A6: RELATIONS

There is no branch of mathematics, however abstract, whic may not some day be applied to phenomena in real world. — Nicolai Ivanovitch Lobachevsky (1792-1856)

Course: CS 5002
Fall 2018
Due: 21 Oct 2018, Midnight

OBJECTIVES

After you complete this assignment, you will be comfortable with:

- Relations and their properites
- Matrix representations of relations
- Equivalence relations and partial orderings
- Closures of relations
- n-ary relations

RELEVANT READING

Rosen:

- 9.1: Relations and Their Properties
- 9.2: n-ary Relations and Their Applications
- 9.3: Representing Relations
- 9.4: Closures of Relations
- 9.5 Equivalence Relations
- 9.6 Partial Orderings

NEXT WEEK'S READING

- Lists, stacks and queues

EXERCISES

Problem 1: Definition of a relation

Let's consider the following **congruence modulo 3** relation R, defined from the set of integers, $\mathbb{Z}$ to the set of integers $\mathbb{Z}$ as follows:

$$m \ R \ n \iff 3|(m-n)$$

(a) (1 point) Is $10 \ R \ 1$? Please explain why or why not.

(a) ____________________

(b) (1 point) Is $(8,1) \in R$? Please explain why or why not.

(b) ____________________

(c) (1 point) List five integers n such that $n \ R \ 0$.

(c) ____________________

(d) (1 point) List five integers n such that $n \ R \ 2$.

(d) ____________________

Problem 2: Definition of a relation

Let A be the set of all strings of a's and b's of length 4. Let's define a relation R on A as follows: For all $s,t \in A$,

$$s \ R \ t \iff s \text{ has the same first two characters as } t.$$

(a) (1 point) Is $abaa \ R \ abba$?

(a) ____________________

(b) (1 point) Is $aabb \ R \ bbaa$?

(b) ____________________

(c) (1 point) Is $aaaa \ R \ aaab$?

(c) ____________________

Problem 3: Properites of relations

Let R be the "greater than or equal to" relation on the set of real numbers, formally defined as follows:

for all $x, y \in \mathbb{R}, x \ R \ y \iff x \geq y$.

Please show your work to determine whether or not the given relation is:

(a) (1 point) Reflexive:

(b) (1 point) Symmetric:

(c) (1 point) Anti-symmetric:

(d) (1 point) Transitive:

Problem 4: Properites of relations

Let A be a Cartesian product $\mathbb{R} \times \mathbb{R}$, and let F be a relation defined on A as follows:

For all (x_1, y_1) and $(x_2, y_2) \in A$: $(x_1, y_1) \ F \ (x_2, y_2) \quad \Longleftrightarrow \ x_1 = x_2$

Please show your work to determine whether or not the given relation is:

(a) (1 point) Reflexive:

(b) (1 point) Symmetric:

(c) (1 point) Anti-symmetric:

(d) (1 point) Transitive:

Problem 5: Properties of relations

Let R be a relation operating on the set of all Web pages, defined as follows: `everyone who visits Web page` a `has also visited Web page` b.

Please show your work to determine whether or not the given relation is:

(a) (1 point) Reflexive:

(b) (1 point) Symmetric:

(c) (1 point) Anti-symmetric:

(d) (1 point) Transitive:

Problem 6: Combining Relations

Let A be the set of all ALIGN students on our campus, and let B be the set of all books available in the Northeastern University libraries. Let relation R_1 consist of all ordered pairs (a, b), where student a is required to read book b in a course. Similarly, let relation R_2 consist of all ordered pairs (a, b), where student a has read book b.

Describe (in words) the ordered pairs in each of the combined relations:

(a) (1 point) $R_1 \cup R_2$

(a) ____________________

(b) (1 point) $R_1 \cap R_2$

(b) ____________________

(c) (1 point) $R_1 - R_2$

(c) ____________________

(d) (1 point) $R_2 - R_1$

(d) ____________________

Problem 7: Combining relations

Let R be the relation $\{(2,5),(2,6),(3,6),(3,7),(4,5)\}$, and let S be the relation $\{(3,5),(4,5),(4,6),(5,6)\}$. Find the composition $S \circ R$.

Problem 8: n**-ary relations**

List all triples in the relation $\{(a, b, c) | a, b, c$ are integers such that $0 < a < b < c < 5\}$.

Problem 9: Matrix representation of a relation

Represent each of these relations on the set $\{1, 2, 3\}$ with a matrix, such that the elements of the given set are listed in an increasing order:

(a) (1 point) $\{(1, 1), (1, 2), (1, 3)\}$

(b) (1 point) $\{(1,2),(2,1),(2,2),(3,3)\}$

(c) (1 point) $\{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$

(d) (1 point) $\{(1,3),(3,1)\}$

Problem 10: Matrix representation and properties of relations

Represent each of these relations on the set $\{1, 2, 3\}$ with a matrix, such that the elements of the given set are listed in an increasing order:

Use matrix representation of a relation to determine whether the following relations are reflexive and symmetric.

$\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

(a) (1 point) Matrix representation?

(a) ____________________

(b) (1 point) Reflexive?

(b) ____________________

(c) (1 point) Symmetric?

(c) ____________________

$\{(1, 3), (3, 1)\}$

(a) (1 point) Matrix representation?

(a) ____________________

(b) (1 point) Reflexive?

(b) ____________________

(c) (1 point) Symmetric?

(c) ____________________

Problem 11: Closures

Let R be the relation on the set $\{2, 4, 6, 8\}$ containing ordered pairs $(2, 4)$, $(4, 4)$, $(4, 6)$, $(6, 2)$, $(6, 6)$ and $(8, 2)$. Please show your work to find:

(a) (2 points) Reflexive closure of R

(b) (2 points) Symmetric closure of R

Problem 12: Equivalence relations

Consider the following relations defined on set $\{0, 1, 2, 3\}$. Show your work to determine whether or not the given relations are equivalence relations.

(a) (3 points) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

(b) (4 points) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Problem 13: Equivalence relations

Consider the following relations defined on the set of all people. Show your work to determine whether or not the given relations are equivalence relations.

(a) (4 points) $\{(a, b) | a$ and b are the same age$\}$

(b) (4 points) $\{(a, b) | a$ and b have the same parents$\}$

(c) (4 points) $\{(a, b) | a$ and b speak a common language$\}$

Problem 14: Equivalence relations

Consider the following relations defined on the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$. Show your work to determine whether or not the given relations are equivalence relations.

(a) (4 points) $\{(f, g) | f(1) = g(1)\}$

(b) (4 points) $\{(f, g) | f(0) = g(1) \text{ and } f(1) = g(0)\}$

Problem 15: Partitions

Which of the given collections of subsets are partitions of the set $\{-3, -2, -1, 0, 1, 2, 3\}$?

(a) (1 point) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

(a) ____________________

(b) (1 point) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$

(b) ____________________

(c) (1 point) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

(c) ____________________

(d) (1 point) $\{-3, -2, 2, 3\}, \{-1, 1\}$

(d) ______________________________

Problem 16: Partial ordering

Consider the following relations on set $\{0, 1, 2, 3\}$. Show your work to determine which of the given relations are partial orderings.

(a) (4 points) $\{(0,0), (1,1), (2,2), (3,3)\}$

(b) (4 points) $\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,2), (3,3)\}$

PROBLEMS

Problem 17: Equivalence relations

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

Problem 18: Equivalence relations

Suppose that R_1 and R_2 are equivalence relations on the set S. Show your work to determine whether the intersection combination of R_1 and R_2, $R_1 \cap R_2$, is an equivalence relation.

PROGRAMMING PROBLEMS

Problem 19: Reflexivity of a relation

Write a simple Python program that, given the matrix representing a relation on a finite set, determines whether the given relation is reflexive.

Problem 20: Symmetry of a relation

Write a simple Python program that, given the matrix representing a relation on a finite set, determines whether the given relation is symmetric.

Question	Points	Score
Definition of a relation	4	
Definition of a relation	3	
Properites of relations	4	
Properites of relations	4	
Properties of relations	4	
Combining Relations	4	
Combining relations	2	
n-ary relations	2	
Matrix representation of a relation	4	
Matrix representation and properties of relations	6	
Closures	4	
Equivalence relations	7	
Equivalence relations	12	
Equivalence relations	8	
Partitions	4	
Partial ordering	8	
Equivalence relations	5	
Equivalence relations	5	
Reflexivity of a relation	5	
Symmetry of a relation	5	
Total:	100	

SUBMISSION DETAILS

Things to submit:

- Submit the following on Blackboard for Assignment 6:
  - The written parts of this assignment as a .pdf named “CS5002_[lastname]_A6.pdf”. For example, my file would be named “CS5002_Bonaci_A6.pdf”. (There should be no brackets around your name).
  - Make sure your name is in the document as well (e.g., written on the top of the first page).