CS3000: Algorithms & Data — Summer '25 — Laney Strange

Some Useful Algo Math

At various points this semester, we're going to rely on some ol' algebra and discrete math that you probably already know. We don't expect you to remember it all at once, so this document is a quick summary of some useful things. (Many of these are also in CLRS, but we wanted you to have them all in one place :)

Polynomials

You can always do polynomials by hand or look them up, but here are a few we might encounter:

$(a + b)(c + d) = ac + bc + ad + bd$ (FOIL!)

$(a + b)^2 = a^2 + 2ab + b^2$

$(k - 1)(k - 2) = k^2 - 3k + 2$

$(k - 1)(k + 2) = k^2 + k - 2$

Exponents and Logarithms

We'll mostly use $\log_2$, which we'll denote as lg. A logarithm $\log_x y$ is asking the question: x to the power of what turns out to be y?? For example, $\lg 16$ is asking: 2 to the power of what turns out to be 16? That's why $\lg 16 = 4$

$a^b \cdot a^c = a^{b+c}$

$\frac{a^b}{a^c} = a^{b-c}$

$(\frac{a}{b})^c = \frac{a^c}{b^c}$

$(ab)^c = a^c \cdot b^c$

$a^{b^c} = a^{bc}$

$\log_c(ab) = \log_c a + \log_c b$

$\log_b a = \frac{1}{\log_a b}$

$\log_c(ab) = \log_c a + \log_c b$

Exponent to a Log

A common use-case: $2^k = n$, and we need to solve for k. Take the log base 2 of both sides and we get $k = \lg n$.

A common use-case: $n = 2^{\lg n}$. Here's the general rule:

$$a = b^{\log_b a}$$

And another we might see $k^{\lg n} = n^{\lg k}$. Here's the general rule:

$$a^{\log_b c} = c^{\log_b a}$$

Summations

Sum of the first n positive integers

$\sum_{i=1}^{n} i = 1 + 2 + 3 + ... + n$ by definition.
$= \frac{(n)(n+1)}{2}$ by a formula we can prove (with induction!)
$= \Theta(n^2)$ when we put a bound on it

What if it starts at 2 instead of 1?

$\sum_{i=2}^{n} i = \frac{(n)(n+1)}{2} - 1$
$\sum_{i=2}^{n} (i-1) = \frac{(n)(n-1)}{2}$

General arithmetic summation

$\sum_{i=1}^{n} a_i = (a + di) = \frac{(n)(\mathit{first}+\mathit{last})}{2}$

Sum of a geometric series for $x \neq 1$
$\sum_{k=0}^{n} x^k = 1 + x + x^2 + x^3 + ...$ by definition.
$= \frac{x^{n-1}-1}{x-1}$ by a formula

Summation Math

Addition inside the sigma
$\sum_{i=1}^{n} (k+i) = (k+1) + (k+2) + ... + (k+n)$

$= nk + \sum_{i=1}^{n} i$
$= nk + \frac{(n)(n+1)}{2}$

Multiplication inside the sigma
$\sum_{i=1}^{n}(ki) = k + 2k + ... + nk$
$= k \cdot \sum_{i=1}^{n} i$
$= k \cdot \frac{(n)(n+1)}{2}$

Splitting a summation
$\sum_{i=1}^{n}(ai + bi) = \sum_{i=1}^{n} ai + \sum_{i=1}^{n} bi$

Re-indexing a summation
$\sum_{k=0}^{n} a_{n-k} = \sum_{j=0}^{n} a_j$