CS5002

Jan 31^{st} – week 4 :)

Admin

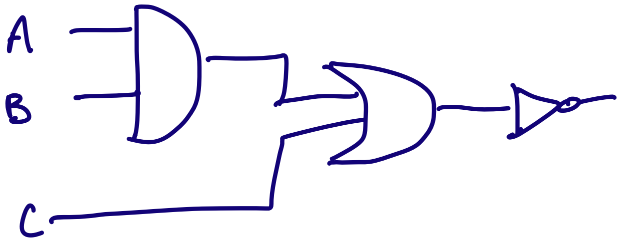
- LCA 4 due 2/1 6pm
- Hw2 <u>was</u> due 6pm
- Hw3 due 2/7 6pm
- Hw Solns
- Exam #1 2/21 during class
  - 2/7 – post practice questions
  - 2/14 – review practice solutions

Agenda

1. Sets
2. Set operations
3. Set Functions
4. LCA #4
5. Set equality

later... slides

0. Review



$\neg((A \wedge B) \vee C)$

Base 10 → base b

Euclid's division algorithm

Convert 21_{10} to two's complement

need to know #bits

add 2 two's comp numbers, get an extra bit

not overflow!

1. Sets

Set - unordered collection of distinct objects

unordered, distinct / no dupes

$S = \{1, 3, 5, 7, 9\} = \{5, 3, 1, 9, 7\} = \{9, 7, 1, 5, 3\}$

$= \{1, 1, 1, 3, 3, 5, 7, 9\}$

$\in$ is an element of

ex $3 \in S$ logic! True

$4 \in S$ False

Define a set - roster notation

ex $S = \{1, 3, 5, 7, 9\}$ $T = \{1, 3, 5, \ldots, 99\}$

set builder notation - x arbitrary element of set

ex $T = \{x \mid x > 0 \wedge x \leq 99 \wedge x \text{ is odd}\}$

Domain = integers

$\forall x$ }

$\forall x \in \mathbb{Z}$

Common Sets

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

$\{\}, \emptyset$ empty set

U universe set

$U = \{x \in \mathbb{Z} \mid 0 < x < 10\}$

$S = \{x \mid 2x \in U \wedge x \div 2 \in \mathbb{Z}\}$

$T = \{2x \mid x \in S\}$

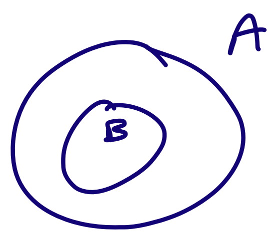
	$2x \in U$	$\wedge$	$x \div 2 \in \mathbb{Z}$	
$4 \in S$	T	$\wedge$	T	T
$8 \in S$	F			F
$3 \in S$	T	$\wedge$	F	F
$9 \in S$	T	$\wedge$	T	T

$8 \in T \quad 2x \mid x \in S$

$4 \in T \quad 2x \mid x \in S$

$A = \{1, 3, 5, 7, 9\}$

$B = \{1, 3\}$



$\in$ is an element of

something included in set

$\subseteq$ subset

$\subset$ proper subset

$B \subset A$

$\boxed{x \in B \Rightarrow x \in A \wedge A \neq B}$

$B \subseteq A$

$\boxed{x \in B \Rightarrow x \in A}$

$B \subset A$

Any sets .... every set $S \subseteq S$

$\{\} \subseteq$ every set S

$\boxed{5 \in A}$

$\boxed{\{5\} \subseteq A}$

$A = \{1, 3, 5, 7, 9\}$

$\{1, 3\} \subseteq A$ $\{3, 9\} \subseteq A$ $\{5\} \subseteq A$ $A \subseteq A$

