CS3000: Algorithms & Data — Summer 2025 — Laney Strange

Recitation 3
Date: May 27th, 2025

Name: Sample Solution

- Recitation problems are for practice only. We'll go over the solutions during your scheduled recitation on Tuesday!
- We will provide `.tex` starter files for recitations, just as we do for homeworks. For most recitations, we strongly encourage you to work out your solution in LaTeXto practice with typesetting.
- Collaboration is strongly encouraged during recitation!

Problem 1. *A Suboptimal Strategy*

Consider the rod-cutting problem from class today.[1] Use a counterexample to show that the following strategy does not always determine an optimal way to cut rods:

- Define the density (value per inch) of a rod of length i to be $d_i = p_i/i$.
- Starting with a length n rod, cut off a piece of length i which has the maximum density d_i for all values $1 \leq i \leq n$.
- Repeat the process of cutting off pieces which have maximum density d_i (for valid i) among all remaining pieces until the rod is not cut into smaller pieces anymore.

Solution:

Here is a counterexample for which the above strategy would not yield an optimal solution.

Lengthl i (in inches)	1	2	3	4
Prices p_i (in dollars)	1	20	33	36
Densities $d_i = p_i/i$	1	10	11	9

Let the initial rod length is 4. On following the strategy above, we could first cut a rod of length 3 for a price of 33. This leaves us with a rod of length 1, of price 1. The total price following this strategy would be 34.

However, the optimal solution would be to cut the rod into two rods of length 2, each fetching 20 for a total of $40 > 34$.

[1] Given a rod of n inches, as well as prices for rod's of length $1 \leq i \leq n$, find an optimal cutting of the rod into smaller pieces to maximize revenue r_n.

Problem 2. *DP Value and Solution*

Below is a modified version of the auxiliary function MEMOIZED-CUT-ROD-AUX that we discussed in class today. It accepts four parameters: price list p, rod length n, array of values r, and array of sizes s.

The key difference between this and the original version is that it completes the array of sizes s such that $s[i]$ contains the value j indicating that an optimal cut for a rod of length i is j inches.

Modify today's oversight function MEMOIZED-ROD-CUT (which you can find on the course website) to (1) use the function below to construct a solution, and (2) print out the actual cuts in an optimal solution, in addition to its value.

MEMOIZED-CUT-ROD-AUX-V2(p, n, r, s)

```
1  if r[n] ≥ 0
2      return (r[n], s)
3  if n == 0
4      q = 0
5  else
6      q = −∞
7      for i = 1 to n
8          (val, s) = MEMOIZED-CUT-ROD-AUX-V2(p, n − i, r, s)
9          if q < p[i] + val
10             q = p[i] + val
11             s[n] = i
12 r[n] = q
13 return (q, s)
```

Solution:

MEMOIZED-CUT-ROD-V2(p, n)

```
1  let r[0 : n] and s[0 : n] be new arrays
2  for i = 0 to n
3      r[i] = −∞
4  (val, s) = MEMOIZED-CUT-ROD-AUX-V2(p, n, r, s)
5  print val
6  j = n
7  while j > 0
8      print s[j]
9      j = j − s[j]
```