A1: LOGIC AND BASE CONVERSIONS

Contrariwise, if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic. —Lewis Carroll

Course: CS 5002
Fall 2018
Due: 16 Sept 2018, Midnight

OBJECTIVES

After you complete this assignment, you will be comfortable with:

- Interpreting logic statements
- Generating a truth table for a logic statement and determining logical equivalence.
- Converting a number from one base to another
- Writing negative binary numbers

RELEVANT READING

- Rosen, Ch 1, sections 1.1 through 1.5.
- Optional: Discrete Math: Chapter 3 (Intro), 3.1 (Propositional Logic), (http://discrete.openmathbooks.org/dmoi/ch_logic.html)

LOGIC PROBLEMS

Problem 1: Which is a proposition?

Which of the following are propositions? If the statement is a proposition, note the truth value of that proposition.

(a) What time is it? (½)

(a) ______________________

(b) The sun is hot. (½)

(b) ______________________

(c) It rains every day in Seattle. (½)

(c) ______________________

(d) Go see a Seahawks game. (½)

(d) ______________________

Problem 2: Sentence from logic statements

Let p be "It is cold" and let q be "It is raining". Write a sentence that describes each of the following statements:

(a) $\neg p$ (1)

(a) ______________________________________

(b) $p \wedge q$ (1)

(b) ______________________________________

(c) $p \vee q$ (1)

(c) ______________________________________

(d) $q \vee \neg p$ (1)

(d) ____________________

(e) $\neg p \wedge \neg q$ (1)

(e) ____________________

(f) $\neg(\neg q))$ (1)

(f) ____________________

Problem 3: Logic statement from sentence

Let p be "The student is tired" and let q be "The student is having fun".

Write a logic statement that describes the following sentences.

(a) The student is tired and having fun. (1)

(a) ____________________

(b) The student is not having fun and tired. (1)

(b) ____________________

(c) The student is either tired, or having fun, or both. (1)

(c) ____________________

(d) Either the student is having fun or the student is tired, but the student is not having fun if the student is tired. (1)

(d) ____________________

(e) That the student is not tired is necessary and sufficient for the student to have fun. (1)

(e) ____________________

Problem 4: Truth Values

The *truth value* of a compound statement (that is, a statement composed of multiple statements) is determined by the truth values of the substatements together with how the substatements are combined to form the compound statement.

Determine the truth value for each of the following statements. Hint: your answer for each statement will be either true or false.

(a) Paris is in France and $2 + 2 = 4$. (1)

(a) ____________________

(b) *Paris is in France* and $2 + 2 = 5$. (1)

(b) ____________________

(c) *Paris is in England* and $2 + 2 = 4$. (1)

(c) ____________________

(d) *Paris is in England* and $2 + 2 = 5$. (1)

(d) ____________________

Problem 5: Comparing phones

Suppose the following [1]:

- Phone A has 256 MB RAM, 32 GB of ROM and a 8 MP camera.

[1] Based on Rosen, Exercise 1.6

- Phone B has 288 MB RAM, 64 GB of ROM and a 4 MP camera.
- Phone C has 128 MB RAM, 32 GB of ROM and a 5 MP camera.

Determine the truth value of the following propositions.

(a) Phone B has the most RAM of the phones. (1)

(a) ______________________

(b) Phone C has more ROM or a higher resolution camera than Phone B. (1)

(b) ______________________

(c) Phone B has more RAM, more ROM and a higher resolution camera than Phone A. (1)

(c) ______________________

(d) If Phone B has more RAM and more ROM than Phone C, then it also has a higher resolution camera. (1)

(d) ______________________

(e) Phone A has more RAM than Phone B if and only if Phone B has more RAM than Phone A. (1)

(e) ______________________

Problem 6: Truth Tables, Method 1

There are two different approaches to creating a truth table for a proposition or compound logical statement. Both consist of incrementally evaluating the logic statement for different values of the substatements or variables.

In this approach, fill each row from the left to the right. The first row is computed for you.

Fill out the empty spots in the truth table for the following statement:

$$\neg(p \wedge \neg q)$$

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	*F*	*F*	*T*
T	F			
F	T			
F	F			

After computing the intermediate steps, the final truth table is usually presented on its own. Use the table you filled out above to finish the table below.

Final truth table:

p	q	$\neg(p \wedge \neg q)$
T	T	
T	F	
F	T	
F	F	

Problem 7: Truth Tables, Method 2

Truth tables, Method 2. In this approach, each column represents a symbol in the logic statement, going in order from left to right. However, you fill in the rows in a different order, starting with the inputs (p and q), and then following operators order of precedence.

Fill out the empty spots in the truth table for the following statement: Fill out the columns in the order specified in the "Step" row. The first column is filled out for you.

$$\neg(p \wedge \neg q)$$

p	q	$\neg$	$(p$	$\wedge$	$\neg$	$q)$
T	T		*T*			
T	F		*T*			
F	T		*F*			
F	F		*F*			
$Step$		4	1	3	2	1

Final truth table:

p	q	$\neg(p \wedge \neg q)$
T	T	
T	F	
F	T	
F	F	

Problem 8: Truth Tables

For each of the expressions below, provide a truth table.

