CS3000: Algorithms & Data — Summer '23 — Laney Strange

Karatsuba Algorithm (fast mulitplication)

We went over the divide-and-conquer Karatsuba algorithm in class; this handout is so you can see the function typeset in the CLRS pseudocode style. We use n to represent the number of digits in the numeric inputs, w and y. This algorithm does a version of multiplication that works especially well on large numbers and has a run-time of $\Theta(n^{\lg 3}) = \Theta(n^{1.59})$.

KARATSUBA(w, y, n)

if $n == 1$
        return $w \cdot y$
$m = \lceil (n/2) \rceil$
split w into a and b
split y into c and d
$ac =$ KARATSUBA(a, c, m)
$bd =$ KARATSUBA(b, d, m)
$bacd =$ KARATSUBA$(b - a, c - d, m)$
return $10^{2m} \cdot ac + 10^m \cdot (bacd + ac + bd) + bd$

Here's how we typeset the above function in CLRS style:

```
\begin{codebox}
\Procname{$\proc{Karatsuba}(w, y, n)$}
\li \If $n == 1$
\Then
\li \Return $w \cdot y$
\End
\li $m = \lceil(n/2)\rceil$
\li split $w$ into $a$ and $b$
\li split $y$ into $c$ and $d$
\li $\id{ac} = \proc{Karatsuba}(a, c, m)$
\li $\id{bd} = \proc{Karatsuba}(b, d, m)$
\li $\id{bacd} = \proc{Karatsuba}(b-a, c-d, m)$
\li \Return $10^{2m}\cdot ac + 10^m \cdot (bacd + ac + bd) + bd$
\end{codebox}
```