$\{5, 7\} \subseteq A$ $\{\} \subseteq A$ $\{1, 5, 7, 9\} \subseteq A$

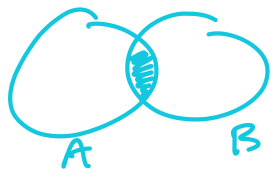
$\{\}$ contains no elements

12. Operations

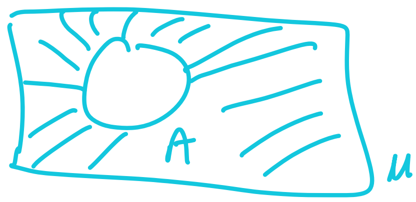
- Combine multiple sets into a new set

Union $A \cup B = \{x \mid x \in A \vee x \in B\}$

Intersect $A \cap B = \{x \mid x \in A \wedge x \in B\}$

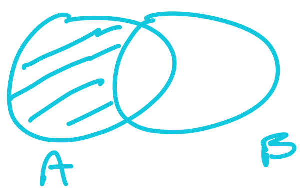


Complement $\bar{A} = A^c = \{x \mid x \notin A\}$

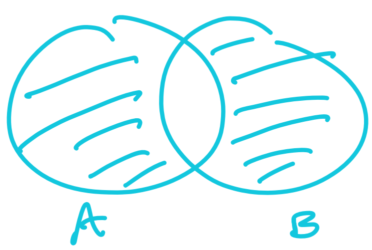


want, not need

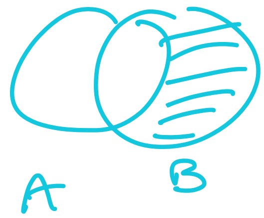
difference $A - B = A \cap \bar{B}$



Symmetric difference $A \Delta B$



$B - A$



7:37

$\boxed{\text{3. Set Functions}}$

- cardinality $|S|$ = # of distinct elements in S

ex $\mathcal{U}$ = everyone here $|\mathcal{U}| = 51$

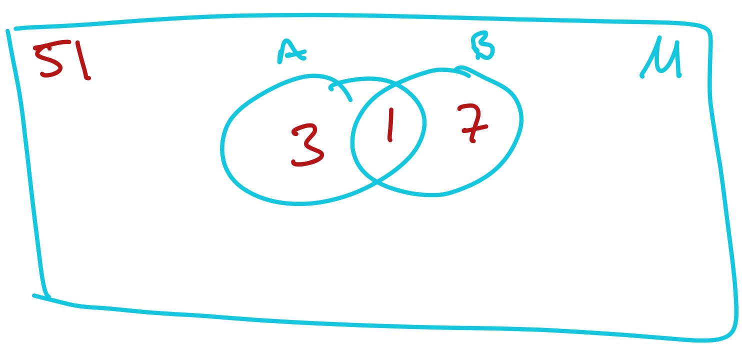
$A = \{x \mid \text{first name } \cancel{}\ M\}$ $|A| = 3$ $|A \cap B| = 1$

$B = \{x \mid \text{last name } S\}$ $|B| = 7$

Venn diagram w/ cardinalities

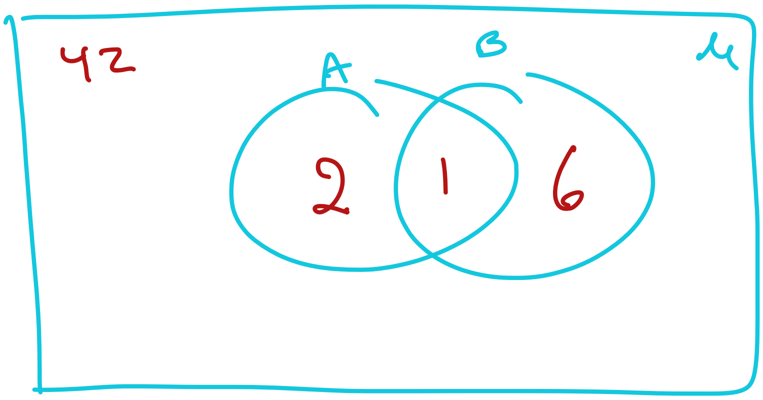
↳ every component has cardinality, no overlaps

sum of all components $= |\mathcal{U}|$



XXXX

$3 + 1 + 7 + 51 = 62$



$42 + 2 + 1 + 6 = 51 = |U|$

Powerset

$$P(S) = \{A \mid A \subseteq S\}$$

↳ subset of S

ex $S = \{a, b\}$

$$P(S) = \{\{\}, \{a\}, \{b\}, \{a,b\}\}$$

↳ set of sets

$\{a\} \subseteq S$ ~ same structure

$\{a\} \in P(S)$

$\{\{b\}\} \subseteq P(S)$ ~ same structure

$A = \{2, 4, 6\}$

$\{2,4\} \in P(A)$ T

$\{2,4\} \subseteq P(A)$ F

$\{\{2\}\} \subseteq P(A)$ T

$|P(A)| = 3$ F ~ $|P(A)| = 2^{|A|} = 2^3 = 8$

$$|P(A)| = 2^{|A|}$$

$S = \{1, 2\}$ $\quad P(S) = \{\{\}, \{1\}, \{2\}, \{1,2\}\}$

Cartesian product

$$A \times B = \{\underbrace{(a, b)}_{\text{ordered pair}} \mid a \in A \wedge b \in B\}$$

