CS3000
6/20-Tues.

Admin

- exam back today / solutions
- make-up exam question 6/22
- Recitation 5 today → last recitation :)
  - 6/20 9pm → submit Rec 3-5
- Second trace HW due 6/20 9pm
- Short HW4 out now due 6/22 9pm

} no extensions

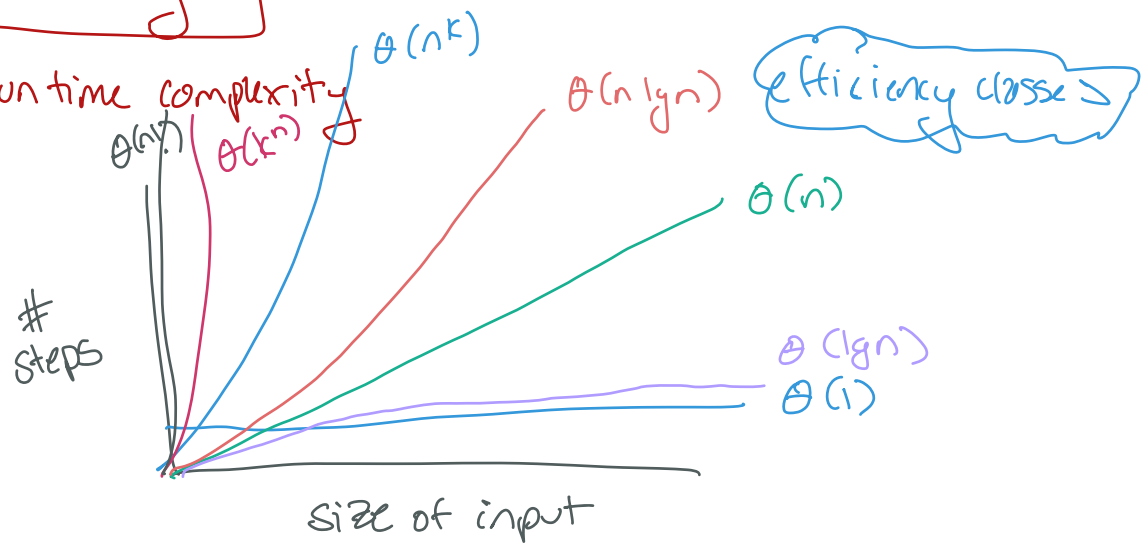
- Form for tomorrow's topics
- Please do TRACE :)

Agenda

1. Intractability
2. P vs. NP

1. Intractability

↳ run time complexity



$\theta(1)$

$\theta(\lg n)$

$\theta(n)$ ($\theta(V+E)$)

$\theta(n \lg n)$ (comparison based sorting $\Omega(n \lg n)$)

$\theta(n^k)$ (polynomial) $\theta(V \cdot E)$

tractable :)

$\theta(k^n)$ exponential

$\theta(n!)$ factorial

'intractable' :/

↳ may be a solution, but can't really run on a computer

Traveling Salesman Problem (TSP)

person

Salesman starts in Boston
wants to visit n cities → optimal way

like SSSP/APSP but visit all n before home

- only known solution is to visit all n! permutations
  pick the best one

Run-time: $\Theta(n!)$

What does n! run-time mean?

- ex: computer does $2^{16} = 65,536$ operations per second

n	n!	wall-clock time
4	24	24/65536 = .00036
8	40320	.615 seconds
16	20,922,789,888,000	3695 days
32	?	1.271×10^{20} millenia

TSP

- no known $\Theta(n^k)$ or better solution
- But, don't know if one exists or not

NP complete

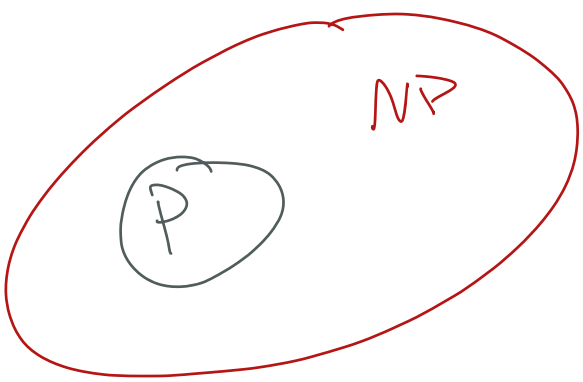
millenial problems
$1m prize to solve either way

2. P vs NP

P – solvable in $\Theta(n^k)$ time
by a deterministic Turing machine

NP – solvable in $\Theta(n^k)$ time
by a non-deterministic Turing machine

(Brute force solution)



class P
- poly time soln
- Brute force soln

class NP
- Brute force solution
- poly time soln may exist
- solution can be verified

in $\Theta(n^2)$ time

(not all problems in CS belong to NP)

NP/P ⇝ decision problems

- possible answers: yes/no
- optimization → decision
  - ex. SSSP: Is there a path from s to v of length $\leq l$?
- P vs. NP has big implications for optimization problems

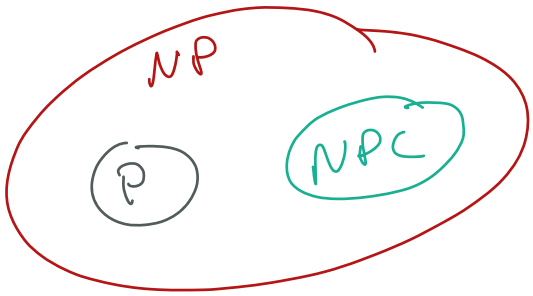
P vs. NP problems

- SSSP ⇝ $\in P$
- SSLP (general graph) ⇝ $\in NP$
  $\in P$?? no known poly time soln

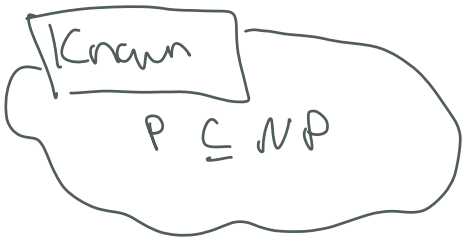
- Eulerian path (edges) ⇝ $\in P$
- Hamiltonian path (vertices) ⇝ $\in NP$
  $\in P$?? no known poly time soln

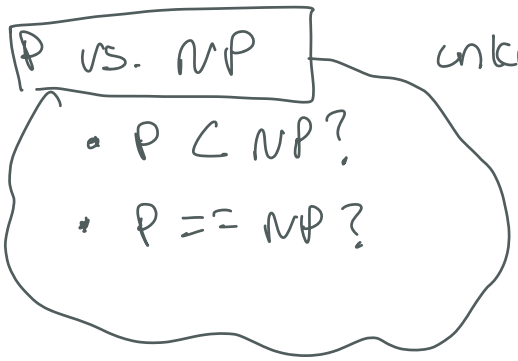
NP complete:
- in NP
- not known to be in P

} TSP, SSLP, Hamilton

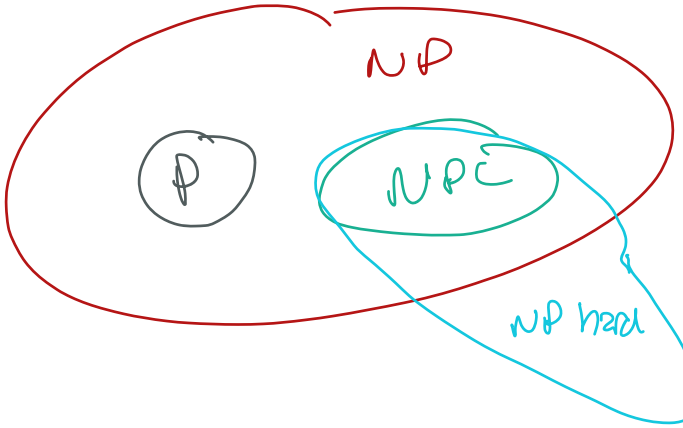


$ question:
- what is relationship between P, NP?
- All NPC are related to each other





NP-hard
- as hard as any NP problem
- all NPC are NP-hard
- some NP-hard problems not in NP



- NP hard might not even have a brute force solution!