Assignment 9

• Requirements Document
• Design Document
• Due TONIGHT! Due Sunday night.
Assignment 10

- Implementation of Assignment 9
- Due Wednesday, December 4, 2013
Amortized Time for Tree Insertion

In an amortized analysis, the time required to perform a sequence of data-structure operations is averaged over all the operations performed. (Cormen et al.)
General techniques for improving the performance of programs

[Clinger]

• Don't compute things that don't need to be computed.

• Don't recompute things if you can help it.

• Use more efficient representations and algorithms.
Example of Precomputation

- FSetString
- size()
Example of Caching

- FSetString
- size()
fib(n) {
    if n is 0, return 0;
    if n is 1 or 2, return 1;
    return fib(n-1) + fib(n-2);
}
Example of Memoization

allocate array for memo;
set all elements of memo to zero;

fib(n) {
    if n is 0, return 0;
    if n is 1 or 2, return 1;
    if memo[n] is not zero, return memo[n];
    memo[n] = fib(n-1) + fib(n-2);
    return memo[n];
}
Example of Dynamic Programming [Clinger]

• Memoization is a top-down technique.

• Dynamic programming uses the same idea, but is a bottom-up technique.
Divide-and-Conquer Algorithms
GUI

JavaPowerTools
QuickCheck
File Input and Output

Slides from:

Building Java Programs: A Back to Basics Approach, 2nd edition by Stuart Reges and Marty Stepp

http://www.buildingjavaprograms.com/slides/2ed/ch06.ppt
Proof: Theorem for Polynomial Function Big-O
Theorem

If \( f(x) \) is a polynomial function of \( x \) of degree \( k \), with positive leading coefficient and restricted to non-negative \( x \), then

\[ O(f) = O(x^k). \]
Lemma

If $f \in O(g)$ and $g \in O(f)$, then

$$O(f) = O(g).$$
Lemma: If $f \in O(g)$ and $g \in O(f)$, then $O(f) = O(g)$.

Suppose $f \in O(g)$ and $g \in O(f)$. Then there exist constants $c_0$, $c_1$, $c_2$, and $c_3$ such that:

- $c_1 > 0$
- $c_3 > 0$
- $\forall x: f(x) \leq c_0 + c_1 * g(x)$
- $\forall x: g(x) \leq c_2 + c_3 * f(x)$
Lemma

If $f(x)$ is a polynomial function of $x$ of degree $k$, restricted to non-negative $x$, then

$$f \in O(x^k).$$
Lemma: If $f(x)$ is a polynomial function of $x$ of degree $k$, restricted to non-negative $x$, then $f \in O(x^k)$.

Let

$$f(x) = a_k x^k + a_{k-1} x^{k-1} + \ldots + a_0 \text{ with } a_k > 0.$$ 

Let $c_0'$ be the maximum value of $f(x)$ taken over all $x \in [0, 1]$.

Let $c_0 = \max(0, c_0')$.

Let $c_1$ be the sum of the absolute values of the coefficients $a_i$. 
Lemma

If \( f(x) \) is a polynomial function of \( x \) of degree \( k \), with positive leading coefficient and restricted to non-negative \( x \), then

\[
(x^k) \in O(f).
\]
Lemma: If $f(x)$ is a polynomial function of $x$ of degree $k$, with positive leading coefficient and restricted to non-negative $x$, then $(x^k) \in O(f)$.

Let

\[ f(x) = a_k x^k + a_{k-1} x^{k-1} + \ldots + a_0 \]

\[ g(x) = |a_{k-1}| x^{k-1} + \ldots + |a_0| \]

\[ h(x) = a_k x^k - g(x) \]

with $a_k > 0$.

$g \in O(x^{k-1})$ so there exist positive constants $c_0$ and $c_1$ such that for all $x \geq 0$

\[ g(x) \leq c_0 + c_1 x^{k-1} \]

Let $c_2$ be the maximum value of $g(x)$ over all $x \in [0, 2 * c_1/a_k]$.

Let $c_3 = \max(0, (2 * c_0/a_k), (1/a_k) * c_2)$.

Let $c_4 = 2/a_k$. 
Theorem

If $f(x)$ is a polynomial function of $x$ of degree $k$, with positive leading coefficient and restricted to non-negative $x$, then

$$O(f) = O(x^k).$$
Code Review of Red-Black Trees