SOLUTIONS

CS5002
Spring 2024 - Boston
Exam #1 Practice Questions

This practice exam contains some sample questions on the same topics that will appear on Exam #1. Please complete the questions on your own, and we'll go through the solutions in class on February 14th and will also publish them on Piazza.

Exam #1 takes place on February 21st, 2024. It will be administered on paper, during our usual lecture time.

The exam is designed to be approximately 90 minutes long, but you may use the entire class time to complete it.

For the exam, you may bring one 8.5x11-inch piece of paper, with anything written or typed on it (one side only). You will submit this cheat sheet along with your exam, and you will not be permitted to use any other materials or notes during the exam.

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Part I: Logic and Truth Tables

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Practice #1 Logic - Expressing Natural Language

Consider the statements P, Q, R, and T below.

P = It is hot
Q = Molly Seidel makes the Olympic team
R = It is raining
T = Aliphine Tuliamuk makes the Olympic team

Translate each of the following English sentences into logical statements, using only the symbols $\neg$, $\wedge$, $\vee$, and/or $\Rightarrow$.

A Aliphine Tuliamuk makes the Olympic team only if Molly Seidel doesn't

Solution: $T \Rightarrow \neg Q$

B Either Molly Seidel or Aliphine Tuliamuk makes the Olympic team

Solution: $(Q \wedge \neg T) \vee (\neg Q \wedge T)$

Translate **the negation** of each of the following English sentences into logical statements, using only the symbols $\neg$, $\wedge$, $\vee$, and/or $\Rightarrow$. Simplify your response as much as possible.

C Neither Molly Seidel nor Aliphine Tuliamuk makes the Olympic team

Statement: $\neg Q \wedge \neg T$
Negation: $\neg(\neg Q \wedge \neg T) \equiv Q \vee T$

D If it is raining, then it is not hot

Statement: $R \Rightarrow \neg P$
Negation:

$\neg(R \Rightarrow \neg P)$	
$\neg(\neg R \vee \neg P)$	definition of implication
$\neg\neg R \wedge \neg\neg P$	Demorgan
$R \wedge P$	Double negation

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Practice #2 Logic - Laws of Logical Equivalence

Use the laws of logical equivalence to show that $(p \wedge q) \vee \neg p \equiv \neg p \vee q$.

Solution

$(p \wedge q) \vee \neg p$	
$\neg p \vee (p \wedge q)$	commutative
$(\neg p \vee p) \wedge (\neg p \vee q)$	distributive
$T \wedge (\neg p \vee q)$	complement
$(\neg p \vee q)$	identity

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Practice #3 Logic - Truth Table

Use a truth table to show that $(p \land q) \Rightarrow r \equiv p \Rightarrow (\neg q \lor r)$. For full credit, keep the steps small -- apply just one operation per column.

Solution

p	q	r	$p \land q$	$(p \land q) \Rightarrow r$	$\neg q$	$\neg q \lor r$	$p \Rightarrow (\neg q \lor r)$
T	T	T	T	T	F	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	F	T	T
F	T	F	F	T	F	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

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Practice #4 - Predicate Logic

Our universe is all students at Northeastern. Consider the following predicates

- Khoury(x).... x is a Khoury student
- knows(x, y)... Student x knows student y

Using these predicates, quantifiers ∀, ∃ and logical symbols ∨, ∧, ¬ and/or ⇒ express each English statement below in propositional logic:

A Some student knows every Khoury student.

Solution:
$\exists x\ \forall y\ Khoury(y)\ \Rightarrow knows(x,\ y)$

B Every Khoury student knows at least one Northeastern student who is not in Khoury

Solution:
$\forall x\ \exists y\ Khoury(x)\ \Rightarrow \neg Khoury(y)\ \wedge\ knows(x,\ y)$

C There is a student who doesn't know any Khoury students

Solution:
$\exists x \forall y\ \ Khoury(y) \Rightarrow \neg knows(x,\ y)$

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Practice #5 - Predicate Logic

Assume x and y are integers, and either explain why the following statement is true, or provide a counterexample: $\forall x \, \exists y \;\; y^2 = x$

Solution
It's not true.

Translated to English:

- For all integers x, there exists an integer y such that $y^2 = x$, i.e., y is x's square root.

Counterexample: if $x = 2$, then there is no integer y which is its square root.

