As you get settled...

- Get out your notes
- Get out a place to do today's ICA (7)
  - Please remember to write your name, my name, the ICA #, and the date!

Warm-up 0: do the matrix multiply defined below

$$A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & 4 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 0 & 4 \end{bmatrix}$$

$$AB = \underbrace{\begin{bmatrix} 0+6+0 & 0-6+8 \\ 1+8+0 & -1-8-4 \end{bmatrix}}_{2\times 2} = \begin{bmatrix} 6 & 2 \\ 9 & -13 \end{bmatrix}$$

Vector spaces

ICA Question 1: build a matrix transform in $f: \mathbb{R}^{2x1} \rightarrow \mathbb{R}^{2x1}$ that...

a) halves the magnitude of x_0

b) rotates x_1 clockwise by π → yes, this is an odd concept, but will let us practice more! :)

To solve this:

1) draw a picture of the domain and the codomain with at least two points showing their original and transformed locations

2) solve the equations for the column vectors of A

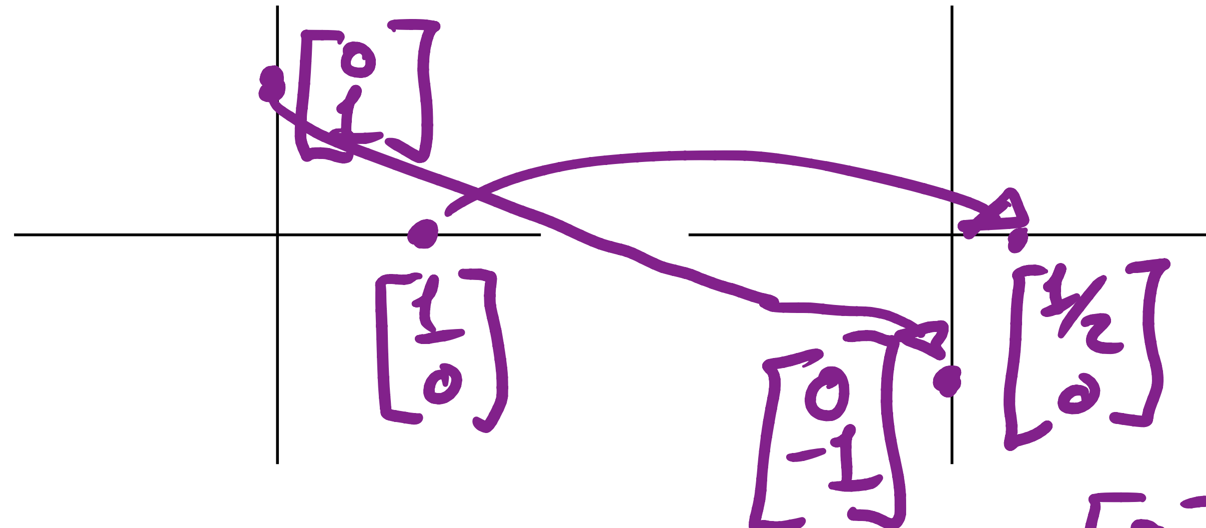
Matrix Transforms

A: 2x2 HW 2

ICA Question 1: build a matrix transform in $f : \mathbb{R}^{2x1} \rightarrow \mathbb{R}^{2x1}$ that...

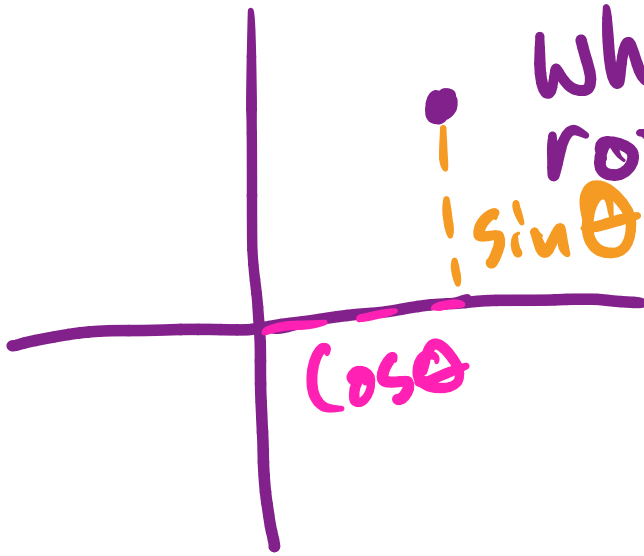
a) halves the magnitude of x_0

b) rotates x_1 clockwise by π



$$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1/2\\0\end{bmatrix} = \begin{bmatrix}1 & 1\\ a_0 & a_1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = a_0$$

$$A\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix}$$

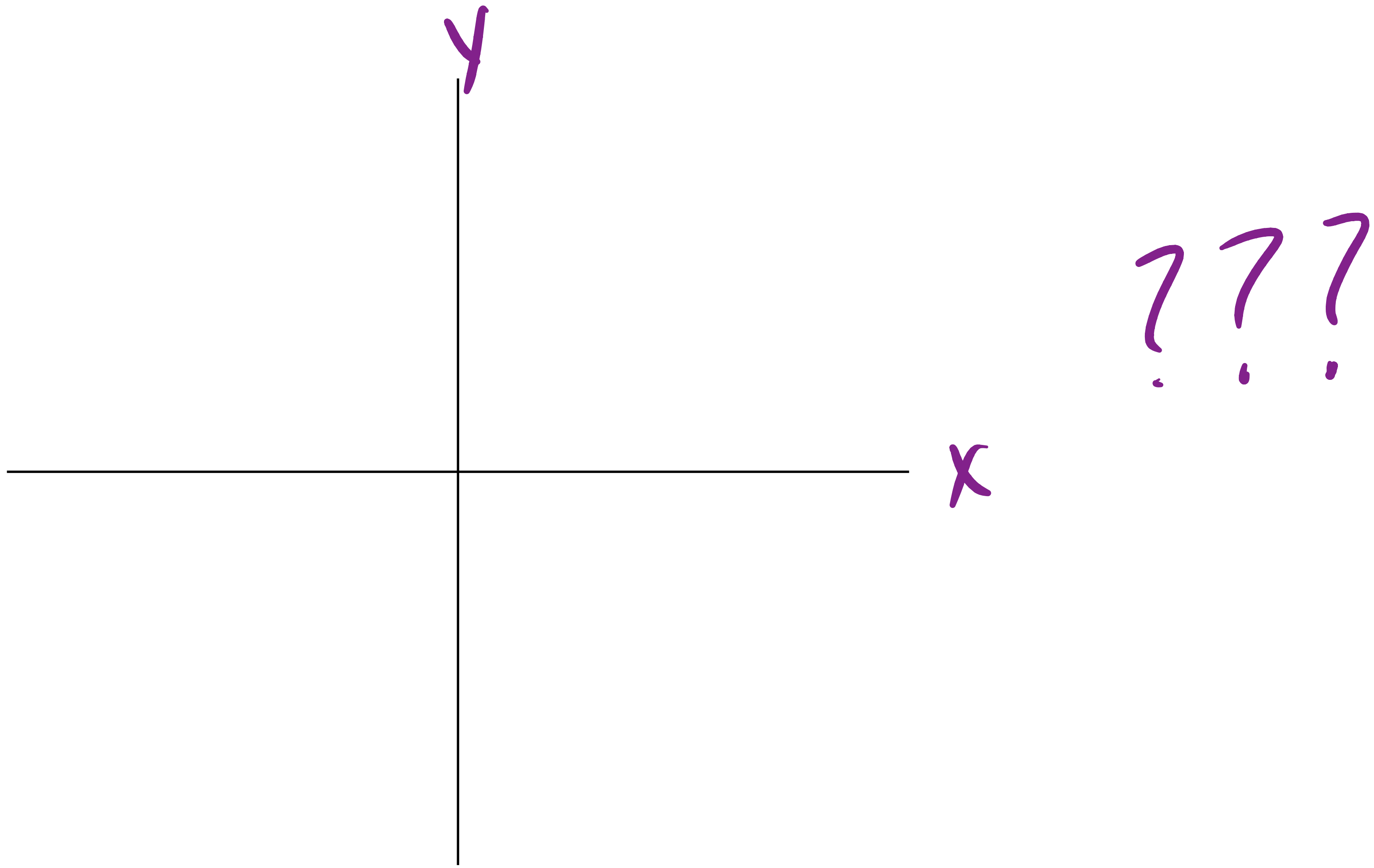


when we're rotating

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} \rightarrow \begin{bmatrix}0\\-1\end{bmatrix}$$

