CS 5002
Jan 17th - week 2

Admin

- HW1 due 1/24 6pm
  - late deadline 2/23
  - rubric (draft)
- ICA #2 today due 1/18 6pm

Agenda

1. Implications
2. Predicate Logic
3. ICA #2
4. Laws of Logical Equivalence

0. Review

- one thing we learned $\boxed{\text{and/or}}$
- one question

bit.ly/5002_lecture_q

Demorgan's $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Truth table

P	Q	P ∨ Q
T	T	T
T	F	T
F	~~F~~ T	T
F	F	F

Logic statement:
- declarative
- truth value

Logical operators
$\vee$
$\wedge$
$\neg$

ex Negation?

All Bostonians love Dunkin'

negation → It is not the case that all Bostonians love Dunkin' ✓

All Bostonians hate Dunkin' ✗✗✗

At least one Bostonian doesn't love Dunkin' ✓

(ex) either/or

logical: $\vee$

English: thing1 or thing 2 - thing1, thing2, or both

either/or - thing1, thing2, not both

logical: $\oplus$ XOR

shortcut!

(ex) $P \oplus Q$ ... $(P \wedge \neg Q) \vee (\neg P \wedge Q)$

↳ by definition

P	Q	$P \oplus Q$	$\neg(P \oplus Q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

1. Implications

- $\vee$, $\wedge$, $\neg$ is all we need
- $\oplus$ is shortcut
- $\Rightarrow$ is shortcut ... If, then

$P \Rightarrow Q$ "p implies q"

"If P, then q"

ex PWHL, Boston has a team

$5 bet on championship

truth value needs to respect the original statement

P = Boston wins

Q = Laney gets $7.50

$P \Rightarrow Q$

If Boston wins, Laney gets $7.50

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Boston wins, Someone didn't pay!

Eng: If Boston, Laney $, otherwise Laney −$

$P \Rightarrow Q \wedge \neg P \Rightarrow R$

ex We get cake if it's my birthday $\equiv$ If it's my bday, then we get cake

P = we get cake

Q = its my birthday

$P \Rightarrow Q$? $Q \Rightarrow P$?

P	Q	$Q \Rightarrow P$
T	T	T
T	F	T
F	T	F
F	F	T

no cake on my bday :(

$\boxed{\text{if and only if}}$

$P \Longleftrightarrow Q \equiv P \Rightarrow Q \wedge Q \Rightarrow P$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

express in logic $\wedge, \vee, \neg$

- look at False outcomes
- then negate!

$P \wedge \neg Q$ $P \Rightarrow Q$ F

$\neg(P \wedge \neg Q)$ negation!

$\neg P \vee \neg\neg Q$ demorgan's

$\boxed{\neg P \lor Q}$ definition of implication

If ~~~, then ~~~   ~~~ if ~~~

if and only if   P only if Q   P if Q

If P, then Q

$P \Rightarrow Q$?

$Q \Rightarrow P$?

2. Predicate Logic

not quite a logic statement...

$x+2=3$ what is x? definition

Bostonians love Dunkin Quantity

Predicate is a generalization
- not a logic statement, no truth value

$P(x)$ $x+1=3$
$P(x,y)$ $4+x<3+y$ } no truth values :(

Turn a predicate into a logic statement.

1. Definition - plug in a value

$P(x)$ $x+1=3$ ✗

$P(2)$ $2+1=3$ ✓

$P(5)$ $5+1=3$ ✓

2. Quantifiers

$\forall$ for all

$\exists$ there exists } need to know the universe ex universe: integers

$\forall x$ ~ "for all integers x"

$\forall x \quad x+1=3$ — logic statements! But False

$\forall x \quad P(x)$

$\exists x$ — "there exists an integer x"

$\exists x \quad x+1=3$ — logic statements! True

$\exists x \quad P(x)$

7:25

5:53 PM Wed Jan 17 100%

Done ICA #2 Screenshot 2024-01-17 at 12.56.30 PM (4 of 5)

Problem #1

Consider the following English statement and its related propositions:

- You can access the website only if you have paid the subscription fee.
- P = you can access the website
- Q = you have paid the subscription fee

Write out a truth table with columns for $P \Rightarrow Q$, $Q \Rightarrow P$, and explain which implication best respects the original statement and why.

P only if Q

$P \Rightarrow Q$?

$Q \Rightarrow P$?

Side Quest

If P, then Q $\quad P \Rightarrow Q$

P if Q $\quad Q \Rightarrow P$

P if and only if Q $\quad P \Leftrightarrow Q$

Problem #1

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Write out a truth table with columns for $P \Rightarrow Q$, $Q \Rightarrow P$, and explain which implication best respects the original statement and why.

