

CS5002

Feb 14th - week 6 ♡

Admin

- HW4 due 2/15 6pm

- late deadline 2/23 HW4-4

- exam #1 2/21 during class

- HW5 out 2/20

Agenda

1. stars and bars

2. breaking down problems [lecture 9: overcounting]

3. practice problems

6. Review

• one thing we learned

• still have a question?

product rule n^k

sum rule $n+m$

permutation $P(n,k)$ $\frac{n!}{(n-k)!}$

combination $C(n,k)$ $\frac{n!}{k!(n-k)!}$
 $\binom{n}{k} \binom{n}{k}$

How many bit strings of length 12 with 9 zeroes?

- Combo $n=12$ $\binom{12}{9}$
 $k=9$

length 12 with 3 zeroes $\binom{12}{3}$

length 12 with 9 ones $\binom{12}{9}$

1. Stars and Bars

| | |
|---|--|
| order matters rep ok n^k | order matters no rep $P(n,k)$ |
| order doesn't matter rep ok $\binom{n+k-1}{k}$? | order doesn't matter no rep $\binom{n}{k}$ |

(x) lot time... two js cookies
 real life... boxes of js cookies



indistinguishable objects
 - type 1 \neq type 2
 - can't tell different type 1's apart

5 flavors of cookies
 C, L, T, P, M

I want 6 cookies
 - order doesn't matter
 - rep ok

combination
 (... but what's n?)

$C(n, k)$
 $C(_, 6)$
 $\hookrightarrow n=5?$ no, because $C(5, 6) = ???$



\hookrightarrow 2 C's, 1 L, 1 T, 2 P



\hookrightarrow one of each, 2 mints



\hookrightarrow 4 C, 2 P

bar (|) separate categories
 star (*) one selection

check in #1: no valid outcomes can be repeated

check in #2: rep ok

check in #3: order doesn't matter

Every outcome:

- 6 stars
- 4 bars
- 10 characters

Need to count # outcomes

* = 0 Bar = 1

0 0 1 0 1 0 1 0 0 1 outcome #1
 0 1 0 1 0 1 0 1 0 0 outcome #2
 0 0 0 0 1 1 1 0 0 1 outcome #3

How many bit strings of length 10 have 6 zeros?
 $C(10, 6) = 210$

Generalize...

- rep ok
- order doesn't matter

$$C(-, k) = C(\#bars + \#stars, \#stars)$$

| | | | |
|--------------------------------|--------------------------------|--------------------------------|-----------|
| order matters rep ok | n^k | order matters no rep | $P(n, k)$ |
| order doesn't matter rep ok | $C(\#stars + \#bars, \#stars)$ | order doesn't matter no rep | $C(n, k)$ |

⊗ roulette



ball lands on 2 number in a circle

Numbers 1-36

Larry bets on evens
Keith bets on odds
- order doesn't matter
- rep ok

How many ways to win/lose 5 spins?

$C(-, 5)$ $k=5$
 $n=6$
 $\#stars + \#bars$
 $5 + 1$

| Larry | Keith | |
|----------------|---------|------------------------------|
| x x | xx | $L \times 3$ $K \times 2$ |
| *x*x*x* | | $L \times 5$ $K \times 0$ |
| *x*x*x* | x | $L \times 4$ $K \times 1$ |
| x* | x*x* | |
| x | x*x*x* | |
| | *x*x*x* | |

How many bit strings of length 6 have 5 zeros?

$$C(6, 5) = 6$$

Mini LCA

- 50 people
- 100 identical cookies

How many ways to distribute 100 cookies to 50 people?

$\#bars = 49$
 $\#stars = 100$

$$C(\#stars + \#bars, \#stars) = C(149, 100)$$

How many ways to distribute 100 cookies to 50 ppl, if everyone gets at least one?

- give 1 cookie to everyone!

- now, there are 50 cookies left \rightarrow #bars = 49
#stars = 50

$$C = (49, 50)$$

$$\binom{99}{7} = 11$$

2. Breaking Down Problems

• "and" - multiply

• "or" - add

• cases

strings of length 2-5?

cases don't overlap \star what if they do?

• "at least" / "at most"

• start by listing some valid outcomes, invalid outcomes

\hookrightarrow d1 d2 d3 d4 d5

\hookrightarrow d1 d3

• does order matter?

QS, QP = QP, QS?

if same, then order doesn't matter - count it once \star overcounting

• is repetition ok?

dogs: d1 d1? no

roulette wheel: 3, 3? yes

ex cases overlapping?

How many $\mathbb{Z}^+ \leq 100$ are divisible by 7 or by 30?

- case 1: div's by 7 = 14

{7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84,

- case 2: div's by 30 = 3

{30, 60, 90}

91, 98}

$$\binom{\text{case 1}}{\cup} \binom{\text{case 2}}$$

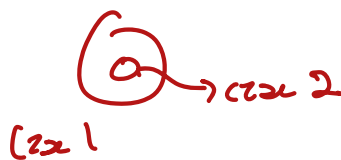
$$= |\text{case 1}| + |\text{case 2}| = 14 + 3 = 17$$

How many $\mathbb{Z}^+ \leq 100$ are divisible by 15 or 30?

- case 1: divs by 15 = 6
- case 2: divs by 30 = 3

{15, 30, 45, 60, 75, 90}
{30, 60, 90}

IEP



$$|case 1| + |case 2| - |case 1 \cap case 2|$$

$$= 6 + 3 - 3 = 6$$

How many $z^+ \leq 100$ are divisible by 2 or 7?

- case 1: divs by 2 = 50
- case 2: divs by 7 = 14

50 + 14? no, that's overcounting



$$|case 1| + |case 2| - |case 1 \cap case 2|$$

$$50 + 14 - 7 = 57$$

Overcounting \rightarrow (on purpose)

mini ICA

- How many ways are there to arrange the letters in the word KHOURY?

$$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 6!$$

- How many ways are there to arrange the letters in the word HELLO?

$$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 5!$$

E L O L H
L L

E L O L H
L L

} counted as 2 atoms, 2ndly 1

to fix the overcounting

$$\frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

ex) 4 cookies C, L, T, P

Valid outcomes

↳ I want to pick 2 to eat before class and 2 to eat after class

| | | | | |
|---------|----|----|----|-----|
| Before: | CL | CT | LP | |
| | | | | ... |
| After: | TP | LP | CT | |

↳ I want to put my cookies in 2 piles of equal size

| | | |
|----|----|----|
| CL | CT | LP |
| TP | LP | CT |

✓
Same outcomes.

8:02