(a) $p \to \neg p$ (1)

(b) $p \oplus (p \vee q)$ (2)

(c) $p \leftrightarrow \neg p$ (2)

(d) $(p \vee q) \wedge \neg r$ (3)

Problem 9: Logically Equivalent

Use truth tables to determine if the following statements are logically equivalent. Use whichever method you prefer.

$$(p \vee q) \wedge r \; ?? \; p \vee (q \wedge r)$$

Final truth table:

p	q	r	$(p \vee q) \wedge r$	$p \vee (q \wedge r)$
T	T	T		
T	F	T		
F	T	T		
F	F	T		
T	T	F		
T	F	F		
F	T	F		
F	F	F		

Problem 10: If-Then

Write each of these statements in the form "If p, then q" in English[2].

(a) It is necessary to wash the boss's car to get promoted. (1)

(a) ____________________

(b) Winds from the south imply a spring thaw. (1)

(b) ____________________

(c) A sufficient condition for the warrant to be good is that you bought the computer less than a year ago. (1)

(c) ____________________

(d) Willy gets caught whenever he cheats. (1)

(d) ____________________

(e) You can access the website only if you pay a subscription fee. (1)

(e) ____________________

Problem 11: Quantifiers

Determine the truth value for each of the following statements, when the universe of discourse is $\Re$, the set of real numbers.

(a) $\forall x, |x| = x$ (1)

(a) ____________________

(b) $\exists x, x^2 = x$. (1)

(b) ____________________

(c) $\forall x, x + 1 > x$ (1)

(c) ____________________

(d) $\exists x, x + 2 = x$ (1)

(d) ____________________

(e) $\exists x (x^4 < x^2)$ (1)

(e) ____________________

[2] adapted from Rosen, 1-22.

(f) $\forall x(2x > x)$ (1)

(f) ____________________

Problem 12: Quantifiers in English

Let $W(x)$ be the statement "x as visited Washington". The domain (or, *universe of discourse*) consists of the students enrolled at Northeastern-Seattle. Express these quantifications in English.

(a) $\exists x W(x)$ (1)

(a) ____________________

(b) $\forall x W(x)$ (1)

(b) ____________________

(c) $\neg \exists x W(x)$ (1)

(c) ____________________

(d) $\exists x \neg W(x)$ (1)

(d) ____________________

(e) $\forall x \neg W(x)$ (1)

(e) ____________________

Problem 13: Statements into logical statements

Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives.

(a) Something is not in the correct place. (1)

(b) All tools are in the correct place and are in excellent condition. (1)

(c) Everything is in the correct place and are in excellent condition. (1)

(d) One of your tools is not in the correct place, but it is in excellent condition. (1)

Problem 14: Prolog: Logic Programming

(Adapted from Rosen, p51).

You are learning the programming language Python this semester; another language (that you won't learn too much about apart from this, but it has characteristics similar to ones we will learn about later) is one called Prolog. The name comes from *Pro*gramming *Log*ic. It's called a *declarative* programming language, where instead of writing instructions to the computer, your program consists of a series of facts and rules— a bunch of declarations. Prolog facts are simply predicates, as we've already seen. Prolog rules are used to define new predicates using those defined by the Prolog facts.

Let's say we have a Prolog program with the defined predicates $instructor(p, c)$ and $enrolled(s, c)$ that indicates Professor p teaches class c, and student s is enrolled in class c, respectively.

We could then state a series of facts as such:

```
instructor(chan,math273)
instructor(patel,ee222)
instructor(grossman,cs301)
enrolled(kevin,math273)
enrolled(juana,ee222)
enrolled(juana,cs301)
enrolled(kiko,math273)
enrolled(kiko,cs301)
```

We add in another predicate $teaches(p, s) : -instructor(p, c), enrolled(s, c)$, which is true if there exists a class c such that professor p teaches that class and student s is enrolled in that class.

We can then ask queries such as:

`?enrolled(kevin, math273)`

This produces the result `yes`.

I can also query as such:

`?enrolled(X, math273)`

And get the following response:

`kevin, kiko`

Given these Prolog facts above, what would Prolog return when given these queries?

(a) `?enrolled(kevin,ee222)` (1)

(a) ______________________

(b) `?enrolled(kiko,math273)` (1)

(b) ______________________

(c) `?instructor(grossman,X)` (1)

(c) ______________________

(d) `?instructor(X,cs301)` (1)

(d) ______________________

(e) `?teaches(X,kevin)` (1)

(e) ______________________

Problem 15: Nested Quantifier

Let $P(x, y)$ be the statement "Student x has taken class y". The domain of x consists of all the students in our class (5002), and domain for y consists of all CS courses at Northeastern-Seattle. Express these quantifications in English.

(a) $\exists x \exists y P(x, y)$ (1)

(a) ______________________

(b) $\exists x \forall y P(x, y)$ (1)

(b) ______________________

(c) $\forall x \exists y P(x, y)$ (1)

(c) ______________________

(d) $\forall x \forall y P(x, y)$ (1)

(d) ______________________

Problem 16: Quantifiers with specific instances

Let $C(x, y)$ mean that student x is enrolled in class y, where the domain for x is all students at Northeastern-Seattle, and the domain for y consists of all classes offered this semester. Express each statement as a simple English sentence.