P only if Q

You can access the website only if you've paid the fee

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	
T	T	T	T	
T	F	F	T	*
F	T	T	F	*
F	F	T	T	

P only if Q

$P \Rightarrow Q$

$$P \Rightarrow Q \equiv \neg P \vee Q$$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$P \Rightarrow Q$ If P, then Q Q if Q

$Q \Rightarrow P$ If Q, then P P if Q

PM Wed Jan 17 100%

Screenshot 2024-01-17 at 12.56.35 PM (5 of 5)

For the next two problems, the domain is the set of all the characters on the *legendary* Netflix show [Cobra Kai](). Consider the following two predicates:

- *johnny(x)*, meaning "Johnny fights x"
- *karate(x)*, meaning "x studies karate"

johnny(daniel) ✓

Problem #2

Using only variables, logic symbols (¬, ∧, ∨, ⇒, ∃, ∀) and the predicates *johnny()* and *karate()*, express the following statements:

Johnny doesn't fight everyone who studies karate.

$\exists x \quad karate(x) \wedge \neg johnny(x)$

$\exists x \quad \neg johnny(x) \wedge karate(x)$

- quantifiers on left
- ∨, ∧, ¬ on logic statement (true/false) → true/false

¬johnny(x) ✓

¬x XX

↳ person

- johnny (x)

↳ person

→ nesting :(

johnny(karate(x))

↳ True/False

$\forall x \ \text{karate}(x)$ everyone studies karate

$\forall x \ \text{karate}(x) \wedge \text{johnny}(x)$

everyone studies karate and fights johnny

$\neg(\forall x \ \text{karate}(x))$ it is not the case that everyone karates

$\exists x \ \neg \text{karate}(x)$ Someone exists who doesn't study karate

3. Logical Equivalence

- Logic #1
- Logic #2

↳ simpler
fewer operators
easier to prove

$P \equiv Q$ if and only if they have the same truth values for all possible inputs

How do we prove it?

- truth table (ex: proved De Morgan's law)
  $\neg P \vee \neg Q \equiv \neg(P \wedge Q)$
- laws of logical equivalence (handout)
- To make a proof...
  - apply one law at a time
  - make one change to logic statement
  - repeat until two statements match

Which law when?

- sometimes we hit a dead end (ok!)
- keep end goal in mind
- take tiny steps!

A sampling of laws...

Demorgan's $\quad \neg P \vee \neg Q \equiv \neg(P \wedge Q)$

$\neg P \wedge \neg Q \equiv \neg(P \vee Q)$

Definition of implication $\quad P \Rightarrow Q \equiv \neg P \vee Q$

Definition of xor $\quad P \oplus Q \equiv (\neg P \wedge Q) \vee (P \wedge \neg Q)$

Associative $\quad P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

Distributive $\quad P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

Identity $\quad P \wedge T \equiv P \qquad P \vee F \equiv P$

Complement $\quad P \wedge \neg P \equiv F$

$P \vee \neg P \equiv T$

ex $\quad (P \wedge Q) \Rightarrow R \equiv P \Rightarrow (\neg Q \vee R)$

$P \Rightarrow Q \equiv \neg P \vee Q$

start $\quad (P \wedge Q) \Rightarrow R$

$\neg(P \wedge Q) \vee R$ // defn of implication

$(\neg P \vee \neg Q) \vee R$ // Demorg

$\neg P \vee (\neg Q \vee R)$ // Associative

$P \Rightarrow (\neg Q \vee R)$ // defn of implication

$\boxed{\checkmark}$

ex $\neg(P \wedge (\neg P \vee \neg Q)) \equiv \neg P \vee Q$

Start	$\neg(P \wedge (\neg P \vee \neg Q))$	
	$\neg((P \wedge \neg P) \vee (P \wedge \neg Q))$	distro
	$\neg(F \vee (P \wedge \neg Q))$	complement
	$\neg(P \wedge \neg Q)$	Identity
	$\neg P \vee \neg\neg Q$	De Morg
	$\neg P \vee Q$	double complement

$\boxed{\checkmark}$

<u>Identity law</u>

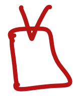
$P \wedge T \equiv P$

$P \vee F \equiv P$

P	T	F	$P \wedge T$	$P \vee F$
T	T	F	T	T
F	T	F	F	F

Implication $\quad P \Rightarrow Q \qquad$ If P, then Q

Q if P



P	Q	$P \Rightarrow Q$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$P \Rightarrow Q \equiv \neg P \vee Q$ by definition of implication

ex $\quad (P \wedge Q) \Rightarrow R \equiv \neg (P \wedge Q) \vee R$

$P \Rightarrow Q$

$\neg P \vee Q$

$\neg (P \wedge Q)$ use DeMorgs!

$(\neg P \vee \neg Q)$ maybe not DeMorg

ex universe = the integers

$P(x,y)$ $x+y=0$

1. $P(3,-3)$ $3+-3=0$

2. $\exists x\, \forall y\ P(x,y)$

there exists an integer x such that for all integers y $x+y=0$

logic statement ✓

truth value False

3. $S(x,y)$ $x \cdot y=0$

$\exists x\, \forall y\ S(x,y)$

logic statement ✓

truth value True

the only ppl johnny fights are ppl who karate

johnny	karate	??
T	T	T
T	F	F
F	T	T
F	F	T

$\forall x \; johnny(x) \Rightarrow karate(x)$