Homework 1

1. The exclusive OR (XOR) of 3 variables A,B,C is defined as:

$$A \oplus B \oplus C \overset{\Delta}{=} (A \oplus B) \oplus C$$

   Prove by truth table the associative property of this operation:

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

2. The exclusive NOR (XNOR, also called coincidence) of 2 variables A, B is defined as:

$$A \overline{\oplus} B \overset{\Delta}{=} \overline{A \oplus B}$$

   and the XNOR of 3 variables A,B,C is defined as:

$$A \overline{\oplus} B \overline{\oplus} C \overset{\Delta}{=} (A \overline{\oplus} B) \overline{\oplus} C$$

   Prove by truth table the associative property of XNOR:

$$(A \overline{\oplus} B) \overline{\oplus} C = A \overline{\oplus} (B \overline{\oplus} C)$$

3. Prove by truth table the following:

$$A \oplus B \oplus C = A \overline{\oplus} B \overline{\oplus} C$$

4. The XOR and XNOR of 4 variables A, B, C, D are respectively defined as:

$$A \oplus B \oplus C \oplus D \overset{\Delta}{=} (A \oplus B \oplus C) \oplus D$$

   and

$$(A \overline{\oplus} B \overline{\oplus} C \overline{\oplus} D) \overset{\Delta}{=} (A \overline{\oplus} B \overline{\oplus} C) \overline{\oplus} D$$

   Represent both expressions using K-map.

5. Consider two 2-bit numbers $A = a_1 a_0$ and $B = b_1 b_0$. The value of a 2-bit number $X = x_1 x_0 0$ is defined as

$$v(X) = x_1 \times 2^1 + x_0 \times 2^0$$

   Assume that A and B are such that $|v(A) - v(B)| \le 2$. A four-variable function $f(a_1, a_2, a_3, a_4)$ is to have value 1 whenever $v(A) \le v(B)$, and value 0 otherwise. Represent this function using truth table. For those minterms representing situations that never occur (i.e. $|v(A) - v(B)| = 3$) you can use “d” (don't care). The function value for such a minterm can be considered as either 1 or 0, for the convenience of K-map simplification. Then simplify the expression of the function and implement it using as few gates as possible.

6. The advantage of Gray code over straight binary code is that Gray code changes by only 1 bit as it sequences from one number to the next. The 3-bit Gray code representations for number 0 through 7 are listed below. Design a decoder to convert a 3-bit binary number into a Gray code number. Your decoder should have three inputs and three outputs and it should convert a binary number, say 011_2 (decimal 3), to a Gray code number, `010`, for all the eight decimal numbers 0 to 7.

Decimal	Binary b_2, b_1, b_0	Gray g_2, g_1, g_0
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

More-challenging problems

1. Use Boolean algebra to check whether the following two statements are equivalent (consistent):
   - Statement A: Those who graduate from HMC with honors are the female engineers and males in other majors;
   - Statement B: The Engineering graduates are composed of males without honors and females with honors.
2. Recall that a 2-variable XOR is the negation of a 2-variable XNOR, and a 3-variable XOR is the same as a 3-variable XNOR. How do you relate the two operations of four variables?

   Can you extrapolate to find the relationship between these two operations with more variables? Justify your answer.
3. A majority function *M(x,y,z)* is equal to 1 when two or three of the variables equal to 1, i.e.:

   $$M(x, y, z) = xy + yz + zx = (x + y)(y + z)(z + x)$$

   Show that

   $$M(a, b, M(c, d, e)) = M(M(a, b, c), d, M(a, b, e))$$
4. A Karnaugh map can be used to simplify functions with as many as six variables. The truth table of a 6-variable function $f(a, b, c, d, e, f)$ has minterms $2^6 = 64$. The K-map contains four maps as shown below:

ef \ cd	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

a'b'

ef \ cd	00	01	11	10
00	16	20	28	24
01	17	21	29	25
11	19	23	31	27
10	18	22	30	26

a'b

ef \ cd	00	01	11	10
00	32	36	44	40
01	33	37	45	41
11	35	39	47	43
10	34	38	46	42

ab'

ef \ cd	00	01	11	10
00	48	52	60	56
01	49	53	61	57
11	51	55	63	59
10	50	54	62	58

ab

This K-map can also be considered as a cube with each of the three dimensions covering two variables (*ab*, *cd* and *ef*), i.e., the four combinations of variables *a* and *b* are represented in the third dimension. The simplification of a 3D K-map is similar to that of a 2D K-map, in that the neighboring 1s in the thrid dimension can also be combined, For example, these minterm pairs can be combined: (0, 16), (16, 48), (48, 32), (32, 0). But minterm pairs (0, 48) (for $a'b'$ and ab) and (16, 32) (for $a'b$ and ab') can *not* be combined.

Simplify a 6-variable function:

$$f(a, b, c, d, e, f, g) = \Sigma(0, 5, 7, 8, 9, 12, 13, 23, 24, 25, 28, 29, 37, 40, 42, 44, 46, 55, 56, 57, 58, 60, 61, 62)$$