

Prim's Algorithm

Prim's algorithm finds a Minimum Spanning Tree for a given graph. It takes in a graph G , a weight function w , and a starting vertex r . Instead of maintaining a set A of edges, it maintains a *key* and π attribute for each vertex. The *key* attribute for a vertex v contains the weight of edge (u, v) , and the π attribute contains the node that immediately preceded that vertex in the spanning tree.

Prim's algorithm uses three Heap operations: INSERT, which inserts an element into the heap and re-heapifies, EXTRACT-MIN, which removes and returns the smallest element and re-heapifies, and DECREASE-KEY, which changes the value of a specific node and re-heapifies.

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PRIM( $G, w, r$ )
1  for each vertex  $u \in G.V$ 
2       $u.key = \infty$ 
3       $u.\pi = \text{NIL}$ 
4   $r.key = 0$ 
5   $H = \{\}$ 
6  for each vertex  $u \in G.V$ 
7      INSERT( $H, u$ )
8  while  $H \neq \{\}$ 
9       $u = \text{EXTRACT-MIN}(H)$ 
10     for each vertex  $v \in G.adj[u]$ 
11         if  $v \in H$  and  $w(u, v) < v.key$ 
12              $v.\pi = u$ 
13              $v.key = w(u, v)$ 
14             DECREASE-KEY( $H, v, w(u, v)$ )

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We typeset the Prim's procedure above with the following L^AT_EX:

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\begin{codebox}
\Procname{$\proc{Prim}(G, w, r)$}
\li \For each vertex $u$ \in $G.V$
\Do
\li $\id{u.key}$ \gets $\infty$
\li $\id{u.\pi}$ \gets $\const{Nil}$
\End
\li $\id{r.key}$ \gets $0$
\li $H$ \gets $\{\}$
\li \For each vertex $u$ \in $G.V$
\Do
\li $\proc{Insert}(H, u)$
\End
\li \While $H$ \ne $\{\}$
\Do

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\li $u \gets \text{proc}\{\text{Extract-Min}\}(H)$
\li \text{For each vertex } $v$ \text{ in } $G.\text{adj}[u]$
\Do
\li \text{If } $v$ \text{ in } $H$ \text{ and } $\text{w}(u, v) < \text{id}\{v.\text{key}\}$
\Then
\li $\text{id}\{v.\text{key}\} \text{ gets } u$
\li $\text{id}\{v.\text{key}\} \text{ gets } \text{w}(u, v)$
\li $\text{proc}\{\text{Decrease-Key}\}(H, v, \text{w}(u, v))$
\end{codebox}

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