CS3000: Algorithms & Data — Summer 2025 — Laney Strange

Recitation 2
Date: May 13th, 2025

Name:

- Recitation problems are for practice only. We'll go over the solutions during your scheduled recitation on Tuesday!
- We will provide `.tex` starter files for recitations, just as we do for homeworks. For most recitations, we encourage you to work out your solution in LaTeXto practice with typesetting.
- Collaboration is strongly encouraged during recitation!

Problem 1. *Proof by Induction - Summation*

Prove that the sum of the first n positive integers is $\frac{(n)(n+1)}{2}$

Predicate $S(n)$ states that $\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$.

Logic Statement $\forall n \in \mathbb{Z}^+, S(n)$

In your solution below, make sure you include the base case, inductive hypothesis (assume true for $S(k)$ for an arbitrary k), and inductive step (show $S(k) \implies S(k+1)$).

Solution:

Problem 2. *Proof by Induction - Correctness*

Consider the pseudocode below for a recursive algorithm.

```
RECURSIVE(n)
1  if n == 1
2        return 1
3  return n + RECURSIVE(n − 1)
```

- What would this function return in the following examples?

  1. RECURSIVE(1)

     Solution:

  2. RECURSIVE(2)
     Solution:

  3. RECURSIVE(3)
     Solution:

  4. RECURSIVE(4)
     Solution:

- Show by mathematical induction that, in general, RECURSIVE(n) returns $n+(n-1)+(n-2)+...+1$ for all positive integers n.

  Solution: