A2: FUNCTIONS AND INTEGER FUNCTIONS

If you think dogs can't count, try putting three dog biscuits in your pocket and then giving Fido only two of them. –Phil Pastoret

Course: CS 5002
Fall 2018
Due: September 23, 2018, Midnight

OBJECTIVES

After you complete this assignment, you will be comfortable with:

- Variables
- Functions, writing and using them
- Domain and codomains of functions
- Injection, surjections and bijections
- Invertible functions
- Composition of functions

RELEVANT READING

- Rosen, 2.3 – Functions
- Online Discrete Math: Chapter 0.1, 0.2, 0.4 (Intro, Functions)

FUNCTION PROBLEMS

Problem 1: Function Defintions

(a) Let $A = \{-5, 0, 5\}$ and $B = \{b, c, d, e\}$. Define a function $f : A \to B$, represented with the arrow diagram in Figure 1:

(b) Let $C = \{5, 6, 7, 8\}$ and $D = \{z, y, x, w\}$. Define a function $g : C \to D$, represented with the arrow diagram in Figure 2:

(c) Let $E = \{-10, 10, 100\}$ and $F = \{00, 10, 11\}$. Define a function $h : E \to F$, represented with the arrow diagram in Figure 3.

Problem 2: Valid functions

Let $X = \{25, 77, 555\}$ and $Y = \{6, 7, 8, 9\}$. Which of the arrow diagrams, depicted in Figures 4–7, represent a valid function $f : X \to Y$?

If an arrow diagram is not a valid function, please explain why is thst.

(a) Arrow diagram represented in Figure 4.

(b) Arrow diagram represented in Figure 5.

(c) Arrow diagram represented in Figure 6.

(d) Arrow diagram represented in Figure 7.

Problem 3: Valid functions

Is f a function from $\mathbb{Z}$ to $\mathbb{R}$ if f defined as:

(a) $f(x) = x$

(b) $f(x) = \pm x$

(c) $f(x) = \sqrt{x^2 + 1}$

Points: _____ out of 12

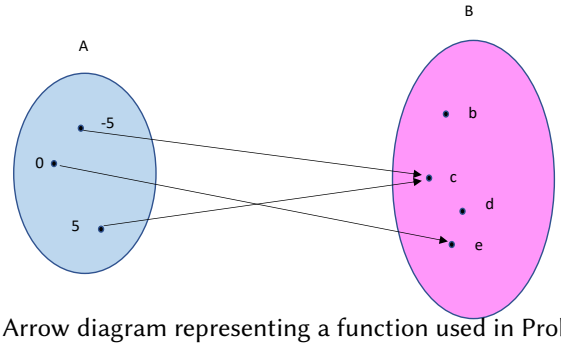

Figure 1: Arrow diagram representing a function used in Problem 1(a).

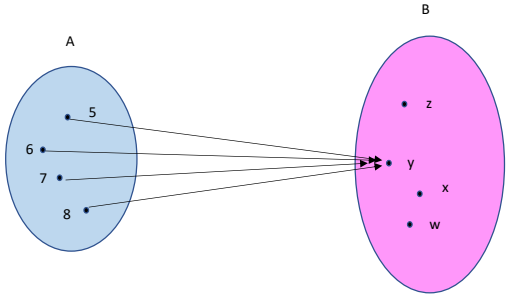

Figure 2: Arrow diagram representing a function used in Problem 1(b).

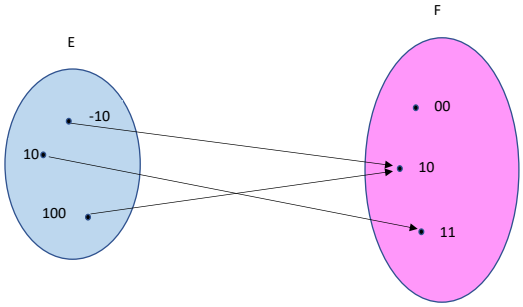

Figure 3: Arrow diagram representing a function used in Problem 1(c).

