

CS 3800, Fall 2015  
Homework 4 (55 points)  
Assigned: Friday, 9 October 2015  
Due: Friday, 16 October 2015

Unless stated otherwise, all languages are over the alphabet  $\{0, 1\}$ .

1. [15 points] For each of the following regular expressions, construct a DFA that recognizes the language described by the regular expression.
  - (a)  $(0^*11^*00)^*$
  - (b)  $(0^*(11^*00)^*)^*$
  - (c)  $(0^* \cup (11^*0)^*)$
  - (d)  $(01100)^*$
  - (e)  $(0^* \cup (11^*) \cup (11^*0) \cup 0)$
  
2. [15 points] For each of the following languages, decide whether the language is regular. If it is, then construct a regular expression that describes the language. If it isn't, construct a CFG that generates the language.
  - (a)  $\{w \mid w \text{ contains more 0s than 1s}\}$
  - (b)  $\{w \mid w \text{ is divisible by 3}\}$
  - (c)  $\{w \mid w \text{ contains an odd number of 1s}\}$
  - (d)  $\{w \mid w \text{ contains at least three 1s}\}$
  - (e)  $\{w \mid w \text{ does not contain two consecutive 1s}\}$
  
3. [10 points] For each of the following languages, state whether the language is regular, context-free but not regular, or not context-free.
  - (a)  $\{w \mid w \text{ contains more 0s than 1s}\}$
  - (b)  $\{0^n1^n \mid n \text{ is a non-negative integer}\}$
  - (c)  $\{ww^R \mid w \text{ is non-empty}\}$
  - (d)  $\{w \mid w \text{ represents a multiple of 2125}\}$
  - (e)  $\{w \mid \text{the length of } w \text{ is a prime number}\}$
  - (f)  $\{w \mid w \text{ contains more 0s than 1s}\}$
  - (g)  $\{w \mid w \text{ (read as a binary numeral) is congruent to 8 mod 26}\}$
  - (h)  $\{0^n1^n0^n \mid n \text{ is a positive integer}\}$
  - (i)  $\{1^n1^n1^n \mid n \text{ is a positive integer}\}$
  - (j)  $\{1^m1^n \mid m \text{ is prime but } n \text{ is not prime}\}$
  
4. [5 points] Which of the following grammars generates a language that is not regular?

- (a)  $S \rightarrow 0 \mid 1S1$
- (b)  $S \rightarrow 1 \mid 11S$
- (c)  $S \rightarrow 0 \mid 1S \mid 0S$
- (d)  $S \rightarrow 0 \mid 1 \mid SS$
- (e)  $S \rightarrow 1 \mid 1S1$

5. [5 points] Which of the following grammars is in Chomsky normal form?

- (a)  $S \rightarrow 0 \mid 1 \mid SS$
- (b)  $S \rightarrow 1 \mid 0S$
- (c)  $S \rightarrow 0 \mid 1 \mid S0S1$
- (d)  $S \rightarrow \epsilon \mid 0 \mid 1$
- (e)  $S \rightarrow 0 \mid 1 \mid S$

6. [5 points] Construct a PDA that recognizes the language generated by

$$\begin{aligned} S &\rightarrow A \\ A &\rightarrow \epsilon \mid 0C \mid 1B \\ B &\rightarrow A0A \\ C &\rightarrow A1A \end{aligned}$$