CS 3800, Fall 2017 (Clinger's section) Homework 6 (40 points) Assigned: Wednesday, 18 October 2017 Due: Wednesday, 25 October 2017

- 1. [8 pts] Use the textbook's mathematical definition of a Turing machine to answer the following true/false questions:
  - (a) Can a Turing machine ever write the blank symbol on its tape?
  - (b) Can the tape alphabet  $\Gamma$  be the same as the input alphabet  $\Sigma$ ?
  - (c) Can a Turing machine's head *ever* be in the same location in two consecutive configurations?
  - (d) Can a Turing machine have only one state?
- 2. [10 pts] Explain why the following idea does not describe an algorithm that solves Hilbert's tenth problem:

Given an input that encodes a Diophantine equation E over variables  $x_1, \ldots, x_k$ , enumerate all possible combinations of integer values for  $x_1, \ldots, x_k$ , evaluating E for each combination of values. If any combination satisfies the equation E, accept. Otherwise reject.

- 3. [15 pts] Show that the collection of Turing-recognizable languages is closed under the following operations:
  - (a) union
  - (b) concatenation
  - (c) star
  - (d) intersection
- 4. [7 pts] Show that the collection of Turing-recognizable languages is not closed under complementation.