$A \times B \neq B \times A$

$(a, b) \neq (b, a)$

$|A \times B| = |A| \cdot |B|$

ex $\quad L = \{\text{WW}, \text{Diplomat}, \text{Rookie}\}$

$T = \{\text{FG}, \text{Archer}\}$

evening in Strangehouse: one Laney show, one Tom show

$L \times T = \{(\text{ww}, \text{fg}), (\text{ww}, \text{arch}),$
$(\text{dip}, \text{fg}), (\text{dip}, \text{arch}),$
$(\text{rook}, \text{fg}), (\text{rook}, \text{arch})\}$

$|L \times T| = 6$

$A = \{2, 4\}$

$B = \{4, 6\}$

$A \times B = \{(2,4), (4,4), (2,6), (4,6)\}$

ICA #4 - First two problems on gradescope

Problem #1

For each subpart below, identify whether the given expression is True or False.

A $Jon\ Knight \in A$

True

B $\{Jon\ Knight\} \in A$ ⊆ instead of ∈ True

False

C $\{Joey\ McIntyre,\ Jon\ Knight\} \subseteq A \cup B$

True

D $Donnie\ Wahlberg \in A \triangle B$ J_ or _K but not JK

False

Problem #2

For each subpart below, list the resulting sets in roster notation.

A $(A - B) \times \{Donnie\}$ {Joey} × {Donnie}

{(j, d)}

B $\overline{(A \cap B)}$ {donnie, danny, joey}

C $(A \cup B) \cap \overline{B}$

A∪B = {Jon, Joey, Jordan}

{Joey} $\overline{B}$ = {Joey, Donnie, Danny}

D $\{x \in U \mid x \in A \land x \in B\}$

{Jon, Jordan}

4. Set Equality

- $A = B$ $\quad x \in A \Rightarrow x \in B \ \wedge \ x \in B \Rightarrow x \in A$

  A and B contain the same elements

A, B can be the result of set operations

Show $A = B$ by proving with laws of set equality

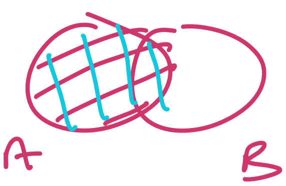
- laws in handout, same as logical equiv.

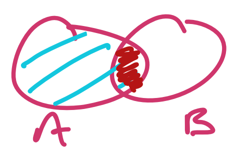
A few examples..

$A - B = A \cap \overline{B}$ def of difference

$A \cap (A \cup B) = A$ Absorption

$A \cup (A \cap B) = A$





$A \cap A = A$ Idempotent

$A \cup A = A$

Goal

$(A \cap \overline{B}) - (B \cap \overline{C}) = A \cap \overline{B}$

- proving true for <u>any</u> A, B, C

start w/ example (not a proof)

$A=\{1,2,4\}$ $\quad B=\{4,7\}$ $\quad C=\{1,2,3,4\}$

$U=\{1,2,3,4,5,6,7\}$

rhs $\quad A\cap\bar{B}$ $\qquad \bar{B}=\{1,2,3,5,6\}$

↳ $\{1,2\}$

lhs $\quad (A\cap\bar{B})-(B\cap\bar{C})$ $\qquad \bar{C}=\{5,6,7\}$

↳ $\{1,2\}$ $\quad - \quad$ ↳ $\{7\}$

$\{1,2\}$

Proof

- apply one law at a time
- label the law

$(A\cap\bar{B})-(B\cap\bar{C})=A\cap\bar{B}$

$(A\cap\bar{B})-(B\cap\bar{C})$

$(A\cap\bar{B})\cap\overline{(B\cap\bar{C})}$ $\qquad$ def. of diff

$(A\cap\bar{B})\cap(\bar{B}\cup\bar{\bar{C}})$ $\qquad$ demorg

$(A\cap\bar{B})\cap(\bar{B}\cup C)$ $\qquad$ double comp.

$A\cap(\bar{B}\cap(\bar{B}\cup C))$ $\qquad$ assoc.

$A \cap \overline{B}$ absorption :) $\boxed{\checkmark}$