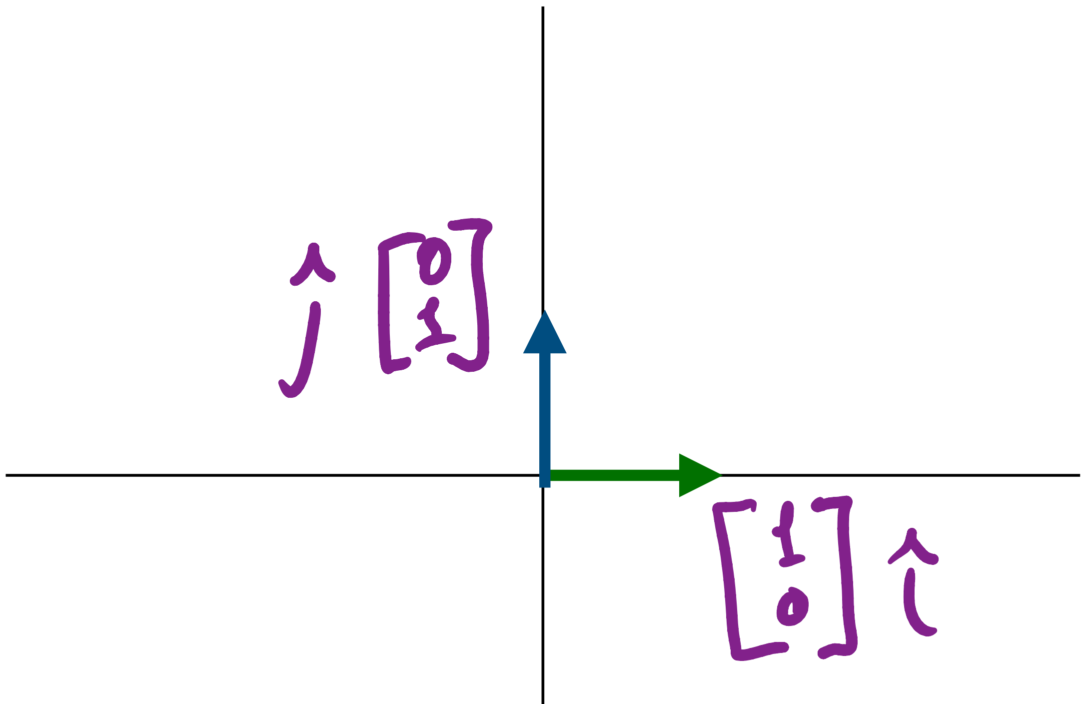
about rotation → End of lec 6 for all the details

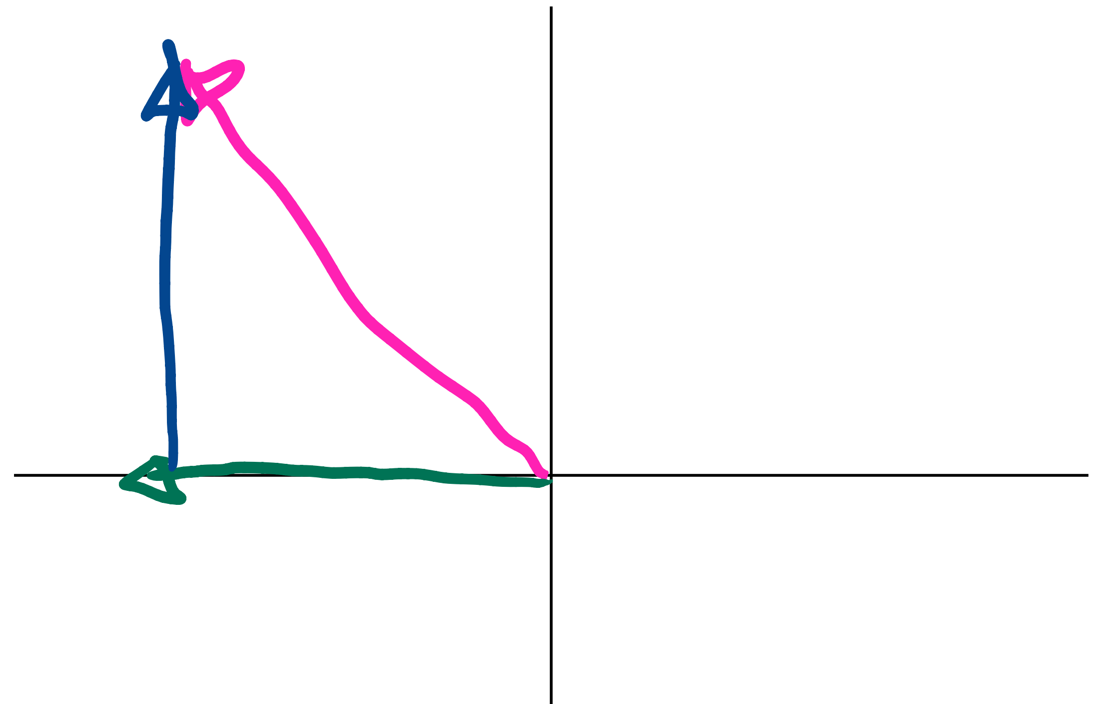
What is a vector space?



Basis vectors

- The basis vectors of a vector space are the vectors that "define it"
- For example, in the x y coordinate system:

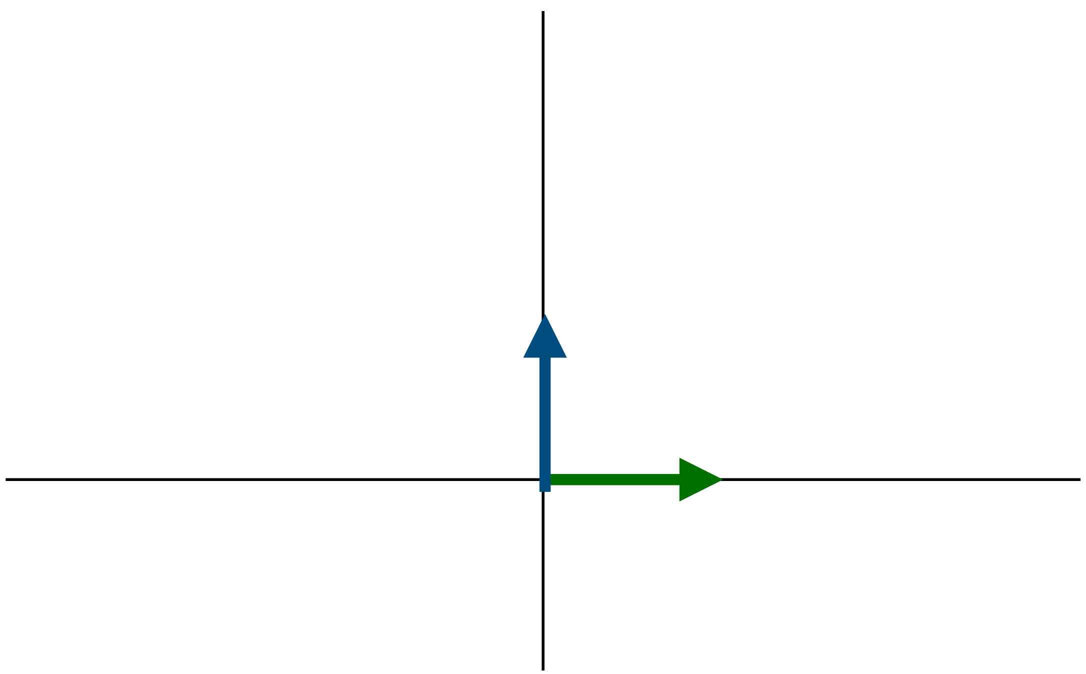


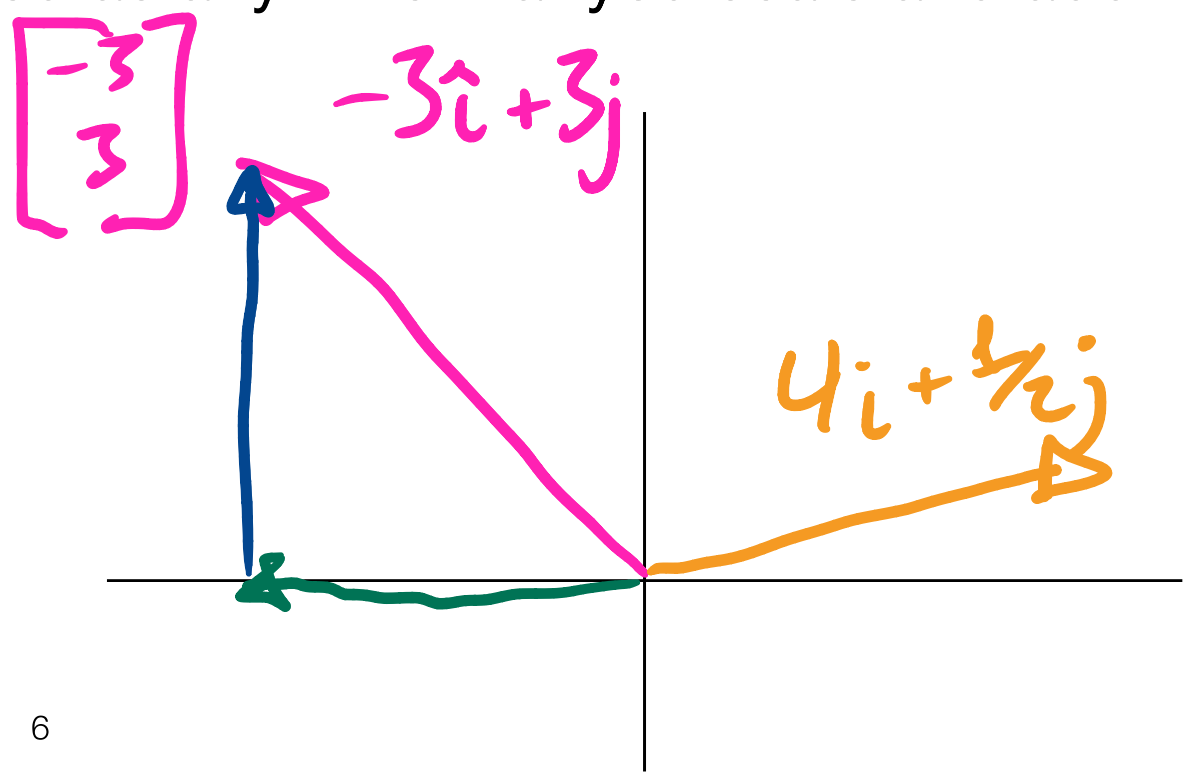


- We can reach any coordinate with a scaled combination of $\hat{i}$ and $\hat{j}$

Linear combinations

- We looked at this definition last time: A **linear combination** of $x_0, x_1, x_2 \ldots$ is $\alpha_0 x_0 + \alpha_1 x_1 + \alpha_2 x_2 \ldots$
- What if we ground this in geometry?
- A **linear combination** can be viewed as any time that you scale and add vectors





Basis vectors

- Every time that we choose to describe a vector as coordinates, we've made a choice of the basis vectors.

