CS 3800, Fall 2017 (Clinger's section)
Homework 4 (70 points)
Assigned: Wednesday, 27 September 2017
Due: Wednesday, 4 October 2017

Unless stated otherwise, all languages are over the alphabet $\{a, b\}$.

1. [5 points] Describe two context-free languages A_1 and A_2 whose intersection $A_1 \cap A_2$ is not context-free.

2. [5 points] Write context-free grammars for both of the languages A_1 and A_2 you described in the previous question.

3. [24 points] Give state diagrams for pushdown automata that generate the following languages over the alphabet $\{a, b\}$.

   (a) $\{a^i b a^j b a^k \mid i = j + k\}$

   (b) $\{w \mid a \text{ occurs in } w \text{ twice as often as } b\}$

   (c) $\{a^i a^j a^k b^k \mid i = j + k\}$

   (d) $\{a^i b^j a^k \mid i < j \text{ and } k = j - i\}$

4. [15 points] For each of the following languages, decide whether the language is regular. If it is, then construct a regular expression that describes the language. If it isn't, construct a CFG that generates the language.

   (a) $\{w \mid w \text{ contains } babab\}$

   (b) $\{w \mid a \text{ occurs an even number of times in } w\}$

   (c) $\{w \mid b \text{ occurs at least three times in } w\}$

   (d) $\{w \mid a \text{ and } b \text{ appear the same number of times in } w\}$

   (e) $\{w \mid w \text{ contains more occurrences of } a \text{ than } b\}$

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5. [12 points] For each of the following languages, state whether the language is regular, context-free but not regular, or not context-free.

   (a) $\{a^n b^n \mid n \text{ is a positive integer}\}$

   (b) $\{a^n b^n c^n \mid n \text{ is a positive integer}\}$

   (c) $\{a^n a^n a^n \mid n \text{ is a positive integer}\}$

   (d) $\{w \mid \text{the length of } w \text{ is a prime number}\}$

   (e) $\{w^{\mathcal{R}} w \mid w \text{ is non-empty}\}$

   (f) $\{a^m b^n \mid m \text{ is prime but n is not prime}\}$

6. [4 points] Which of the following grammars generates a language that is not regular?

   (a) $S \rightarrow \text{b} \mid \text{bb}S$

   (b) $S \rightarrow \text{a} \mid \text{b}S\text{b}$

   (c) $S \rightarrow \text{a} \mid \text{b} \mid SS$

   (d) $S \rightarrow \text{a} \mid \text{b}S \mid \text{a}S$

   (e) $S \rightarrow \text{b} \mid \text{b}S\text{b}$

7. [5 points] Construct a PDA that recognizes the language generated by

$$
\begin{aligned}
S &\rightarrow A \\
A &\rightarrow \epsilon \mid 0B \mid 1C \\
B &\rightarrow A2A \\
C &\rightarrow A3A
\end{aligned}
$$