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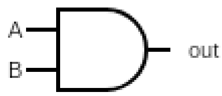
Practice #6 - circuits

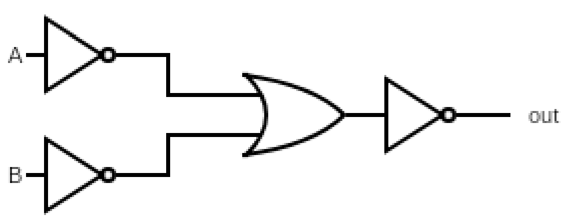
Sometimes, we might not have the particular logic gate we need available. Luckily, we can replicate the behavior of some logic gates using other gates. For the following logic gate, provide a circuit with the same behavior that uses AND, OR, XOR, and/or NOT gates, but **does not use the original gate.**



Start with the gates we can use (OR, NOT, XOR) and see if we can find a column that's either the same as the AND or the exact opposite (because then we can negate it

A	B	$A \wedge B$ (GOAL)	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(\neg A \vee \neg B)$
0	0	0	1	1	1	1	0
0	1	0	1	1	0	1	0
1	0	0	1	0	1	1	0
1	1	1	0	0	0	0	1

Sample Solution



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Practice #7 - representation of numbers

A

Convert 200_{10} to hexadecimal and binary.

Solution (hex):

$200 \div 16 = 12\,R\,8$

$12 \div 16 = 0\,R\,12$

$= C8_{16}$

Solution (binary):

$200 \div 2 = 100\,R\,0$

$100 \div 2 = 50\,R\,0$

$50 \div 2 = 25\,R\,0$

$25 \div 2 = 12\,R\,1$

$12 \div 2 = 6\,R\,0$

$6 \div 2 = 3\,R\,0$

$3 \div 2 = 1\,R\,1$

$1 \div 2 = 0\,R\,1$

$= 11001000_{2}$

B

Convert ABC_{13} to decimal

Solution:

$$ABC_{13} = (12)(1) + (11)(13) + (10)(13)(13) = 1845_{10}$$

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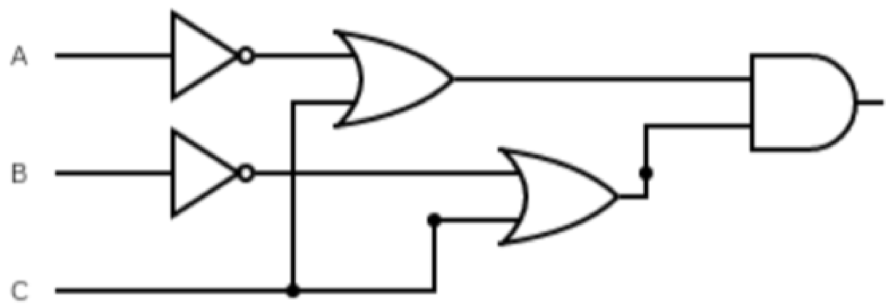
Practice #8 - circuits

Draw a circuit using only AND, OR, and/or NOT gates that directly implements the following logical expression:

$(\neg A \lor C) \land (\neg B \lor C)$

Solution



Practice #9 -- sets

Simplify the following expression by applying the set equality laws.

$(A \cup A) \cap (B \cup \overline{A})$

Solution

$(A \cup A) \cap (B \cup \overline{A})$	
$A \cap (B \cup \overline{A})$	**Idempotent**
$(A \cap B) \cup (A \cap \overline{A})$	**Distributive**
$(A \cap B) \cup \{\}$	**complement**
$(A \cap B)$	**identity**

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Practice #10 -- sets/counting

Dunkin' Donuts conducts a survey to see what kinds of coffee drinks Bostonians like. Here were the results:

- 40 like hot coffee with cream and sugar (aka, "regular")
- 37 like lattes
- 75 like iced coffee
- 19 like both regular and iced coffee
- 13 like both regular and lattes
- 10 like both iced coffee and lattes
- 4 like all three options
- 14 don't like of any of the options

How many people were surveyed?

Solution:

Let's start by defining sets: R is the set of people who like regular, L for lattes, and I for iced coffee.

