CS3000: Algorithms & Data — Summer 2023 — Laney Strange

Homework 3 - Long
Due Wednesday May 31 @ 9pm Gradescope (extra day because of the holiday!)

Name:
Collaborators:

- Put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Wednesday May 31 @ 9pm Gradescope (extra day because of the holiday!). You may submit up to 48 hours late for no penalty, but expect a delay in grading.
- You will have an opportunity to resubmit one short homework and one long homework for new grades, at the end of the semester.
- Solutions must be typeset, preferably in LaTeX. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class, is strictly forbidden.

Problem 1. *Tatte Like Latte Part One (2 + 2 + 2 = 6 points)*

You have $\$D$ to spend on pastries at Tatte. In the bakery display, you see exactly one of each item – each has a price listed next to it, and you've personally assigned a rating of 1-10 as well. You want to spend your money in an optimal way, i.e., you want to maximize the sum of ratings on your items without going over $\$D$.

Here are the items you can buy, along with their prices and your individual ratings:

Item	Price	Your Rating
Chocolate Snail	\$5	9
Chocolate Croissant	\$4	7
Palmier	\$2	5
Monkey Bread	\$8	10

(a) What would an *optimal* solution be if you have \$10 to spend? What is the value of that solution (i.e., what is the sum of all the ratings)?

Solution:

(b) Going by ratings (largest to smallest), what would a Greedy solution be assuming you have \$10 to spend? Is it an an optimal solution?

Solution:

(c) In some versions of this problem, we compute the ratio of value (ranking) to weight (price), as shown in the table below. Using the rating-per-dollar as the way each item is evaluated, what would a Greedy solution be assuming you have \$10 to spend? Is it an an optimal solution?

Item	Price	Your Rating	Rating-Per-Dollar
Chocolate Snail	\$5	9	1.8
Chocolate Croissant	\$4	7	1.75
Palmier	\$2	5	2.5
Monkey Bread	\$8	10	1.25

Solution:

Problem 2. *Tatte Like Latte Part Two (2 + 2 + 4 = 8 points)*

You've eaten your fill of pastries, and now you have a new dollar amount to spend on Tatte's ground coffee beans. Like before, you've assigned a rating to each type of bean. Each type also has a price-per-pound listed, but you don't need to buy an entire pound of beans; instead, you can buy up to one pound of any type. You can also buy multiple types, as long as you don't go over your $\$D$ limit.

For example, here is what the menu might look like now.

Coffee Type	Price (per pound)	Your Rating
Arabica	\$8	5
Liberica	\$10	7
Excelser	\$12	8
Robusta	\$10	10

(a) Going by ratings (largest to smallest), what would a Greedy solution be assuming you have \$15 to spend and can purchase up to one pound of any coffee flavor? What would the total rating be?

Solution:

(b) In some versions of this problem, we compute the ratio of value (ranking) to weight (price), as shown in the table below. Using the rating-per-dollar as the way each item is evaluated, what would a Greedy solution be assuming you have \$15 to spend? What would the total rating be?

Item	Price	Your Rating	Rating-Per-Dollar
Arabica	\$8	5	.625
Liberica	\$10	7	.7
Excelser	\$12	8	.667
Robusta	\$10	10	1

Solution:

(c) Give the pseudocode for a greedy solution that would go by either Rating, or Rating-per-Dollar, whichever is better based on your solutions above. (You can assume you have access to any sorting algorithm we've covered.) Your algorithm should take as parameters: D, the total dollar amount you can spend, an array of coffee prices, and a corresponding array of coffee ratings. It should return the total rating of an optimal solution.

Solution:

Problem 3. *Greedy == Optimal? (2 + 4 = 6 points)*

In either Problem 1 or Problem 2, you found that the greedy strategy did well; your greedy choice was either the rating of an item, or rating-per-dollar. Now your job is to show that the greedy choice you made does yield an optimal solution.

(a) In which problem above did you find that Greedy gave you a pretty good solution? Was your greedy choice rating, or rating-per-dollar?

Solution:

(b) Let's show that your greedy choice would always yield an optimal solution for that problem.

Let S_k be a subproblem of the Tatte problem, and let a_m be the "best" selection in it (highest rating, or highest rating-per-dollar, depending on your answer above). Let A_k be an optimal solution to S_k, and assume you've spent all your money to obtain it. Let us assume that, in A_k, we don't have as much of a_m as possible. We do have some other item, a_j, which is the "best" thing in A_k. Argue that you can safely replace some/all of a_j with the remaining a_m, and end up with a solution at least as good as A_k.

Solution:

Problem 4. *Huffman Codes (4 + 2 + 2 = 8 points)*

(a) Given the follow alphabet and frequencies, use Huffman's algorithm to construct an optimal prefix-free code. Draw a resulting tree.

letter	a	g	h	i	l	m	o	r	t
frequency	12	6	10	17	7	13	24	2	9

Solution:

(b) How would *math* be encoded if we follow your tree?

Solution:

(c) In total, there are 100 characters in the file, so if we used 8 bits (one byte) to represent each character, we'd end up needing 800 bits total. How many bits would we need using Huffman instead?

Solution: