Course Wrap Up

CS 5002: Discrete Math

Tamara Bonaci, Adrienne Slaughter

Northeastern University

December 6, 2018

Section 1

Week 1: Digital Logic and Number Representation

Digital Logic and Number Representation

- Digital logic
  - Statements and propositions
  - Logic operators
  - Evaluating propositions
  - Logical equivalence
  - Predicates and Quantifiers
- Number representations
  - Decimal base
  - Binary base
  - Octal base
  - Hexadecimal base
- Relevance to computers and programming

Section 2

Week 2: Functions and Variables. Integer Functions

Variables and Functions

- Variables
- Functions
  - Domain, codomain and range of a function
  - Partial functions
  - Injection, surjections and bijections
  - Increasing and decreasing functions
  - Inverse functions
  - Composition of a function
- Some important integer functions
  - Floor
  - Ceiling

Section 3

Week 3: Sums and Sequences

Sums and Sequences

- Sequences
  - Geometric sequence
  - Arithmetic sequence
  - Recurrence relations
- Summations
  - Single sums
  - Double sums
- Recurrence relations
  - Closed forms
  - Forward iteration
  - Backward iteration

Section 4

Week 4: Sets

Introduction to Sets and Matrices

- Sets
  - Special sets
  - Set equality, subsets and Venn diagrams
  - Power sets
  - Set cardinality
  - Set operation: union, intersection, complement, disjoint, set difference
  - Membership table
  - Set problems
- Matrices
  - Matrix addition
  - Matrix multiplication

Section 5

Week 5: Introduction to Number Theory

Introduction to Number Theory

- Divisibility and modular arithmetic
  - Divisibility
  - The division algoirthm
  - Congruences
- Prime and composite numbers
  - Unique prime factorization
  - Coprimes
  - Greatest Common Divisor (gcd)
  - Euler Totient Function
  - Euclidean algorithm
- Linear congruences and modular inverses
- Extended Euclidean algorithm
- The Chinese Remainder theorem

Section 6

Week 6: Relations and Recurrences

Introduction to Relations

- Relations and their properties
  - Binary relations
  - Functions as relations
  - Properties of relations - reflexivity, symmetry, anti-symmetry, transitivity
  - Combining relations
- Representing relations
- Closures
- Equivalence relations
  - Equivalence classes
  - Set partitions
- Partial orderings
  - Comparable elements
  - Well-ordered set
  - Lexicographic order
- n-ary Relations

Section 7

Week 7: Introduction to Linear Data Structures

Introduction to Linear Data Structures

- How do computers work?
- What is a data structure?
- Container data structures
  - LIFO order
  - FIFO order
- Stacks
- Queues
- Deques
- Arrays
- Implementation issues

Section 8

Week 8: Midterm

Section 9

Week 9: Introduction to Combinatorics

Introduction to Combinatorics

- Introduction to counting techiniques
  - Basic counting principles: the product, sum, subtraction and division rules
  - The pigeonhole principle
  - The generalized pigeonhole principle
- Permutations and combinations
  - Permutations without repetition
  - Combinations without repetition
  - Binomial coefficients and binomial theorem
  - Pascal identity and triangle
- Generalized permutations and combinations
  - Permuations with repetitons
  - Combinations with repetitions

Section 10

Week 10: Introduction to Algorithms

Introduction to Algorithms

- Growth of functions
- Asymptotic analysis
  - Upper bound
  - Lower bound
  - Tight bound
- Logs, powers and exponents
- Adding functions
- Introduction to algorithms
  - Expressing algorithms
  - Example: linear search
  - Binary search
  - Selection sort
- Algorithm analysis
  - Time and space complexity
  - Correctness
- Representative problems

Section 11

Week 11: Divide and Conquer Algorithms

Divide and Conquer Algorithms

- Introduction to sorting
  - Defining sorting
- Divide and Conquer
  - Merge sort algorithm
  - Merge sort analysis
- Solving recurrences
  - Recursion trees
  - Solving recurrences
- Algorithm correctness
  - Induction proofs
  - Mathematical induction
  - The well-ordering principle

Section 12

Week 12: Algorithms Correctness. Proofs

Introduction to Proof Techniques

- Proof by counterexample
  - Greedy algorithms
  - Interval scheduling problem
- Proof by induction
  - Mathematical induction
  - The well-ordered property
  - Insertion sort
- Proof by loop invariant
  - Insertion sort
  - Bubble sort
  - Merge sort

Section 13

Week 13: Graphs and Trees

Introduction to Graphs and Trees

- Trees
  - Tree definitions
  - Kinds of trees
  - Special trees
    - Binary trees
    - Binary search trees
    - Balanced trees
  - Breath first search
  - Depth first search
  - Tree traversals
    - Pre-order traversal
    - In-order traversal
    - Post-order traaversal

Introduction to Graphs and Trees

- Graphs
  - Graph defintions
  - Directed and undirected graphs
  - Weighted graphs
  - Representing graphs
    - Adjacency list
    - Adjacency matrix
  - Graph traversals: BFS and DFS
  - Shortest path problems - Dijkstra's algorithm

Section 14

Week 14: Introduction to Probability

Introduction to Probability

- Random experiment
- Sample space
- Event
- Probability
  - Axioms of probability
  - Elementary events
  - Equally likely events
- Conditional probability and independence
  - Conditional probability
  - The chain rule
  - The law of total probability
  - The Bayes' rule
  - Independence
  - Mutually exclusive events

Introduction to Probability

- Random variables
- Probability distribution
  - Cumulative distribution function
  - Probability mass function
- Expectation
  - Expectation
  - Linearity of expectation
  - Variance
- Some important discrete probabilities
  - Bernoulli distribution
  - Binomial distribution
  - Geometric distribution
- Random processes