithm

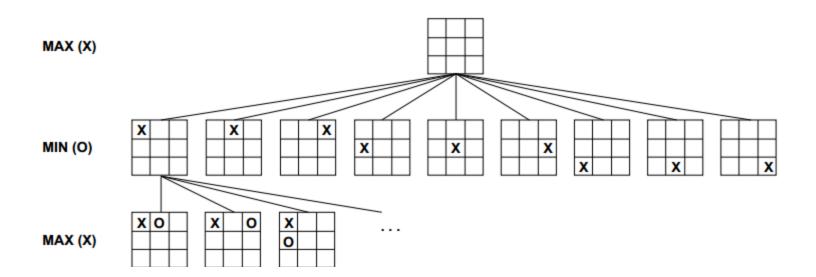
- Two player, adversarial, perfect information game
  - Tic Tac Toe
  - Nim
  - Chess
  - Go

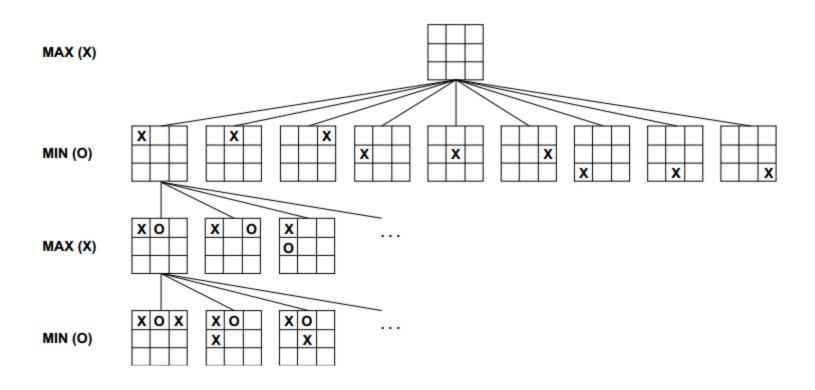
- Two players: MAX and MIN
  - MAX moves first

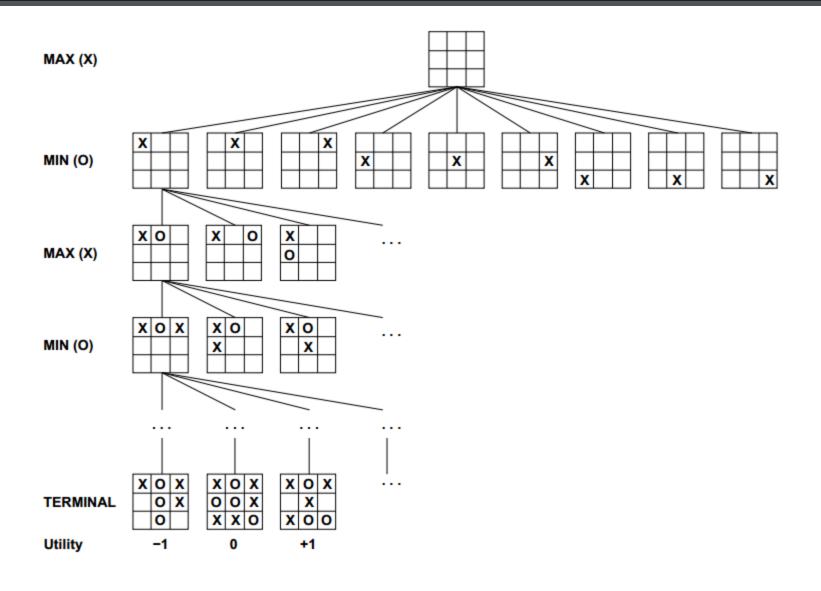
MAX (X)

