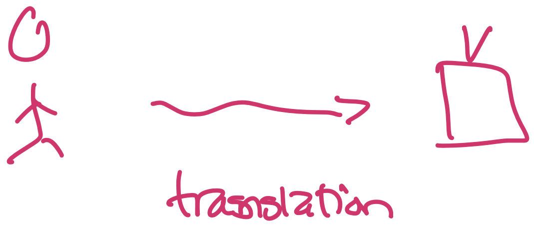
CS5002

Day One

1. Propositional Logic
2. Truth tables

1. Propositional Logic



squishy
ambiguous

clear
unambiguous

What can  understand?

CPU - central processing unit

chip

→ transistor
can have electricity, or not

Boolean logic

everything is True / False

extended ~ 1 / 0
on / off
cat / dog

} how computers are designed

An idea, statement from a human

↳ **Logic statement**

- declarative
- have a truth value — either T/F, no ambiguity

	dec	truth	
P = 7 is prime	✓	✓	True
Q = When is HW due?	x		
R = $4+2=6$	✓	✓	True
S = $1+2=6$	✓	✓	False
T = $-2 < -2$	✓	✓	False
U = $x < -2$	✓	X	what's x?
V = the Pats sucked this year	✓	X	what does it mean?
W = there is life on mars	✓	X	what is life?
X = Laney loves dogs	✓	X	Quantified all dogs 75% of dogs she's met

Sometimes:
- need definitions
- need quantifiers

Above are simple logic statements

↳ compound logic statements ⇝ logical operators

And $\wedge$

$\underset{\text{logic}}{\underline{P}} \wedge \underset{\text{logic}}{\underline{Q}}$

$P \wedge Q$ is a logic statement

ex P = Larry is an Aries

Q = Larry is a horse

$P \wedge Q$

P is True, Q is True $P \wedge Q$ True

otherwise $P \wedge Q$ False

Or $\vee$

$\underset{\text{logic}}{\underline{P}} \vee \underset{\text{logic}}{\underline{Q}}$

ex P = 7 is prime

Q = 8 is prime

$P \vee Q$ has a truth value

P is False, Q is False ... $P \vee Q$ False

Otherwise ... $P \vee Q$ True

not $\neg P$ negation

logic

If P is True, $\neg P$ is False
If P is False, $\neg P$ is True

(ex) P = Laney loves all dogs she's met
$\neg P$ = Laney hates dogs? XXXX
$\neg P$ = Laney does not love all dogs she's met
Laney has met at least one dog she doesn't love

$\wedge$	$\vee$	$\neg$
and	or	not

2. Truth Tables

- specify the truth values of every possible combination of inputs
- show 2 compound statements are equivalent
- one column == one step

Inputs

P	Q	$P \wedge Q$	$P \vee Q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

P	$\neg P$
T	F
F	T

(ex) "no eating or drinking"

P = eating is not allowed

Q = drinking is not allowed

$P \vee Q$ → sign

(True when eating is ok but drinking is not)

P = eating is allowed

Q = drinking is allowed

$\neg P$

$\neg Q$

sign → $\neg P \vee \neg Q$

$\underline{\neg P \wedge \neg Q}$ $\qquad$ $\underline{\neg (P \wedge Q)}$

which one respects the original statement?

$$\boxed{\begin{array}{c} \neg P \wedge \neg Q \\ \neg (P \vee Q) \end{array}}$$

↗ inputs

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	F	T	F	F
T	F	F	T	F	F	T	T
F	T	T	F	F	F	T	T
F	F	T	T	T	F	T	T

$P \vee Q$	$\neg(P \vee Q)$
T	F
T	F
T	F
F	T

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

DeMorgan's Law

For today's ICA, consider the statements *S*, *R*, *U*, and *V* below, about Laney's dogs, Grizz and Carol.

S = Grizz is awake
R = Grizz is hungry
U = Carol plays fetch
V = Carol jumps on Keith

Problem #1

Complete the truth table below for the statement $S \wedge \neg(R \vee U)$. Remember that, in a truth table, we always do one step per column!

S	*R*	*U*	$R \vee U$	$\neg(R \vee U)$	$S \wedge \neg(R \vee U)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

Problem #2

Translate the relevant part of each of the following English sentences into logical statements, using only the symbols ¬, ∧, and/or ∨.

A Carol never jumps on Keith!

$\neg V$ Carol does not jump on Keith

B The calmest time in our house is when Grizz is sound asleep but Carol is playing fetch.

$\neg S \wedge U$

C Grizz is always either hungry or asleep.

$R \vee \neg S$!! ? True if hungry + asleep at the same time

$(S \wedge R) \vee (\neg S \wedge \neg R)$