CS3000: Algorithms & Data — Summer '25 — Laney Strange

DFS and Topological Sort

The algorithm below creates a topological sort of a directed, acyclic graph. A topological sort is a linear ordering of all the vertices such that if G contains an edge (u, v) then u appears before v in that ordering.

This algorithm relies on Depth-First Search, which is a cousin of Breadth-First Search. Instead of visiting all the neighbors of a given node (breadth), DFS explores as far down a path as it possibly can (depth), and backtracks after it has exhausted a path.

TOPOLOGICAL-SORT(G)

call DFS(G) to compute finishing times $v.f$ for each vertex v
return the vertices in descending order of finish time

The run-time of Topological sort is $\Theta(V \lg V)$ because it is bound by the sort of all the finish times. The run-time of DFS on its own is $\Theta(V + E)$.

DFS(G)

for each vertex $u \in G.V$
    $u.color = \text{WHITE}$
    $u.\pi = \text{NIL}$
$time = 0$
for each vertex $u \in G.V$
    if $u.color == \text{WHITE}$
        DFS-VISIT(G, u)

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$time = time + 1$
$u.s = time$
$u.color = \text{GRAY}$
for each $v \in G.Adj[u]$
    if $v.color == \text{WHITE}$
        $v.\pi = u$
        DFS-VISIT(G, v)
$u.color = \text{BLACK}$
$time = time + 1$
$u.f = time$

We typeset the procedures above with the following LaTeX:

```
\begin{codebox}
\Procname{$\proc{Topological-Sort}(G)$}
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\li call $\proc{DFS}(G)$ to compute finishing times $v.f$ for each vertex $v$
\li \Return the vertices in descending order of finish time
\end{codebox}

\begin{codebox}
\Procname{$\proc{DFS}(G)$}
\li \For each vertex $u \in G.V$
\Do
\li $\id{u.color} \gets \const{White}$
\li $u.\pi \gets \const{Nil}$
\End
\li $\id{time} \gets 0$
\li \For each vertex $u \in G.V$
\Do
\li \If $\id{u.color} == \const{White}$
\Then
\li $\proc{DFS-Visit}(G, u)$
\end{codebox}

\begin{codebox}
\Procname{$\proc{DFS-Visit}(G, u)$}
\li $\id{time} \gets \id{time} + 1$
\li $u.s \gets \id{time}$
\li $\id{u.color} \gets \const{Gray}$
\li \For each $v \in G.Adj[u]$
\Do
\li \If $\id{v.color} == \const{White}$
\Then
\li $v.\pi \gets u$
\li $\proc{DFS-Visit}(G, v)$
\End
\End
\li $\id{u.color} \gets \const{Black}$
\li $\id{time} \gets \id{time} + 1$
\li $u.f \gets \id{time}$
\end{codebox}
```