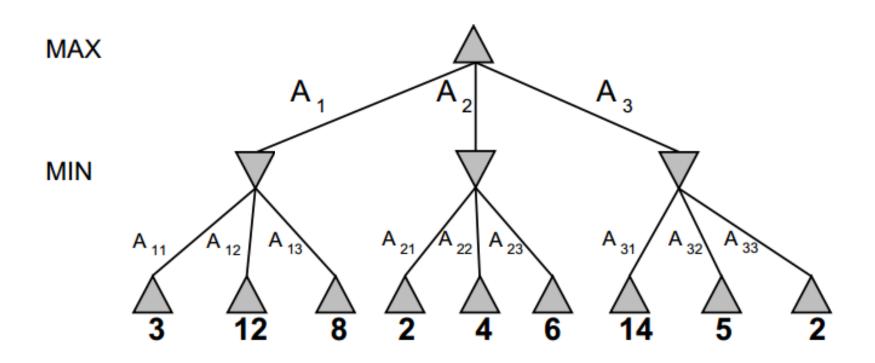


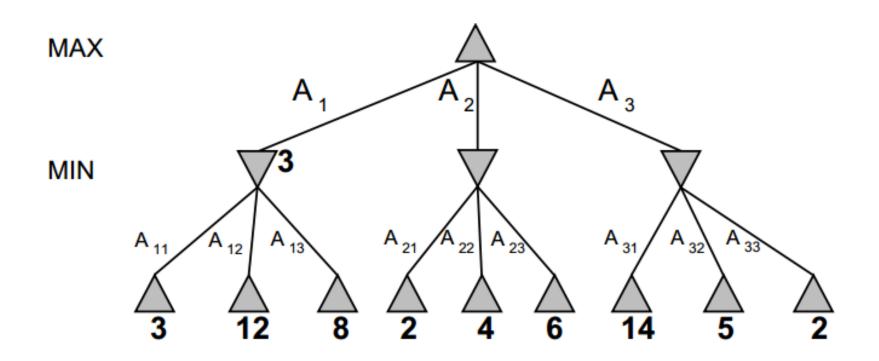


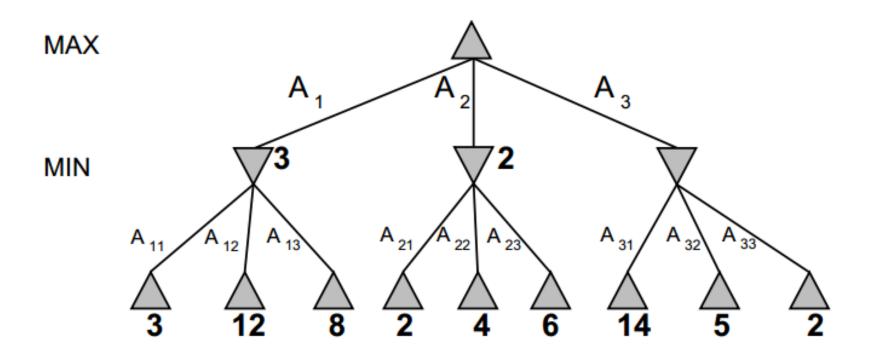


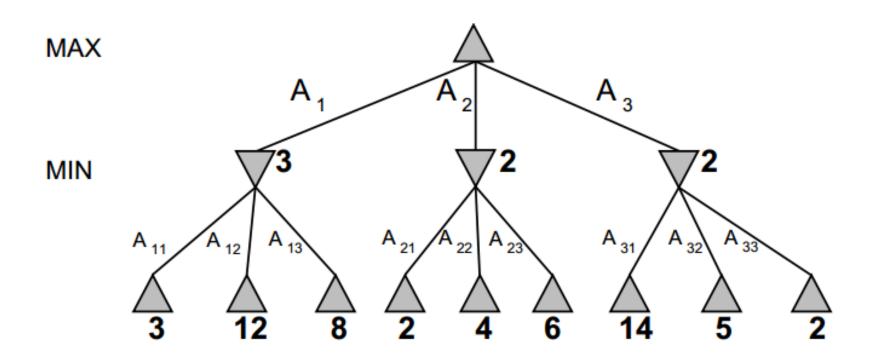


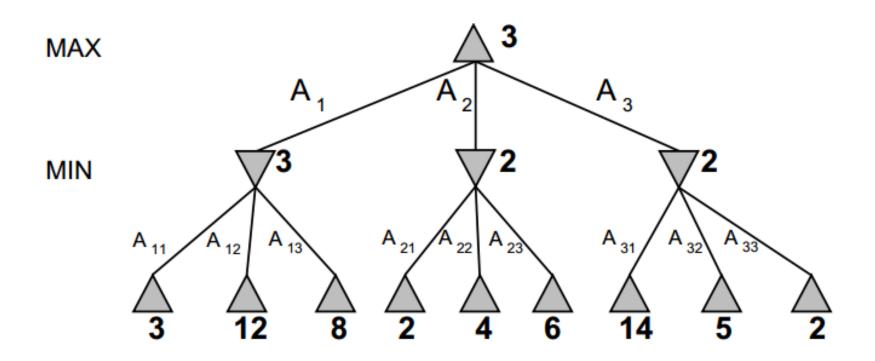