The total number of people who like some coffee, any coffee, is according to the IEP

$$|R \cup L \cup I| = |R| + |L| + |I| - |R \cap L| - |R \cap I| - |L \cap I| + |R \cap L \cap I|$$
$$= 40 + 37 + 75 - 19 - 13 - 10 + 4$$
$$= 114$$

Finally, there are 14 who don't like any of the coffee options, so we get $114 + 14 = 128$

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Practice #11

How many ways can we line up 5 people for picture, of 8 total, if person 2 must be directly to the right of person 1 in every picture person 2 is included in?

Solution -- option 2 -- permutations/sum rule

We could have:

- Person 1 and person 2 not in the picture at all. 6 people are left to arrange and we want to arrange five of them: $P(6,\ 5)$
- ...OR...
- Person 1 is in the picture but person 2 is not. Person 1 could go at any of 5 positions, AND there are 6 people left to arrange in the remaining 4 positions. $5\ \cdot\ P(6,\ 4)$
- ...OR...
- Person 1 and person 2 are both in the picture, and person 2 is directly to the right, so we have a squished-together person, P1P2. They could go in any of 4 positions, AND there are 6 people left to arrange in the remaining 3 positions. $4\ \cdot\ P(6,\ 3)$

Total:

$$P(6,\ 5)\ +\ 5\ \cdot\ P(6,\ 4)\ +\ 4\ \cdot\ P(6,\ 3)\ =\ 3000$$

The cases don't overlap, so we add them all together, no overcounting.

Solution -- option 2 -- combinations to choose the people

Case 1: Person 2 is selected and Person 1 must also be selected

- Choose the remaining 3 people = 6C3
- Arrange the 3 people and the group of Person 1 & 2 = 4!

6C3 * 4! = 480

Case 2: Person 2 is not selected

- Choose the 5 people from 7 people (excluding person 2) = 7C5
- Arrange the 5 people = 5!

7C5 * 5! = 2520

Total: 480 + 2520 = 3000

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Practice #12 -- counting

How many ways are there to choose 5 distinct numbers from the set {1, 2, 3, ..., 300} such that the sum of the numbers is even?

Solution

There are 150 odds and 150 evens. How do we get an even/odd sum?

- Even + even = even
- Odd + odd = even
- Odd + even = odd

Break it into cases. All the ways to get an even sum:

- Case 1. All numbers are evens. C_5^{150}
- Case 2. 3 evens AND 2 odds. $C_3^{150} \cdot C_2^{150}$
- Case 3. 1 even AND 4 odds. $C_1^{150} \cdot C_4^{150}$

Total (cases don't overlap, no overcounting): Case 1 OR case 2 OR case 3

- $C_5^{150} + C_3^{150} \cdot C_2^{150} + C_1^{150} \cdot C_4^{150}$

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Practice #13 -- counting

You're setting the combination on your lock. You have to choose three numbers, each from 1 through 10. For your own good, the lock is constructed in such a way that no number can be used twice in a row but the same number may occur both first and third.

How many different combinations are possible? (*Hint:* Even though it's called a "combination" lock, when you unlock it, you enter the first number, the second number, and then the third number in order.)

Solution: 810

There are three numbers and order matters: __ __ __. It'll definitely require the product rule, because we're choosing a number for the first one, the second one, **and** the third one. So our expression will look like this: __ x __ x __

There are 10 ways to choose the first number. 10 x __ x __

There are 9 ways to choose the second number because it has to be different from the first: 10 x 9 x __

There are 9 ways to choose the third number, because it has to be different from the second but it can be the same as the first: 10 x 9 x 9 = 810

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Practice #14 -- sets

Simplify each of the following expressions by applying the laws of set equality. Note that the U in the first problem represents the universal set.

A $\overline{(\overline{A} \cap \overline{B}\ )} \cap U$

$\overline{(\overline{A} \cap \overline{B}\ )} \cap U$

$=\overline{(\overline{A} \cap \overline{B}\ )}$ Identity

$= \overline{\overline{A}} \cup \overline{\overline{B}}$ Demorgran

$= A \cup B$ Double complement

B $(A \cup A) \cap (B \cup \overline{A})$

$(A \cup A) \cap (B \cup \overline{A})$

$=A \cap (B \cup \overline{A})$ Idempotent

$= (A \cap B) \cup (A \cap \overline{A})$ Distro

$= (A \cap B) \cup \{\}$ complement

$= (A \cap B)$ identity

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