- Implicitly, this is $\hat{i}$ and $\hat{j}$.

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 3\hat{i} + 4\hat{j} \qquad \begin{bmatrix} -4271 \\ 501 \end{bmatrix} = -4271\hat{i} + 501\hat{j}$$

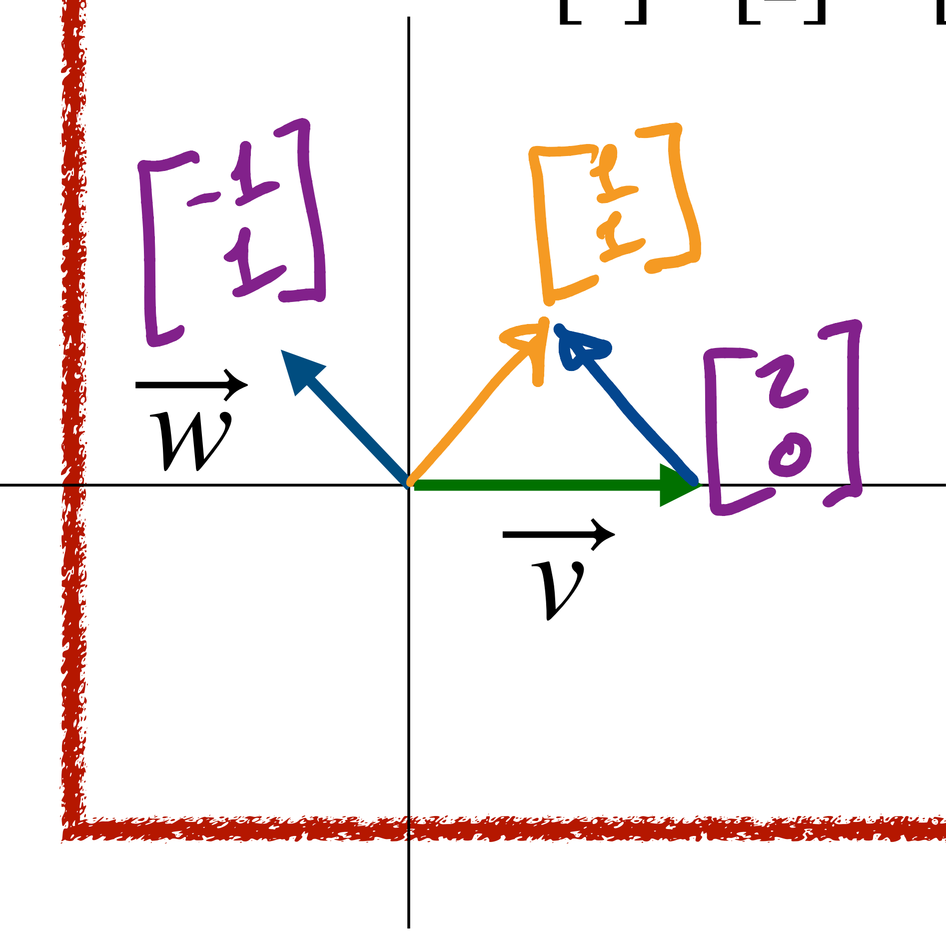
- ... but, what if it wasn't?

1 minute!

Basis vectors

ICA Question ~~1~~ 2: what coordinates can you reach in two dimensions by scaling and adding the following two vectors?

Can you reach $\begin{bmatrix}1\\1\end{bmatrix}$? $\begin{bmatrix}0\\2\end{bmatrix}$? $\begin{bmatrix}-32\\2\end{bmatrix}$?



$$\begin{bmatrix}1\\1\end{bmatrix} = \vec{v} + \vec{w}$$

$$\begin{bmatrix}0\\2\end{bmatrix} = 2\vec{w} + \vec{v}$$

$$\begin{bmatrix}-32\\2\end{bmatrix} = 2\vec{w} + -15\vec{v} = \begin{bmatrix}-2\\2\end{bmatrix} - \begin{bmatrix}30\\0\end{bmatrix}$$

Basis vectors

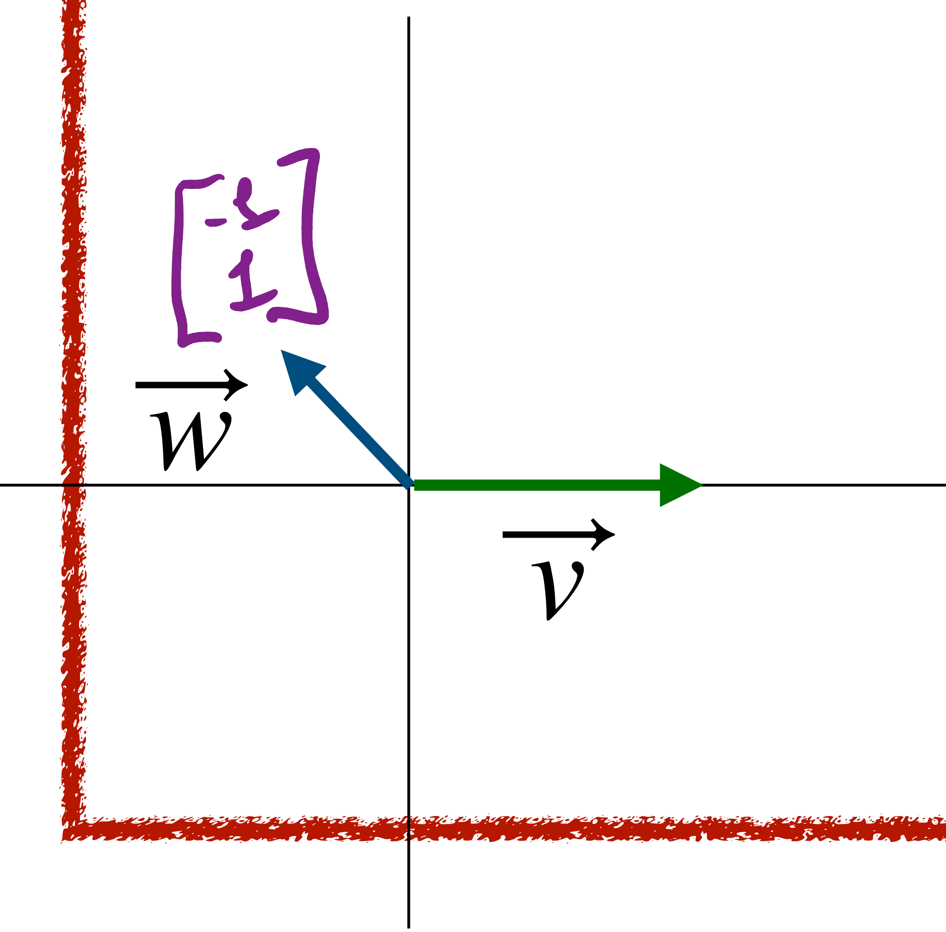
ICA Question 2: what is the equivalent linear combination of $\hat{i}$ and $\hat{j}$ to ....

basis of $\hat{i} + \hat{j}$

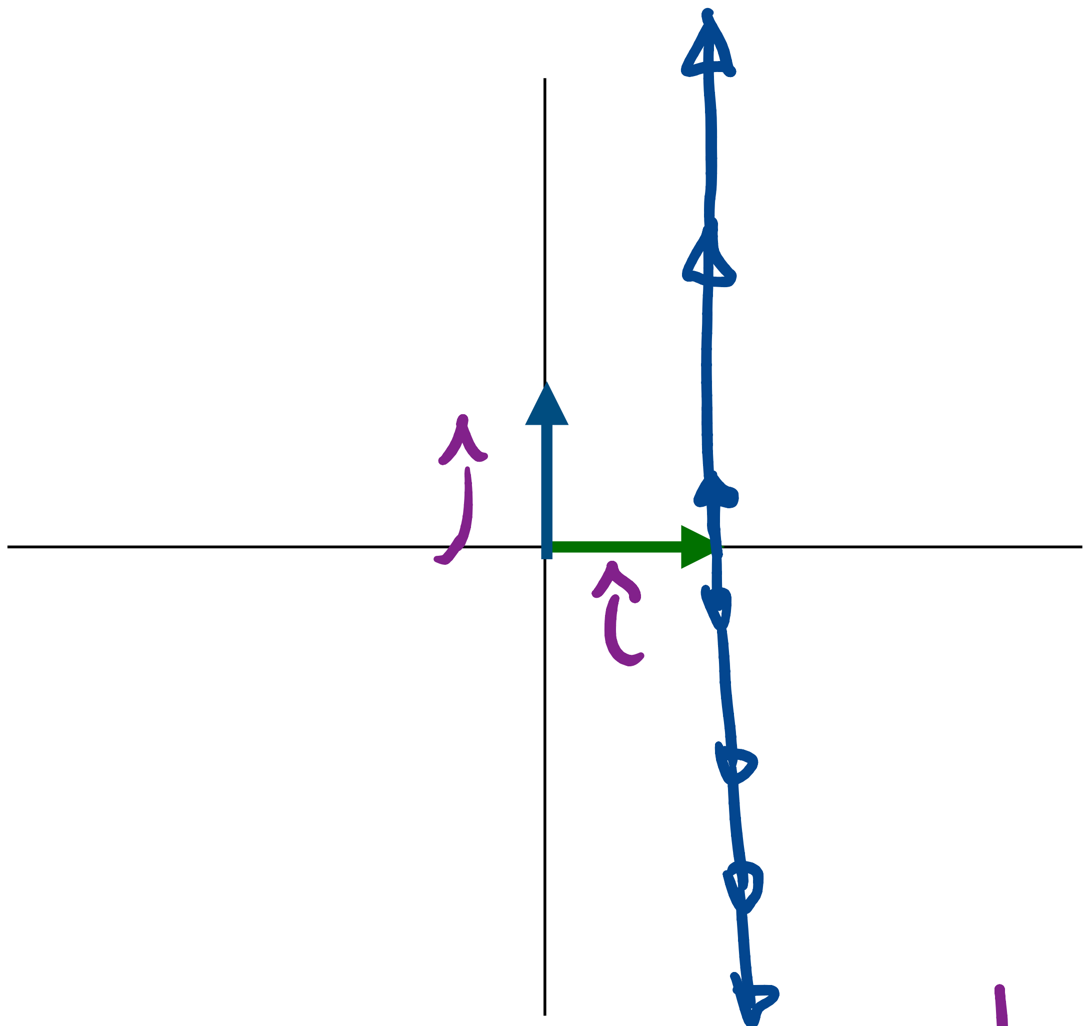
$$37\vec{v} + 23\vec{w} = 37\begin{bmatrix}2\\0\end{bmatrix} + 23\begin{bmatrix}-1\\1\end{bmatrix}$$

