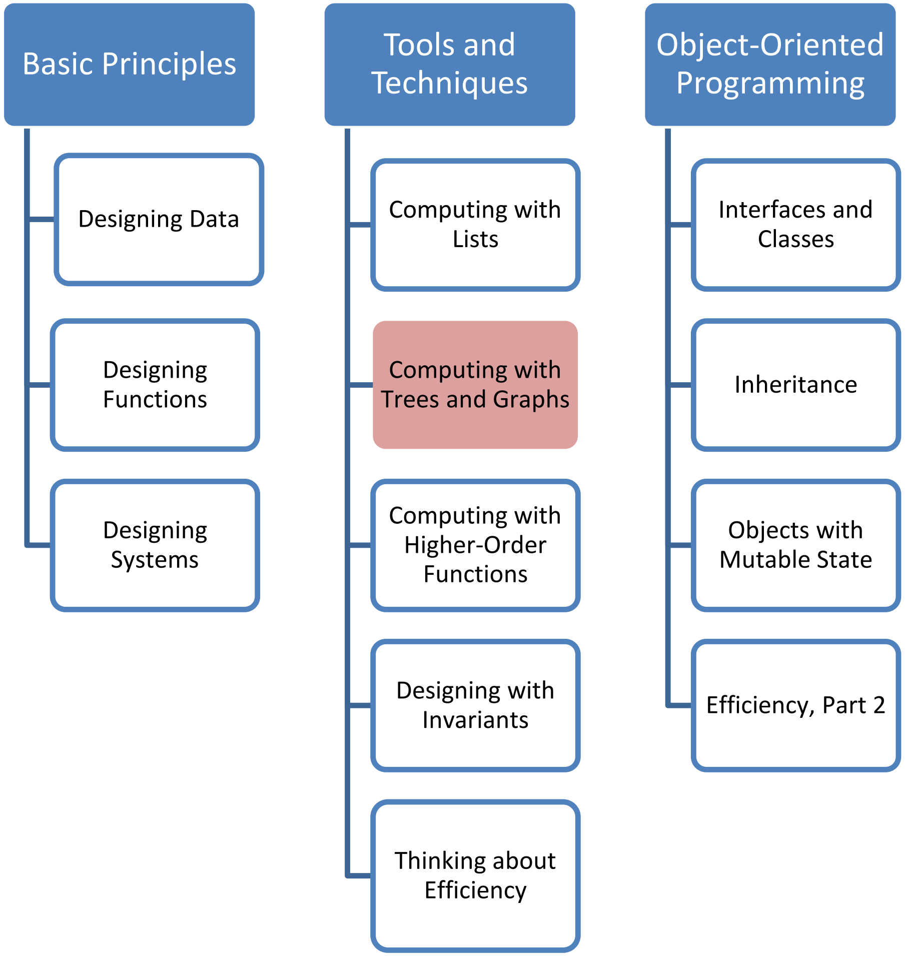
Trees

CS 5010 Program Design Paradigms
“Bootcamp”

Lesson 5.1

Module 05



Basic Principles

- Designing Data
- Designing Functions
- Designing Systems

Tools and Techniques

- Computing with Lists
- Computing with Trees and Graphs
- Computing with Higher-Order Functions
- Designing with Invariants
- Thinking about Efficiency

Object-Oriented Programming

- Interfaces and Classes
- Inheritance
- Objects with Mutable State
- Efficiency, Part 2

Module Introduction

- In this module we will learn about a number of topics having to do with trees and their representation.
- We will learn about
  - branching structures, such as trees
  - mutually recursive data definitions
  - S-expressions
  - How to represent trees and related structures in Java
  - What makes the observer template work in general.

Lesson Introduction

- Many examples of information have a natural structure which is not a sequence, but is rather a tree, which you should have learned about in your data structures course.
- In this lesson, we'll study how to apply the Design Recipe to trees.

Learning Objectives

- At the end of this lesson you should be able to:
  - Write a data definition for tree-structured information
  - Write functions that manipulate that data, using the observer template

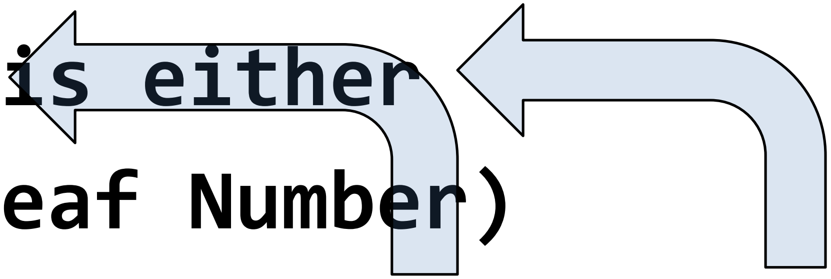
Binary Trees: Data Definition

```
;; A Binary Tree is represented as a BinTree, which is either:
;; (make-leaf datum)
;; (make-node lson rson)

;; INTERPRETATON:
;; datum      : Real       some real data
;; lson, rson : BinTree    the left and right sons of this node

;; IMPLEMENTATION:
(define-struct leaf (datum))
(define-struct node (lson rson))

;; CONSTRUCTOR TEMPLATES:
;; -- (make-leaf Number)
;; -- (make-node BinTree BinTree)
```

There are many ways to define binary trees. We choose this one because it is clear and simple.

Observer Template to follow...