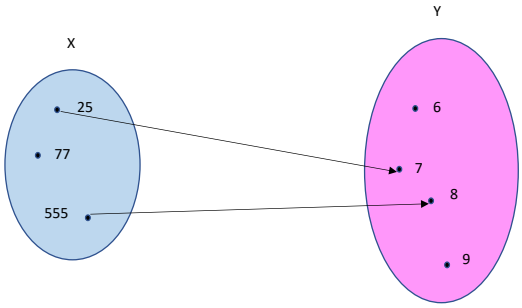

Figure 4: Arrow diagram representing a function used in Problem 2(a).

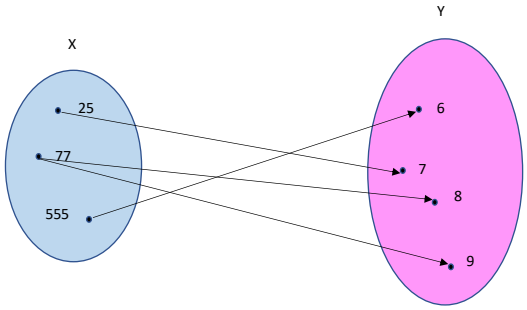

Figure 5: Arrow diagram representing a function used in Problem 2(b).

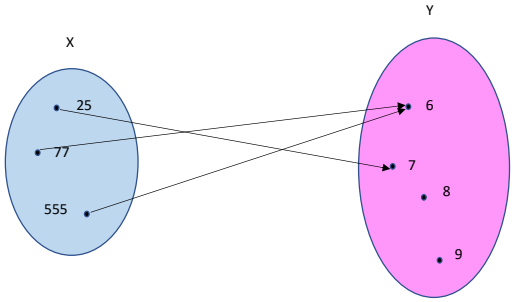

Figure 6: Arrow diagram representing a function used in Problem 2(c).

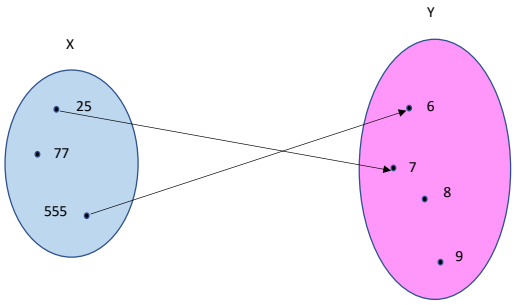

Figure 7: Arrow diagram representing a function used in Problem 2(d).

(d) $f(x) = \frac{1}{x^2-4}$

(e) $f(x) = \frac{1}{x}$

Problem 4: Domain and codomain of a function

Indicate whether or not the following statements are true. Please justify your answer.

(a) If two elements in the domain of a function are equal, then their images in the co-domain are equal.

(b) If two elements in the codomain of a function are equal, then their preimages in the domain are also equal.

(c) A function can have the same output for more than one input.

(d) A function can have the same input for more than one output.

Problem 5: Domain and range of a function

Find the domain and range of these functions.

(a) The function that assigns to each non-negative number its last digit.

(b) The function that asigns the next largest integer to some positive number.

(c) The function that assigns to a bit string the number of bits in the string.

(d) The function that assigns to a bit string the number of ones in the string minus the number of zeros in the string.

(e) The function that assigns to a bit string the number of bits left over when a bit string is split into bytes, which are blocks of eight bits.

(f) The function that assigns to each pair of positive integers the first integer in the pair.

(g) The function that assigns to each positive integer its largest decimal digit.

(h) The function that assigns to a bit string the longest string of ones in the string.

(i) The function that assigns to each pair of positive integers the maximum of these two integers.

(j) The function that assigns to each positive integer the number of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 that do not appear as decimal digits of that integer.

Problem 6: Domain and range of a function

Consider the following functions of two variables, x, y, defined as:

(a) $f : X \times Y \to \mathbb{Z}$, where $X = \{1, 2, 3\}, Y = \{-2, -1, 0\}$, such that $f(x, y) = \frac{x-y}{3}$.

(b) $g : X \times Y \to \mathbb{Z}$, where $X = \{-2, 0, 2\}, Y = \{4, 6, 8\}$ such that $g(x, y) = \frac{x-y}{4}$.

Please find the domain, the codomain and the range for the given functions.

Problem 7: One-to-one functions

Determine whether each of the functions $f : \mathbb{Z} \to \mathbb{Z}$ is a **one-to-one** function.