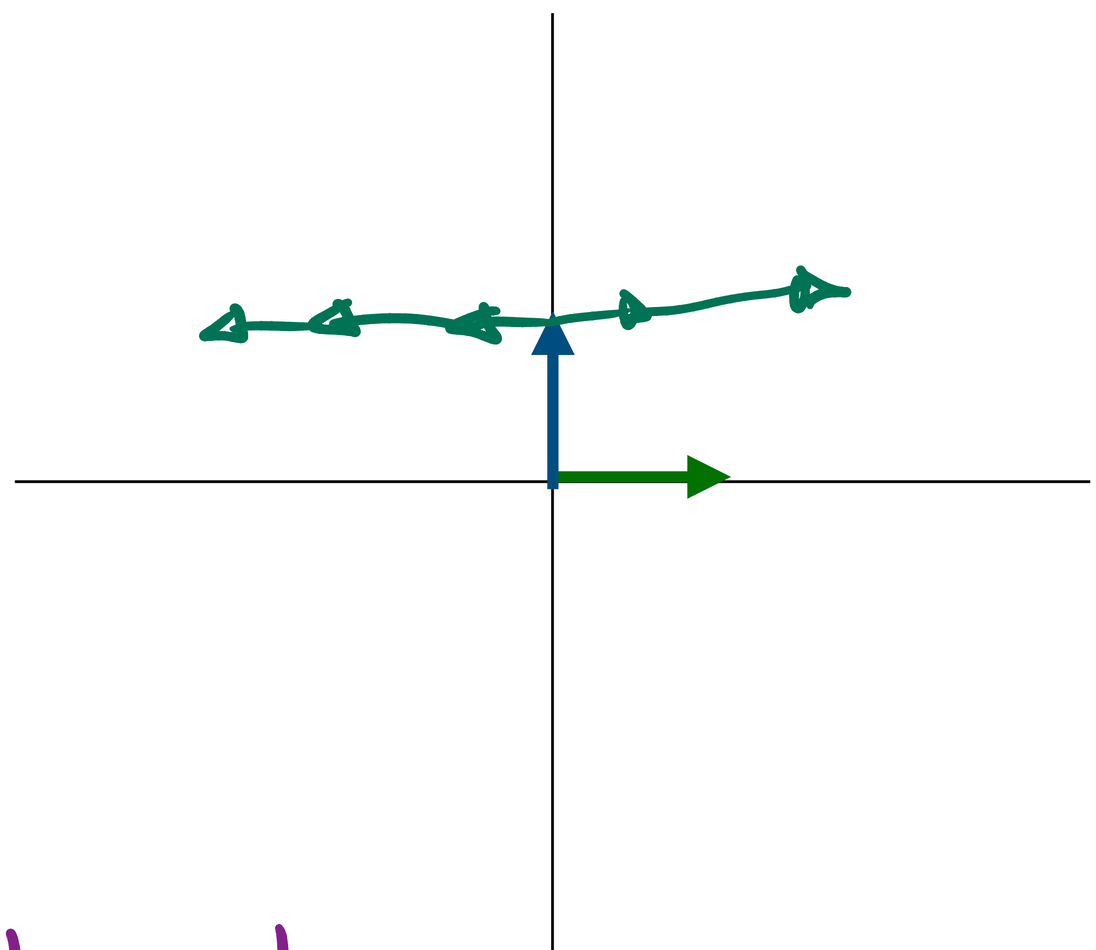
$$\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}51\\23\end{bmatrix}$$

$$(-\hat{i} + \hat{j}) = 51\hat{i} + 23\hat{j}$$



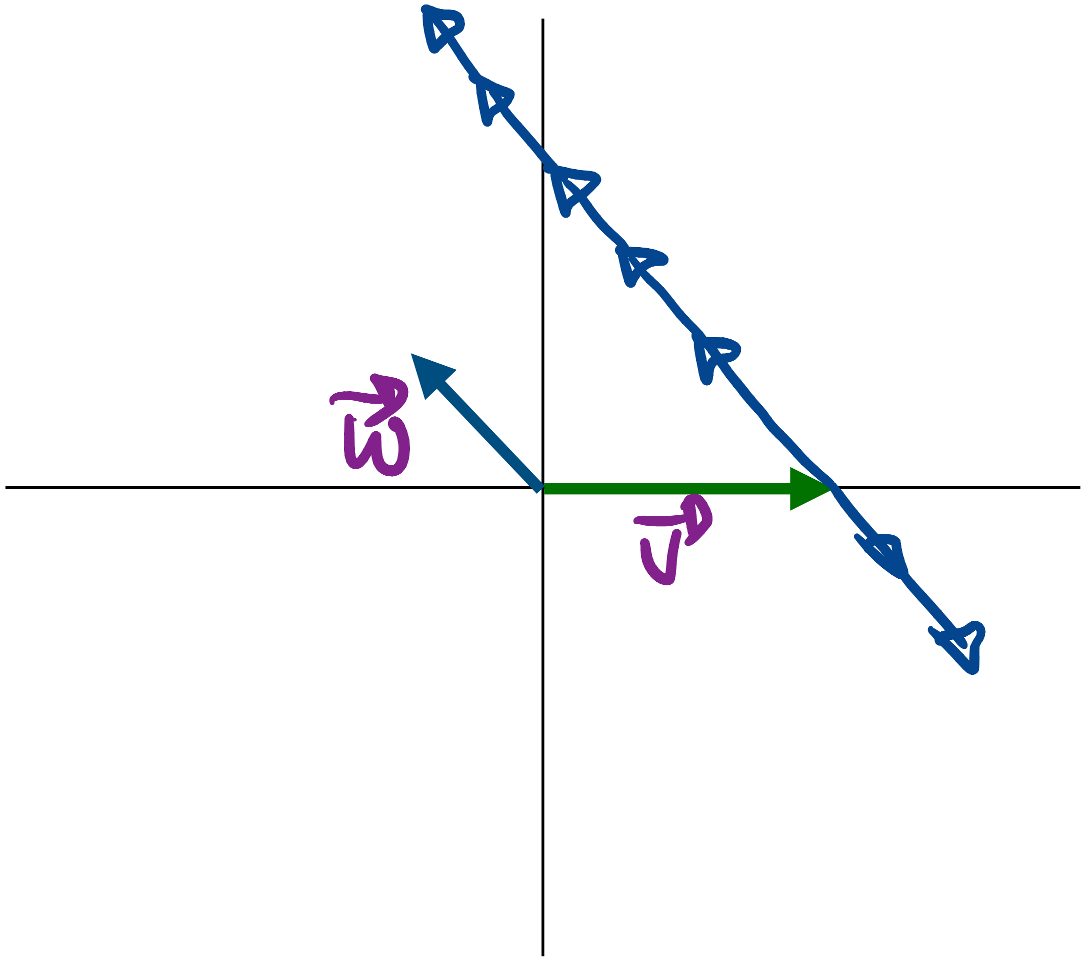
Linear combinations - fix one vector

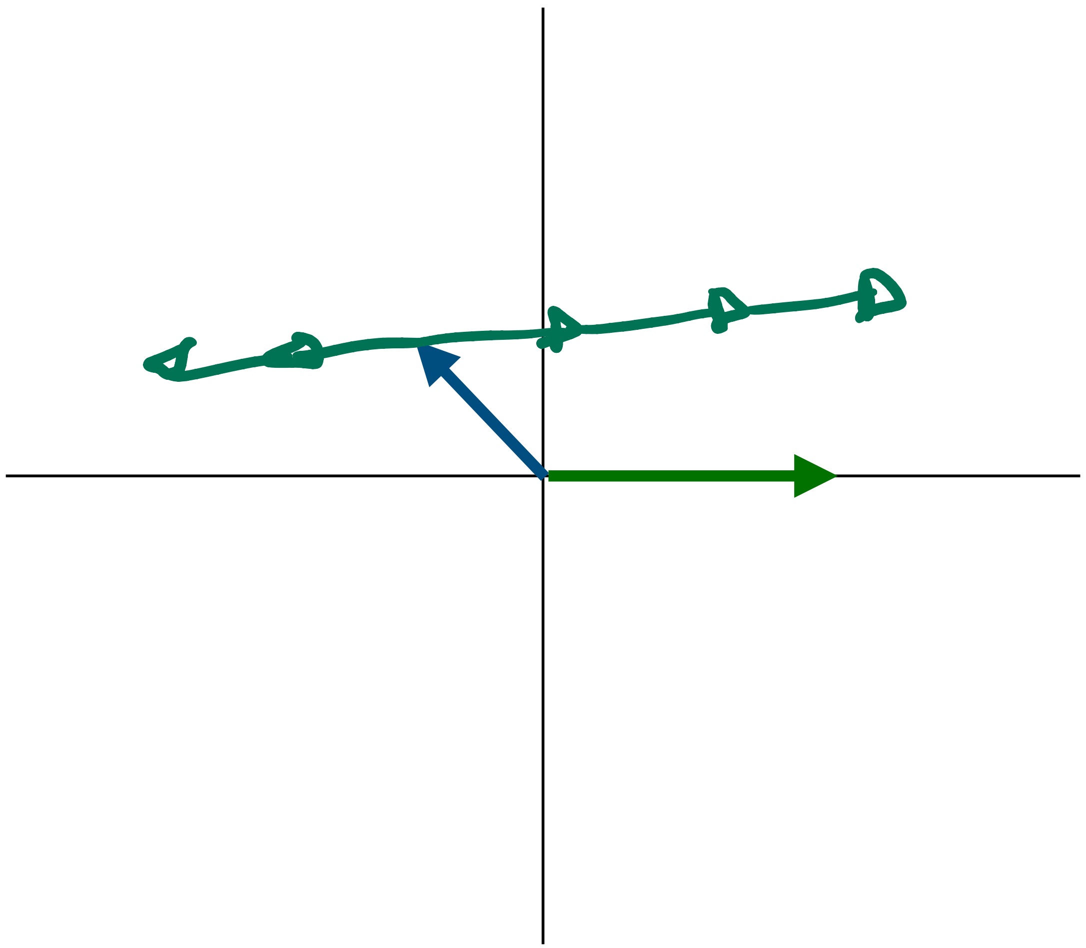




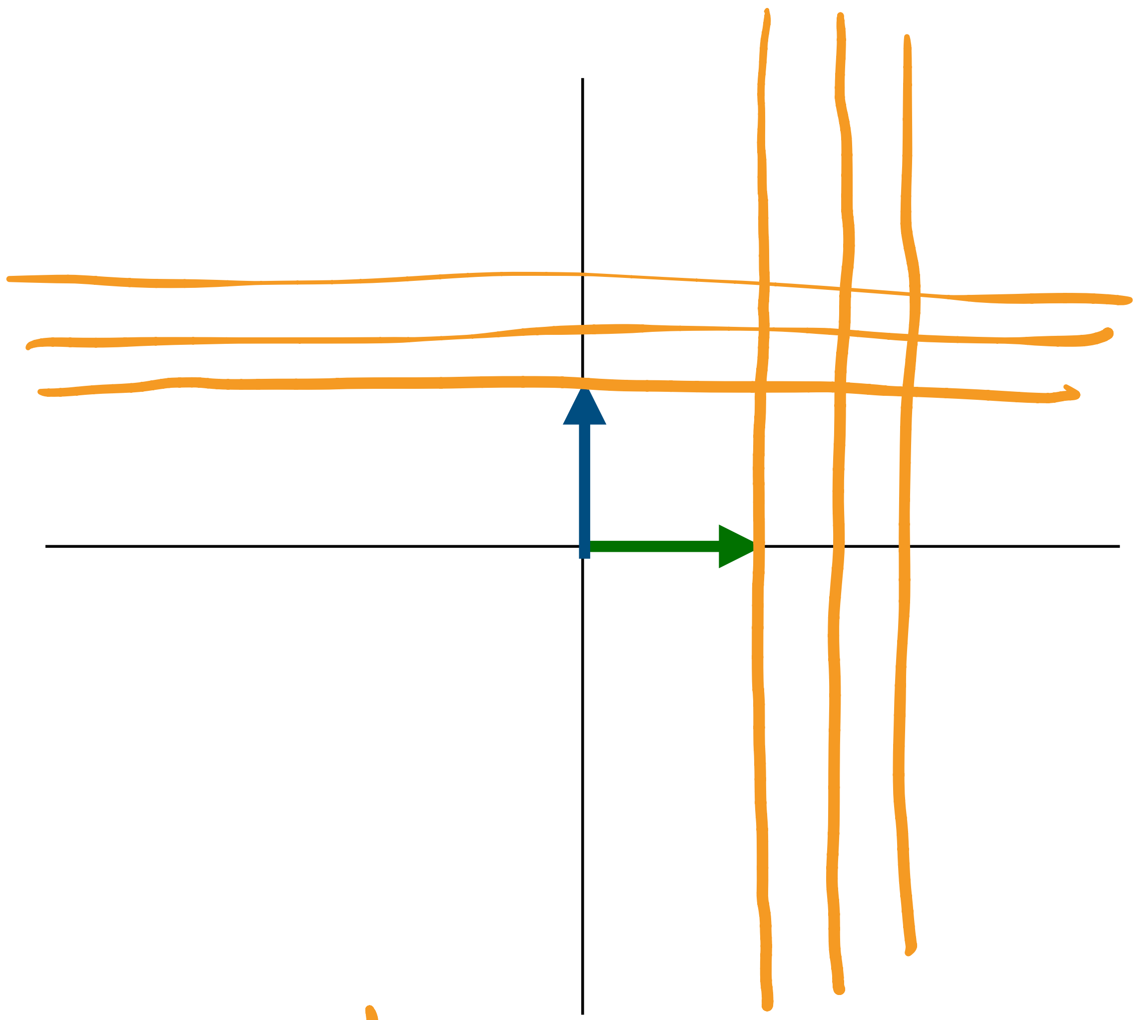
describes a line

Linear combinations - fix one vector

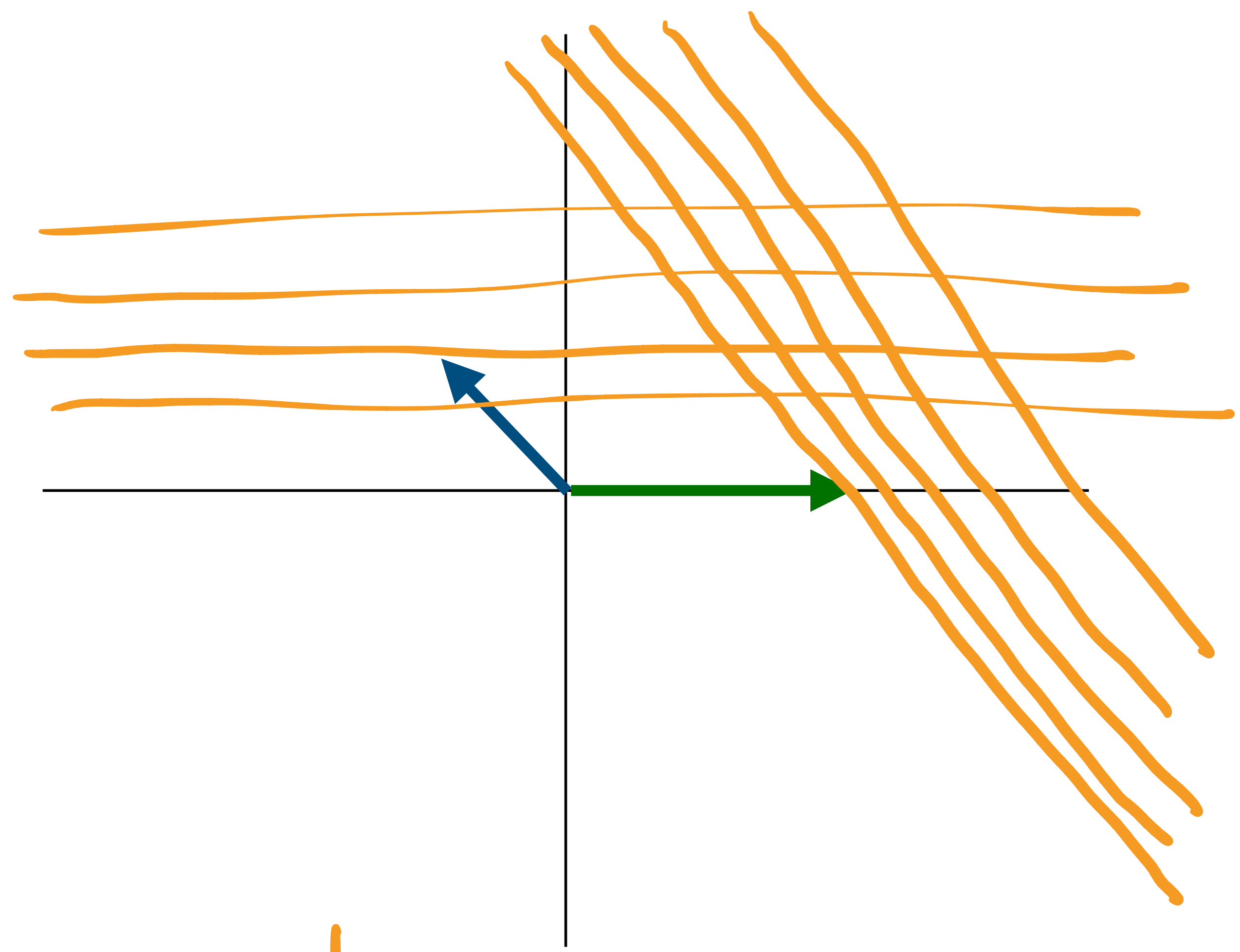




Linear combinations - fix no vectors



a plane



a plane

Spans

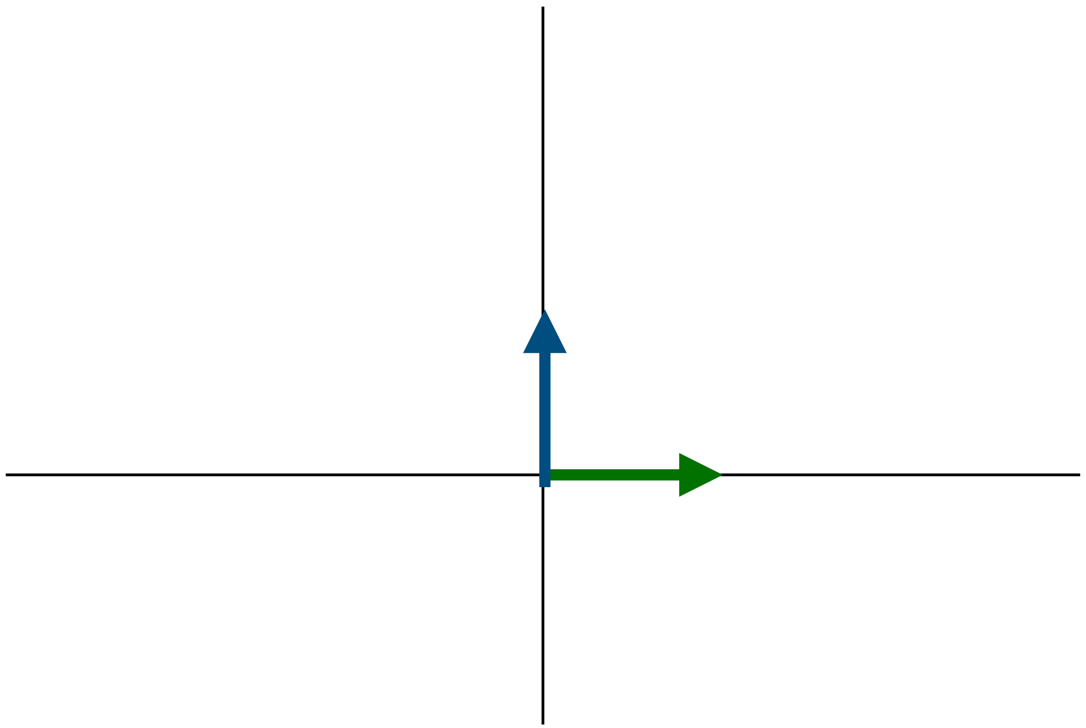
coordinates

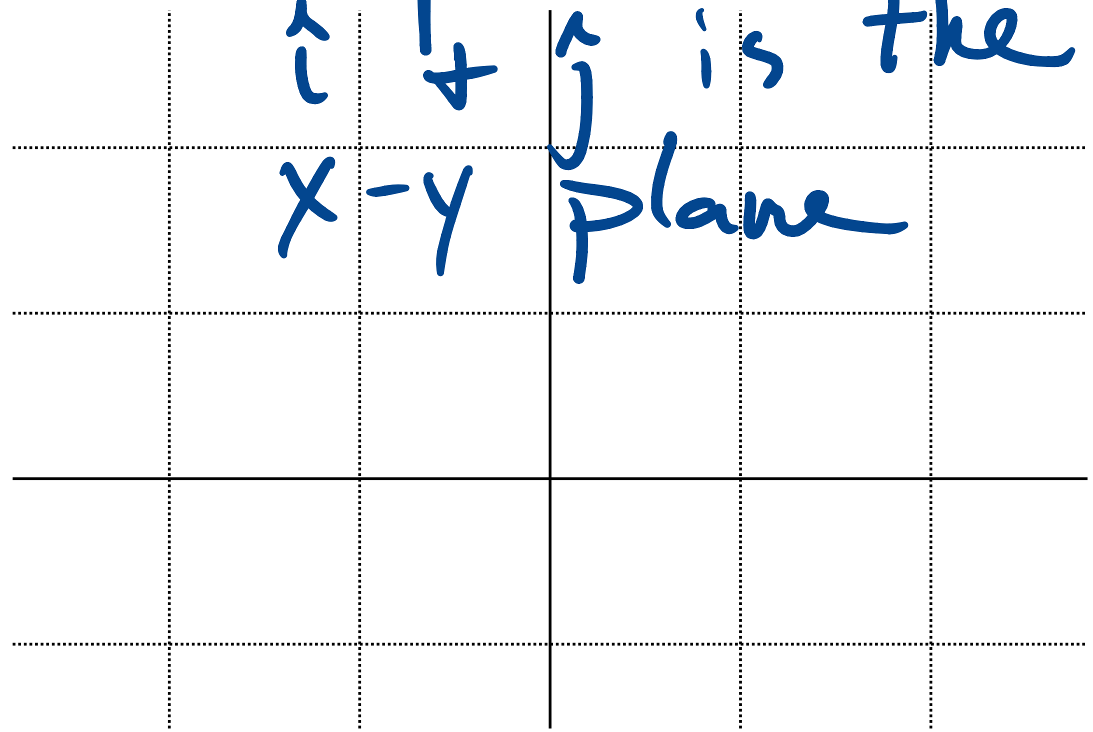
- The **span** of set of vectors is the set of all possible vectors that you can reach with a linear combination of those vectors.

  → the 1st set

- For example, in two dimensions, the **span** of vectors $\overrightarrow{v}$ and $\overrightarrow{w}$ is all vectors that we can define with the equation $\alpha\overrightarrow{v} + \beta\overrightarrow{w}$

Span of $\hat{i} + \hat{j}$ is the x-y plane



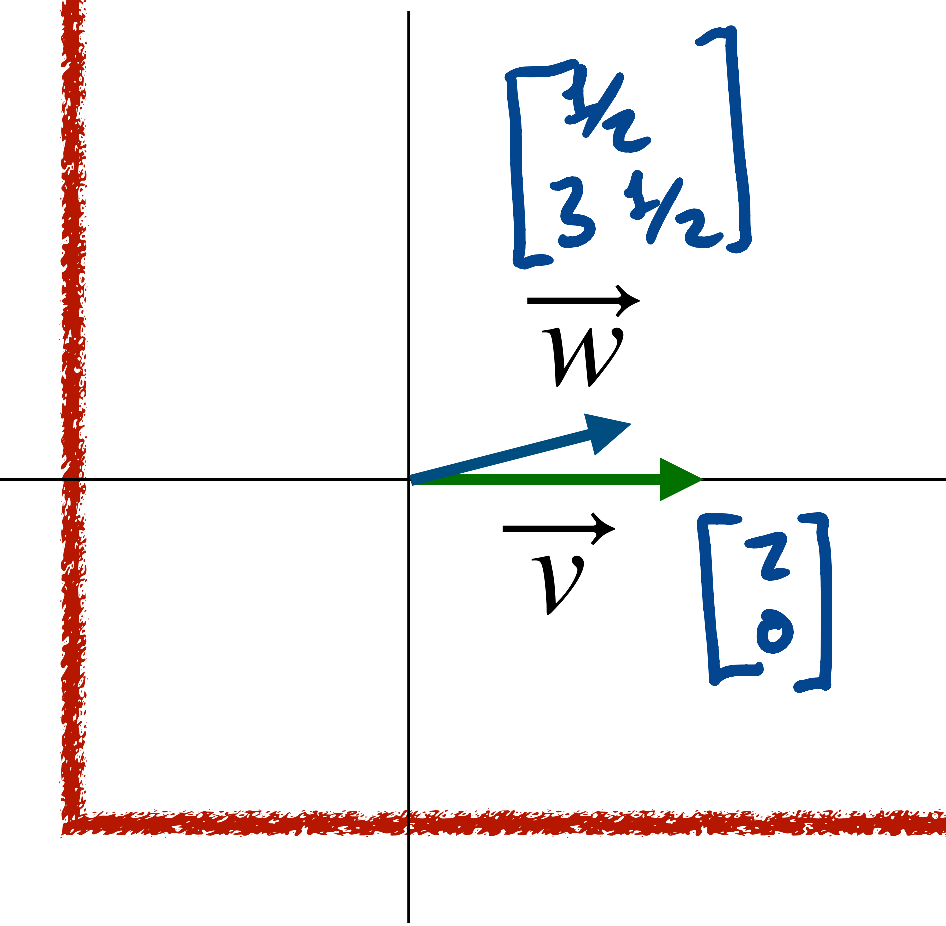


Spans

work until 12:46 (break + think)

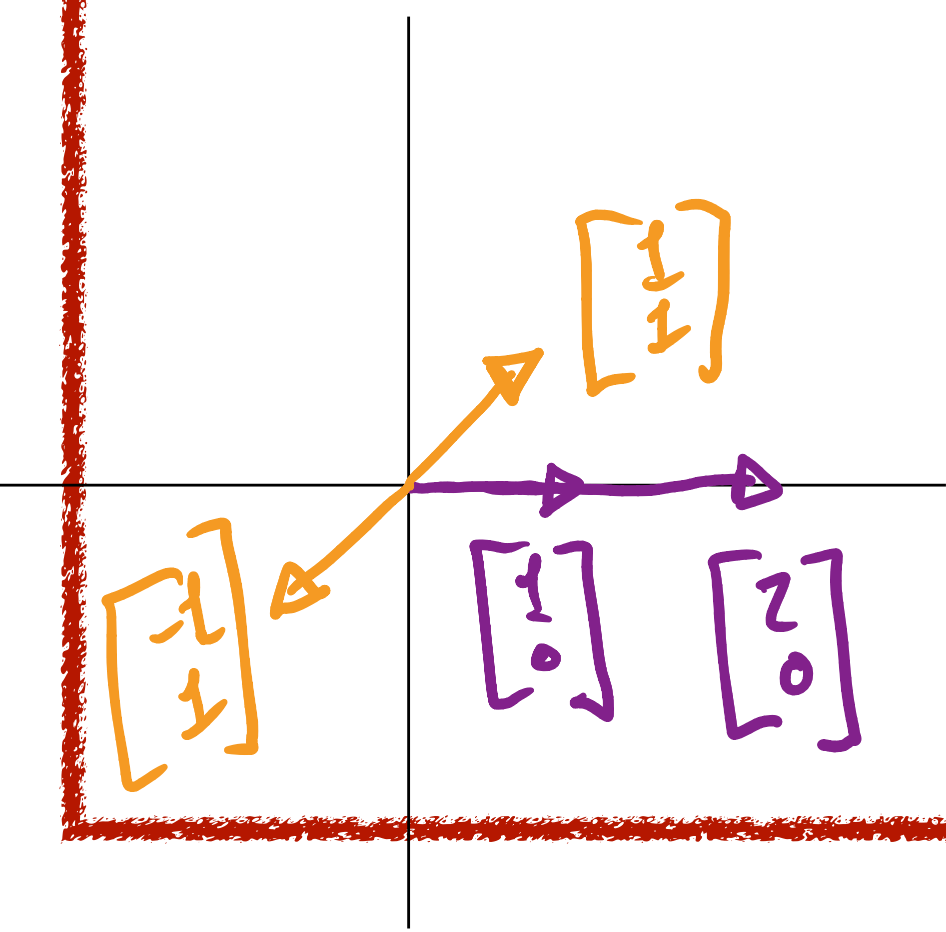
ICA Question 3: what is the **span** of $\vec{v}$ and $\vec{w}$?

2 minutes!
1 minute!
10 seconds



Spans

ICA Question 4: Give an example of two vectors, ($\overrightarrow{v}$ and $\overrightarrow{w}$) whose span is **a line**.



Challenge: 2 vectors w/ no 0s

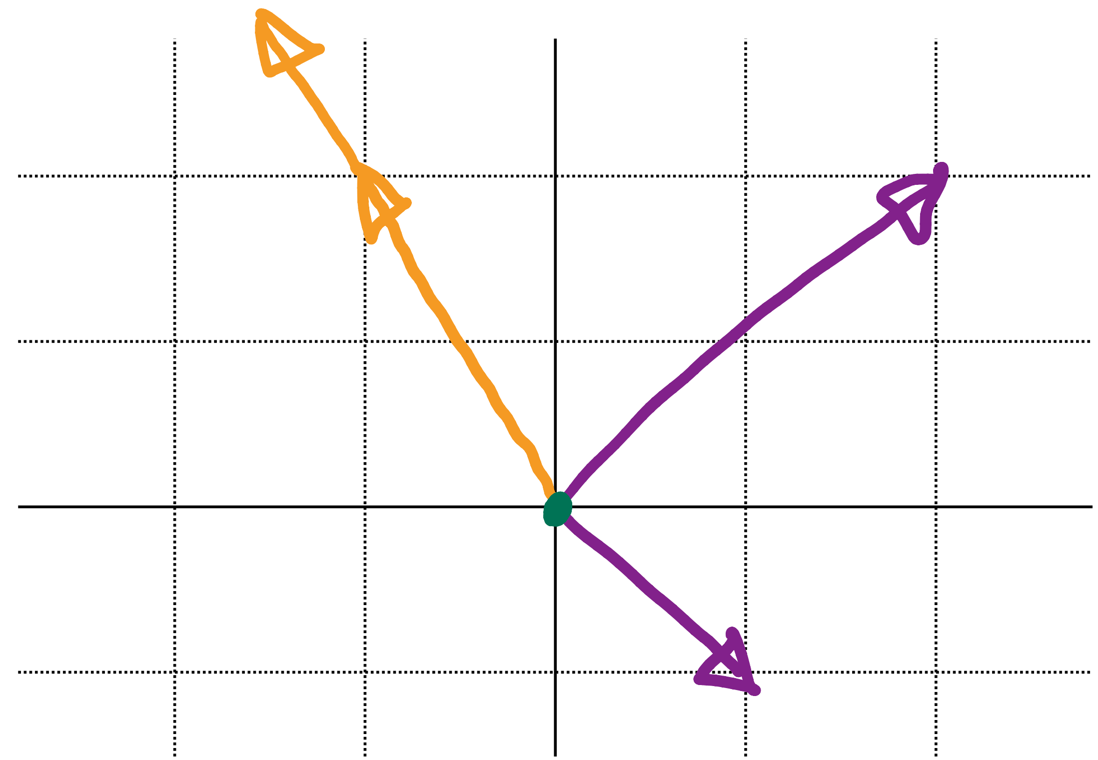
Spans

- The **span** of set of vectors is the set of all possible vectors that you can reach with a linear combination of those vectors.

- In general, we have three cases for the span of any set of 2-d vectors:

  - Every point in the plane
  - A line passing through the origin
  - the origin

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



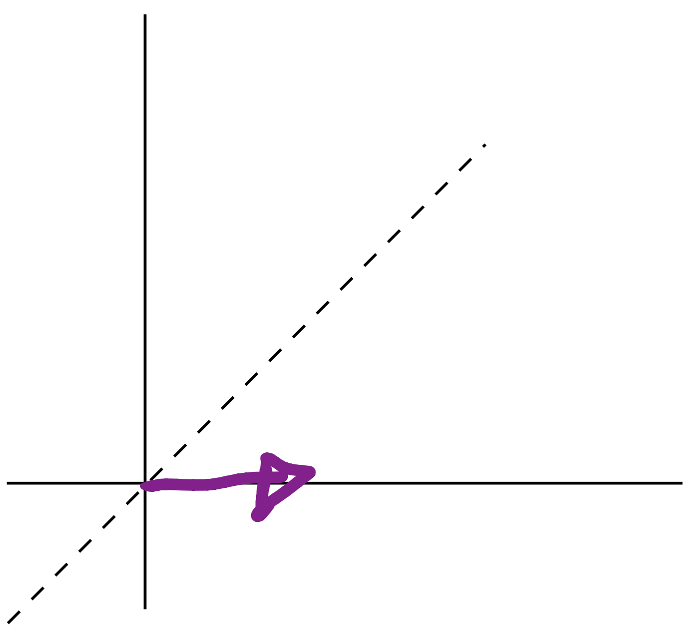
Spans in 3 dimensions

- What are the assumed basis vectors of 3 dimensions?

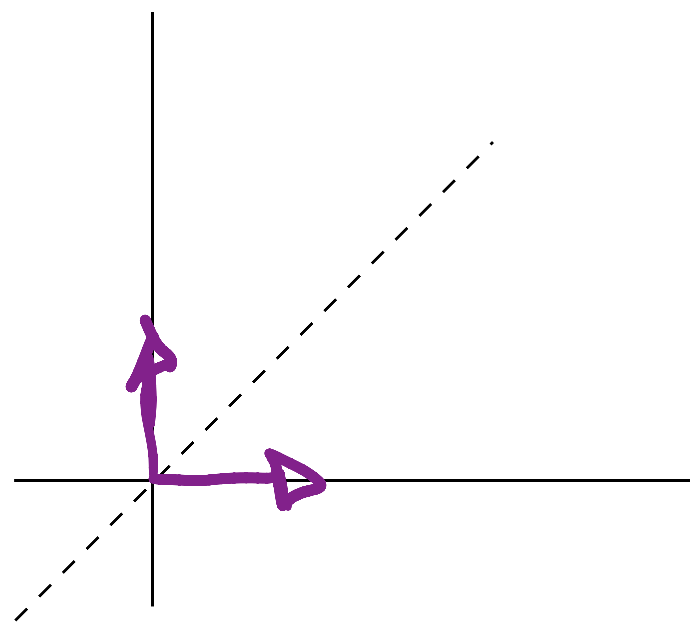
$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Spans in 3 dimensions

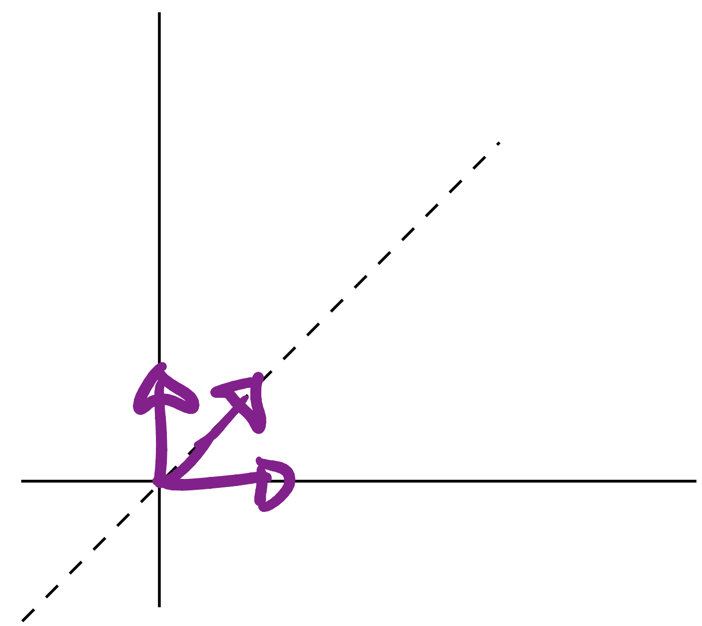
- Building up a set of three vectors in 3 dimensions—how does this change the span?



span: a line, x-axis



span: x-y plane

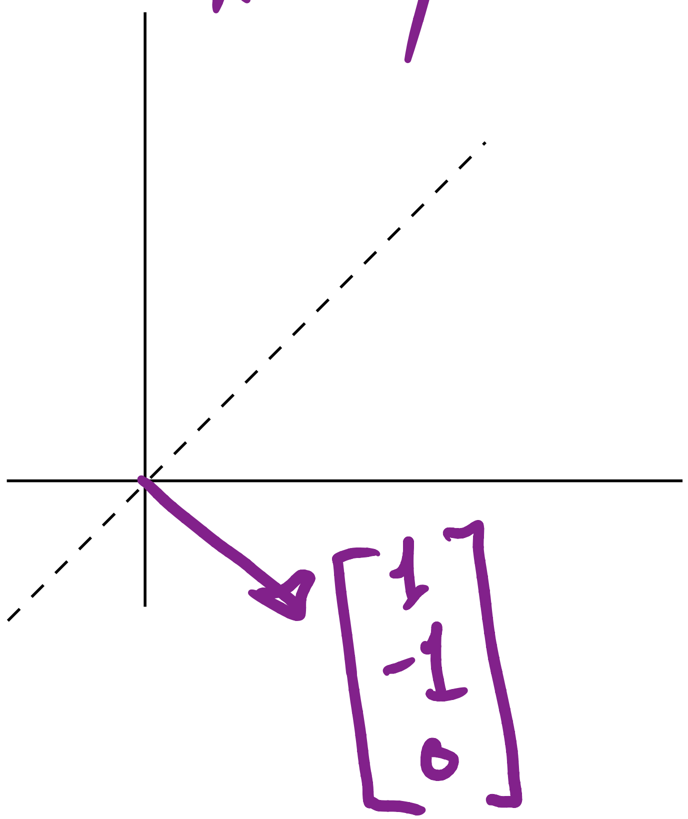


span: 3d space (infinitely expanding cube)