(a) $C(\text{Ben Bitdiddle}, \text{CS5002})$ (1)

(a) ______________________

(b) $\exists x C(x, \text{CS5520})$ (1)

(b) ______________________

(c) $\exists x (C(x, \text{CS5001}) \wedge C(x, \text{CS5004}))$ (1)

(c) ______________________

(d) $\exists x \exists y \forall z ((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z))$ (1)

(d) ______________________

Problem 17: Is the tooth fairy real?

Show that the arguments with premises (hypotheses) "The tooth fairy is a real person" and "The tooth fairy is not a real person" and conclusion "You can find gold at the end of the rainbow" is a valid argument. Does this show that the conclusion is true?

BASE CONVERSIONS

In general, we are most comfortable working with numbers that are base 10, or decimal. In computer science, we also work with numbers in base 2, or binary, or base 16, or hexadecimal. In this section, we'll practice converting numbers from one base to another.

Problem 18: Conversion

(a) $(11011)_2 = (\_\_\_\_\_\_\_\_\_\_)_{10}$ (1)

(a) ____________________

(b) $(11000111)_2 = (\_\_\_\_\_\_\_\_\_\_)_{10}$ (1)

(b) ____________________

(c) $(BADFACED)_{16} = (\_\_\_\_\_\_\_\_\_\_)_2$ (1)

(c) ____________________

(d) $375_{10} = (\_\_\_\_\_\_\_\_\_\_)_2$ (1)

(d) ____________________

Problem 19: 2's Complement

In class, we saw that 2's complement allowed us to express negative numbers without actually using a negation symbol. Convert these numbers to decimal, given the two's complement representation:

(a) 11111 (1)

(a) ____________________

(b) 010101 (1)

(b) ____________________

(c) 1010101 (1)

(c) ____________________

(d) 10000 (1)

(d) ____________________

(e) 01111 (1)

(e) ____________________

(f) 11111111111111111111111111 (1)

(f) ____________________

Problem 20: 1's Complement

There are more than one way to represent negative numbers with out a negative sign. Another way we see this done is called 1's complement. The leftmost bit (most significant) still represents the sign— 1 for negative, 0 for positive. For positive integers, the remaining bits (other than the leading 0) are identical to the binary expansion of the integer. For negative integers, the remaining bits are obtained by first finding the binary expansion of the absolute value of the integer, and then taking the complement of each of these bits, where the complement of a 1 is a 0 and the complement of a 0 is a 1 (that is, just flip the bits).

Find the one's complement representations of the following values, using a bit string length of 6.

(a) -15 (1)

(a) ____________________

(b) 31 (1)

(b) ____________________

(c) -3 (1)

(c) ____________________

(a) What's the largest and smallest number that can be represented in a 6-bit number using 1's complement? (2)

(a) ____________________

What integer (decimal) is represented by the following bit strings using 1's complement?

(a) 101010 (1)

(a) ____________________

(b) 111000 (1)

(b) ____________________

(c) 100000 (1)

(c) ____________________

(d) 111111 (1)

(d) ____________________

(e) 011111 (1)

(e) ____________________

Problem 21: Limits

For the following numbers, provide the smallest number possible to represent, the largest number, and the equivalent values in decimal.

For example, if the provided number is a 2-digit binary number, the smallest number is 00, largest is 11, and the corresponding decimal values are 0 and 3.

(a) A 3-digit decimal number. (1)

(a) ____________________

(b) A 5-digit binary number in two's complement. (1)

(b) ____________________

(c) A 7-digit binary number. (1)

(c) ____________________

Question	Points	Score
Which is a proposition?	2	
Sentence from logic statements	6	
Logic statement from sentence	5	
Truth Values	4	
Comparing phones	5	
Truth Tables, Method 1	4	
Truth Tables, Method 2	5	
Truth Tables	8	
Logically Equivalent	3	
If-Then	5	
Quantifiers	6	
Quantifiers in English	5	
Statements into logical statements	4	
Prolog: Logic Programming	5	
Nested Quantifier	4	
Quantifiers with specific instances	4	
Is the tooth fairy real?	2	
Conversion	4	
2's Complement	6	
1's Complement	10	
Limits	3	
Total:	100	

SUBMISSION DETAILS

Submit your answers to the math problems on Blackboard as a pdf. You can do this by printing out this assignment, hand-writing your answers, and then scanning in, or by using a program such as Preview on the Mac to annotate the pdf. Instructions on how to scan your document using the scanner/copiers on campus are provided on the course website ().

Things to submit:

- Submit the following on Blackboard for Assignment 1:
  - The written parts of this assignment as a .pdf named "CS5002_[lastname]_A1.pdf". For example, my file would be named "CS5002_Slaughter_A1.pdf". (There should be no brackets around your name).
  - Make sure your name is in the document as well (e.g., written on the top of the first page).