This definition is self-referential (recursive)

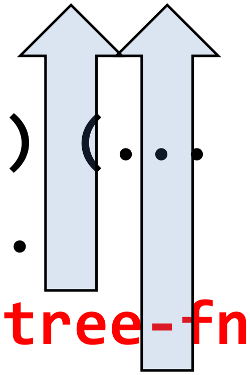


```
;; A BinTree is either
;; -- (make-leaf Number)
;; -- (make-node BinTree BinTree)
```

Observer Template

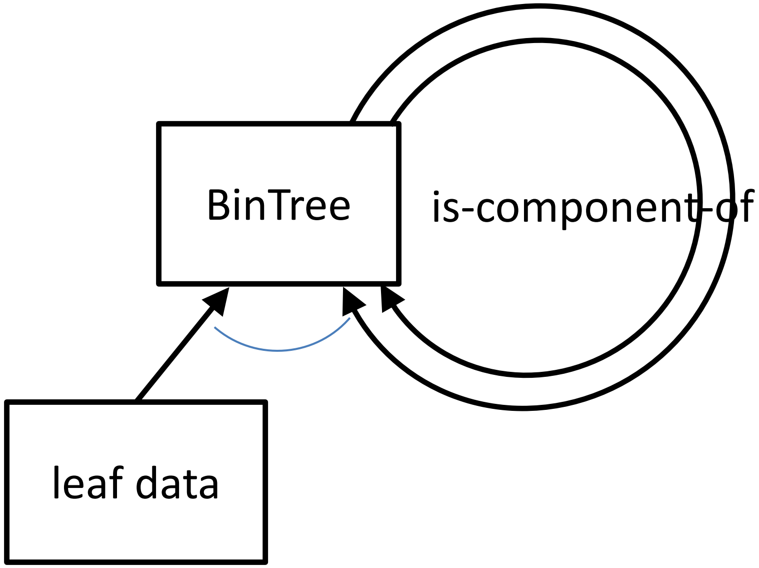
Here's the template for this data definition. Observe that we have two self-references in the template, corresponding to the two self-references in the data definition.

```
tree-fn : BinTree -> ???
(define (tree-fn t)
  (cond
    [(leaf? t) (... (leaf-datum t))]
    [else (...
           (tree-fn (node-lson t))
           (tree-fn (node-rson t)))]))
```



Self-reference in the data definition leads to self-reference in the template; Self-reference in the template leads to self-reference in the code.

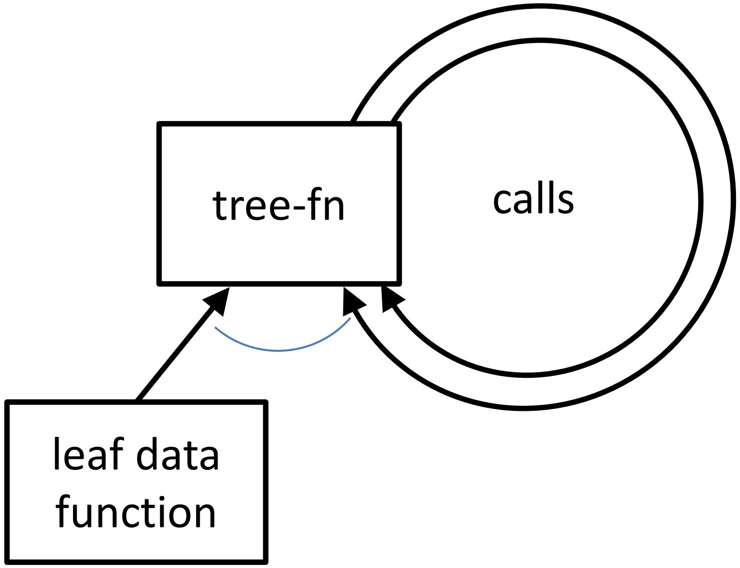
Remember: The Shape of the Program Follows the Shape of the Data



BinTree

is-component-of

leaf data



tree-fn

calls

leaf data function

Data Hierarchy (a **BinTree** is either leaf data or has two components which are **BinTrees**

Call Tree (**tree-fn** either calls a function on the leaf data, or it calls itself twice.)

The template questions

What's the answer for a leaf?

```
tree-fn : Tree -> ???
(define (tree-fn t)
  (cond
    [(leaf? t) (... (leaf-datum t))]
    [else (...
            (tree-fn (node-lson t))
            (tree-fn (node-rson t) ))]))
```

If you knew the answers for the 2 sons, how could you find the answer for the whole tree?

And here are the template questions. When we write a function using the template, we fill in the template with the answers to these questions.

Let’s see how the template questions help us define some functions that observe binary trees.

leaf-sum

What’s the answer for a leaf?

```
leaf-sum : Tree -> Number
(define (leaf-sum t)
  (cond
    [(leaf? t) (leaf-datum t)]
    [else (+
            (leaf-sum (node-lson t))
            (leaf-sum (node-rson t)))]))
```

If you knew the answers for the 2 sons, how could you find the answer for the whole tree?

leaf-max

What’s the answer for a leaf?

```
leaf-max : Tree -> Number
(define (leaf-max t)
  (cond
    [(leaf? t) (leaf-datum t)]
    [else (max
            (leaf-max (node-lson t))
            (leaf-max (node-rson t)))]))
```

If you knew the answers for the 2 sons, how could you find the answer for the whole tree?

leaf-min

What’s the answer for a leaf?

```
leaf-min : Tree -> Number
(define (leaf-min t)
  (cond
    [(leaf? t) (leaf-datum t)]
    [else (min
            (leaf-min (node-lson t))
            (leaf-min (node-rson t)))]))
```

If you knew the answers for the 2 sons, how could you find the answer for the whole tree?

Summary

- You should now be able to:
  - Write a data definition for tree-structured information
  - Write a template for tree-structured information
  - Write functions that manipulate that data, using the template

Next Steps

- Study the file 05-1-trees.rkt in the Examples folder.
- If you have questions about this lesson, ask them on the Discussion Board
- Do Guided Practice 5.1
- Go on to the next lesson