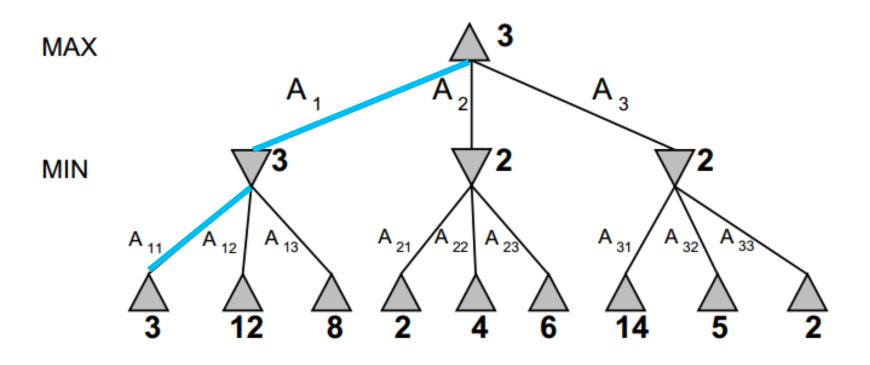










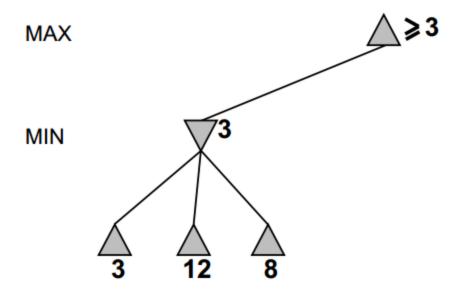


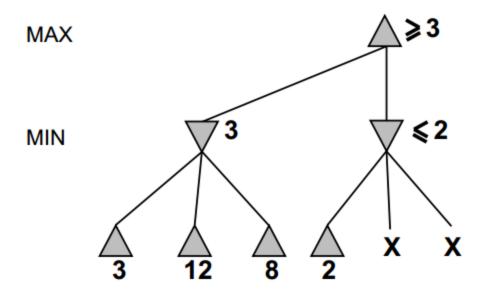
1: X	4: X
2: 0	5: X
3: O	6: X

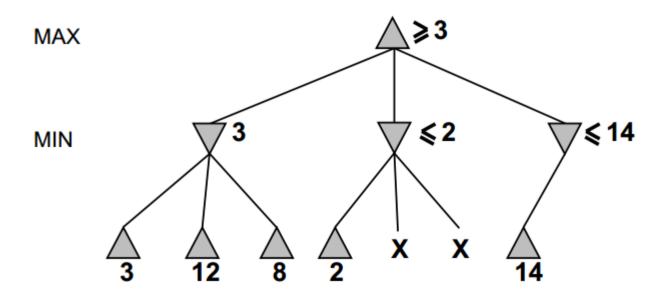
1: X	4:
2:	5:
3:	6:

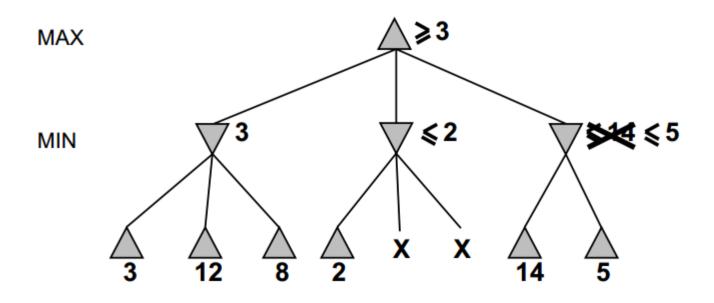
1: X	4:
2: 0	5:
3: O	6:

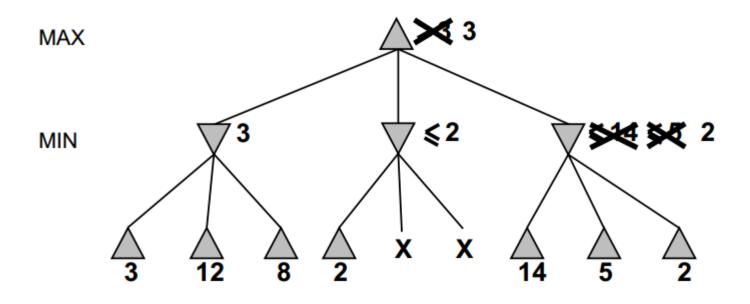
1: X	4: X
2: 0	5: X
3: O	6: X











### What if the tree is too big?

Heuristics!

Quiescence search

### **MIDTERM REVIEW!**

#### **Grade Breakdown**

Overall mean: 84.42%

Overall median: 82.5%

Number of people who took late days: 5

Average for sections:

What is AI?	90.57%
-------------	--------

Propositional Logic 75.09%

First-Order Logic 85.63%

Constraint Solving 78.99%

Planning 78.46%

# Regrading Policy

- If you think I made a mistake:
  - Write down your regrade request on the front of the exam
  - I reserve the right to regrade the whole exam

Regrades are relatively low priority but will get done!

# What is Plagiarism?

 Directly copying text or code from one source and claiming it as your own

It can be accidental!

It doesn't matter what the source is!

Even copying a partial sentence counts!

# Why is Plagiarism Bad?

- Ethical
  - Claiming someone else's work as your own is wrong
  - Damages your reputation (and your employer's...)

- Educational
  - Makes it unclear if you actually understand information or just have basic reading comprehension skills

# Kinds of Plagiarism

Copy-and-paste

Close paraphrasing

Failure to assign credit

Self-plagiarism

# Kinds of Plagiarism

Copy-and-paste

Close paraphrasing

Failure to assign credit (citations)

Self-plagiarism

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

 Bad: "These agents select actions on the basis of the current percept, ignoring the rest of the percept history."

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

 Also Bad: "Simple reflex agents select actions on the basis of current percept, ignoring rest of the percept history."

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

 Still bad: "Simple reflex agents select actions on the basis of the current percept."

# Kinds of Plagiarism

Copy-and-paste

Close paraphrasing

Failure to assign credit (citations)

Self-plagiarism

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

Bad: "The simple reflex agent is the simplest kind of agent. It ignores percept history, and selects actions on the basis of the current percept."

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

• Also Bad: "The simple reflex agent, the simplest kind of agent, chooses its actions based on what it currently knows, ignoring the rest of the percept history."

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

Also Bad: "The simple reflex agent, the simplest kind of agent, chooses its actions based on what it currently knows, ignoring the rest of the percept history."

- Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)
- Okay: "A simple reflex agent works does not keep track of the history of its actions or what it has previously seen. It only knows how to act using simple rules that are triggered by the current percept. [AIMA, p48]"

Source: "The simplest kind of agent is the simple reflex agent. These agents select actions on the basis of the current percept, ignoring the rest of the percept history." (AIMA, p48)

Okay but not good for an exam: "According to Russell and Norvig, the simple reflex agent 'select[s] actions on the basis of the correutn percept, ignoring the rest of the percept history'. [AIMA, p48]"

# Kinds of Plagiarism

Copy-and-paste

Close paraphrasing

Failure to assign credit (citations)

Self-plagiarism

# When and where do I cite something?

If it is a direct quote – next to the quotation

If it is a paraphrase – near the paraphrased paragraph

- Don't need to cite for common knowledge
  - The sky is blue
  - Boston is in Massachusetts

If in doubt, cite it!

