CS3000: Algorithms & Data — Summer 2025 — Laney Strange

APP 3

Due: May 15th, 2025 @ 11:30am via Gradescope

Name: Sample Solution

- APPs will be assigned towards the end of roughly two lectures each week. You'll put together a solution to a short problem that we'll all use in the following lecture. We'll have time set aside to do these in class, or you can work on your own.

- You may handwrite your solutions, or typeset them in LaTeXor another system.

- APPs will be graded on completeness. They must be submitted by 11:30am (just before lecture) on the due date. They will not be accepted late, but we drop 3 of them (out of 8 total).

- Collaboration is strongly encouraged for APPs!

Problem 1.

Below is the pseudocode for Quicksort. Complete the proof we started in class to show the correctness of PARTITION. In particular, use a loop invariant to show that, at the beginning of each iteration of the loop at lines 3-6, for any array index k, if $i + 1 \leq k \leq j - 1$ then $A[k] > x$.

QUICKSORT(A, p, r)

```
1  if p < r
2      q = PARTITION(A, p, r)
3      QUICKSORT(A, p, q − 1)
4      QUICKSORT(A, q + 1, r)
```

PARTITION(A, p, r)

```
1  x = A[r]
2  i = p − 1
3  for j = p to r − 1
4      if A[j] ≤ x
5          i = i + 1
6          swap A[i], A[j]
7  swap A[i + 1], A[r]
8  return i + 1
```

Solution:

- **Initialization:** Prior to the first iteration of the loop, we have $i = p - 1, j = p$. No values lie between $i + 1$ and $j - 1$, so our condition is trivially true.

- **Maintenance:** Consider two cases based on the test in line 4. If $A[j] > x$ then the only thing we do is increment j. After j has been incremented, the condition holds for $A[j - 1]$ and all other entries remain unchanged. If, on the other hand, $A[j] \leq x$, then we increment i and swap $A[i], A[j]$. The item that was swapped into $A[j - 1]$ is, by the loop invariant, greater than x.

- **Termination:** We have $j = r$ when the loop terminates. Therefore the unexamined subarray $A[j : r - 1] = A[r : r - 1]$ is empty, and the values in the array greater than x have been partitioned into the region bounded by positions $i + 1$ and $r - 1$.