(a) $f(x) = x - 1$

(b) $f(x) = x^2 + 1$

(c) $f(x) = x^3$

(d) $f(x) = \lceil \frac{x}{2} \rceil$

Problem 8: One-to-one functions

Consider the following function from the set of ALIGN students, taking CS 5002. Under what conditions is the function **one-to-one**, if it assigns to a student their:

(a) Mobile phone number?

(b) Student identification number?

(c) Final grade in the class?

(d) Home town?

Problem 9: Onto functions

All but two of the following statements are correct ways to express the fact that a function is **onto**. Find the two that are incorrect. Please justify your answer.

(a) f is onto $\leftrightarrow$ every element in its codoman is the image of some element in its domain.

(b) f is onto $\leftrightarrow$ every element in its domain has a corresponding image in its codomain.

(c) f is onto $\leftrightarrow \forall y \in Y, \exists x \in X$ such that $f(x) = y$.

(d) f is onto $\leftrightarrow \forall x \in X, \exists y \in Y$ such that $f(x) = y$.

(e) f is onto $\leftrightarrow$ the range of f is the same as the codomain of f.

Problem 10: Onto functions

Determine whether or not the following function are **onto** function if $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$:

(a) $f(x, y) = x + y$

(b) $f(x, y) = x^2 + y^2$

(c) $f(x, y) = x$

(d) $f(x, y) = |x|$

(e) $f(x, y) = x - y$

Problem 11: One-to-one correspondence

Determine whether or not the following functions are bijections from $\mathbb{R}$ to $\mathbb{R}$:

(a) $f(x) = -3x + 4$

(b) $f(x) = x^2 + 1$

(c) $f(x) = \frac{x^2+1}{x^2+2}$

(d) $f(x) = x^3$

(e) $f(x) = x^5 + 1$

Problem 12: Increasing and decreasing functions

(a) Give an example of a function that is **one-to-one**, but not **onto**.

(b) Give an example of an increasing function from $\mathbb{R}$ to $\mathbb{R}$ that is not **one-to-one**.

Problem 13: Partial and total functions

For each of these partial functions, determine its domain, codomain, domain of defintion, and the set of values for which it is undefined. Also, determine if a function is a **total function**.

(a) $f : \mathbb{Z} \to \mathbb{R}, f(x) = \frac{1}{x}$

(b) $f : \mathbb{Z} \to \mathbb{Z}, f(x) = \lceil \frac{x}{2} \rceil$

(c) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}, f(x, y) = \frac{x}{y}$

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, f(x, y) = x \cdot y$

(e) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}, f(x, y) = x - y,$ if $x > y$

Problem 14: Composition of functions

Consider the following functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$, defined as:

$$\begin{aligned} f(x) &= x + 3 \\ g(x) &= -x \end{aligned}$$

Please find $f \circ g$ and $g \circ f$.

Problem 15: Composition of functions

Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c and d are constants. Determine the necessary and sufficient conditions on the constants a, b, c, and d so that $f \circ g = g \circ f$.

Problem 16: Invertible functions

Prove that some function $f(x) = ax + b$ from $\mathbb{R}$ to $\mathbb{R}$, where a and b are constants, is invertible when $a \neq 0$, and find its inverse.

Problem 17: Integer functions

Let x be a real number. Show that:

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor$$

SUBMISSION DETAILS

- This assignment is due by 11:59 pm PST on Sunday, September 23, 2018.
- Please submit it on Blackboard as a .pdf named "CS5002_[lastname]_A2.pdf". For example, my file would be named "CS5002_Bonaci_A2.pdf". (There should be no brackets around your name).
- Make sure your name is in the document as well (e.g., written on the top of the first page).

HELPFUL HINTS

- Start early!
- Ask clarification questions on Piazza.

Question	Points	Score
Function Defintions	3	
Valid functions	4	
Valid functions	5	
Domain and codomain of a function	4	
Domain and range of a function	10	
Domain and range of a function	4	
One-to-one functions	8	
One-to-one functions	8	
Onto functions	5	
Onto functions	10	
One-to-one correspondence	10	
Increasing and decreasing functions	3	
Partial and total functions	10	
Composition of functions	4	
Composition of functions	4	
Invertible functions	4	
Integer functions	4	
Total:	100	