### **Common Knowledge?!**

- Things that a reasonable person in your audience can be assumed to know
  - The sky is blue
  - Boston is in Massachusetts
- Never applies to data, statistics, or something you read somewhere that you didn't know before!
  - People who are not told that the sky is blue do not necessarily identify it as being blue [Radiolab, NPR, Season 10 Episode 13]
  - Boston's population is 589,141 as of the 2000 US Census, making it the largest city in Massachusetts. [US Census 2000]

#### What do I cite?

- Authors
- Name of paper/book/article
- Where that article was published
  - Newspaper?
  - Conference?
  - Website? (give url and date accessed)
- What date the article was published

# Kinds of Plagiarism

Copy-and-paste

Close paraphrasing

Failure to assign credit (citations)

Self-plagiarism

# Wait, how could I plagiarize myself?!

- Reusing text that you've written in the past without attribution
- Submitting the same paper for two classes
  - Simultaneously
  - Re-use from prior class
- Academic dishonesty: less work for more credit without disclosure to instructor
- Academia: can't pad publication count without doing the extra work!

#### **Plagiarism on Midterm Exam**

Frequent in short-answer questions

- Deducted half-credit where noticed
  - I could have brought them all to the Office of Student Conduct and Conflict Resolution

There are no exceptions to this policy

Please do not do this again!

#### Part 0: What is AI?

Simple reflex agents vs. goal-based agents

Frame problem + example

#### **Part 1: Propositional Logic**

Converting to conjunctive normal form

```
Two sentences are logically equivalent iff true in same models:
     \alpha \equiv \beta if and only if \alpha \models \beta and \beta \models \alpha
                 (\alpha \wedge \beta) \equiv (\beta \wedge \alpha) commutativity of \wedge
                (\alpha \vee \beta) \equiv (\beta \vee \alpha) commutativity of \vee
      ((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) associativity of \wedge
      ((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) associativity of \vee
                  \neg(\neg\alpha) \equiv \alpha double-negation elimination
             (\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha) contraposition
             (\alpha \Rightarrow \beta) \equiv (\neg \alpha \lor \beta) implication elimination
            (\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) biconditional elimination
             \neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta) de Morgan
              \neg(\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta) de Morgan
      (\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) distributivity of \wedge over \vee
      (\alpha \lor (\beta \land \gamma)) \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) distributivity of \lor over \land
```

#### Part 1: Does Jason Knit?

 Intent: use a combination of resolution and proof by contradiction

 Also received full credit for two proofs, one using resolution and one using contradiction

# Part 2: First-Order Logic

 Literals are single predicates (that can be negated) that do not have any variables

Unification

Order of interpretation in prolog matters!

# **Part 3: Constraint Solving**

 Most constrained variable: prunes search tree later by choosing the variable you're most likely to find conflicts with

 Least constraining value: stay flexible by ruling out the fewest remaining values

AnsProlog!

# Part 4: Planning

Planning Richard's Epic Babylon 5 Movie Party

What's the difference between a state and an operator?

#### **Extra Credit**

- Situated actions
  - Actions arise from circumstance
  - Plans are constructed after the fact, to explain what we've done
  - Handling situated behavior within plans (e.g. where does the plan stop and situated action begin?)

Needed to give an example

#### **Up Next: Assignment 4**

Evolutionary algorithms

- Using Processing
  - (optionally)

 Give yourself plenty of time! Relatively simple coding, long time waiting for solutions.



Mona Lisa, after ~162k generations

#### **Genetic Algorithms**

Instead of keeping 1 state, keep a population

- Evolve population based on fitness function
  - Keep the best x% with some probability
  - Crossover the best y%
  - Mutate the remainder