Spans in 3 dimensions

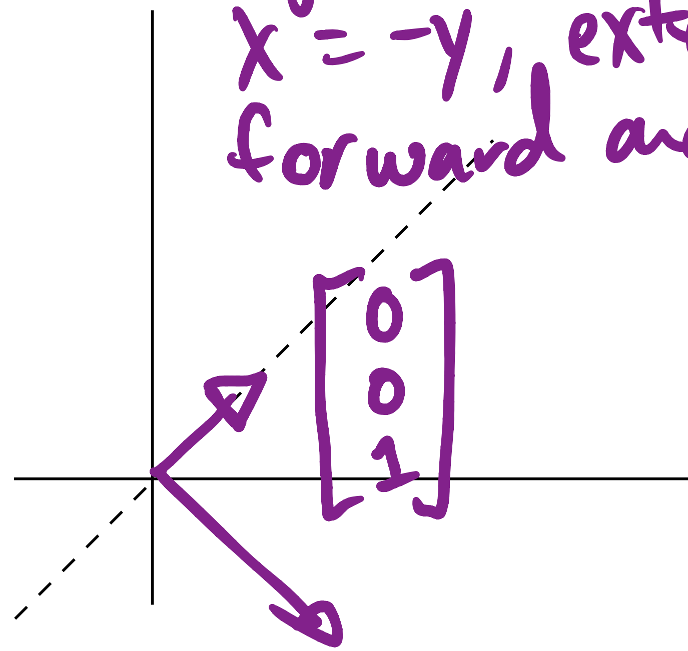
- Building up a set of three vectors in 3 dimensions—how does this change the span?

span: line, $x = -y$



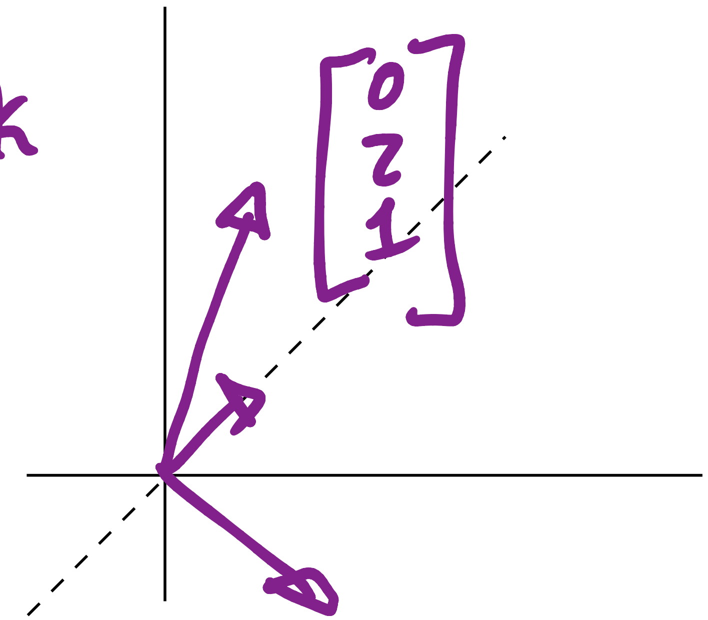
$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

span: a plane, angled at $x = -y$, extending forward and back



$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

span: 3d space



$$\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

Spans in N dimensions

- The **span** of set of vectors is the set of all possible vectors that you can reach with a linear combination of those vectors.

- In general, we have three cases for the span of any set of vectors:

    - Every point in the n-dimensional space

    - A reduced dimensionality space, passing through the origin

    - the origin

Linear dependence

- A set of vectors is **linearly dependent** if one of the vectors is a linear combination of the others
  - (You can think of this as one of the vectors **doesn't** add a dimension to the span of the set)
- $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ + any scaled version

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ → linearly dependent $\vec{c} = 2\vec{a} + 3\vec{b}$

Linear Independence

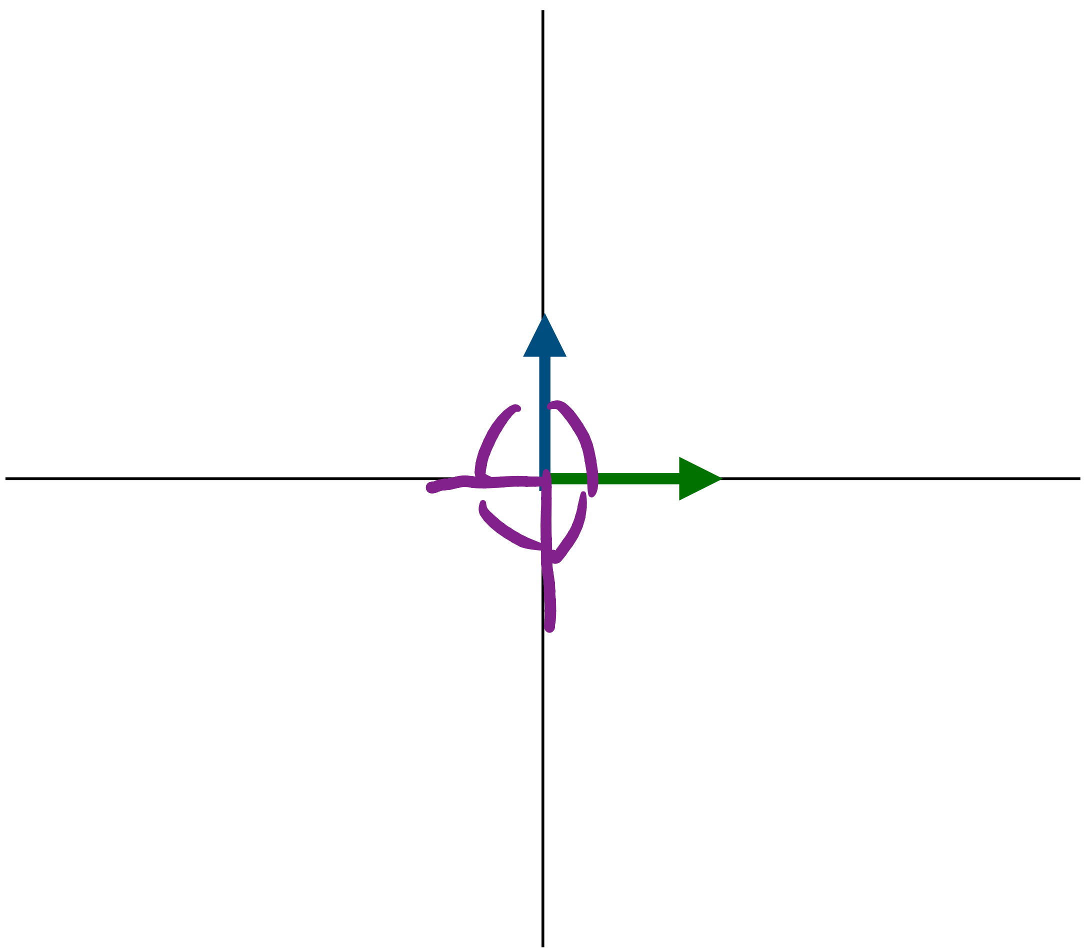
- A set of vectors is **linearly INdependent** if each vector adds a new dimension to the span

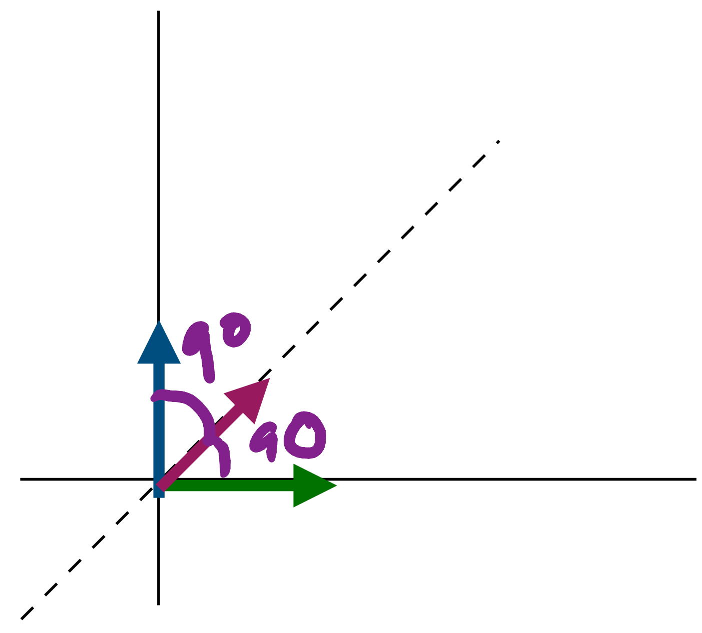
$$\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}?\\ \text{something non-zero}\end{bmatrix}\qquad\begin{bmatrix}37\\524\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$

- Returning to **basis vectors** - these are a set of linearly _independent_ vectors that _span_ the full space

Orthogonality

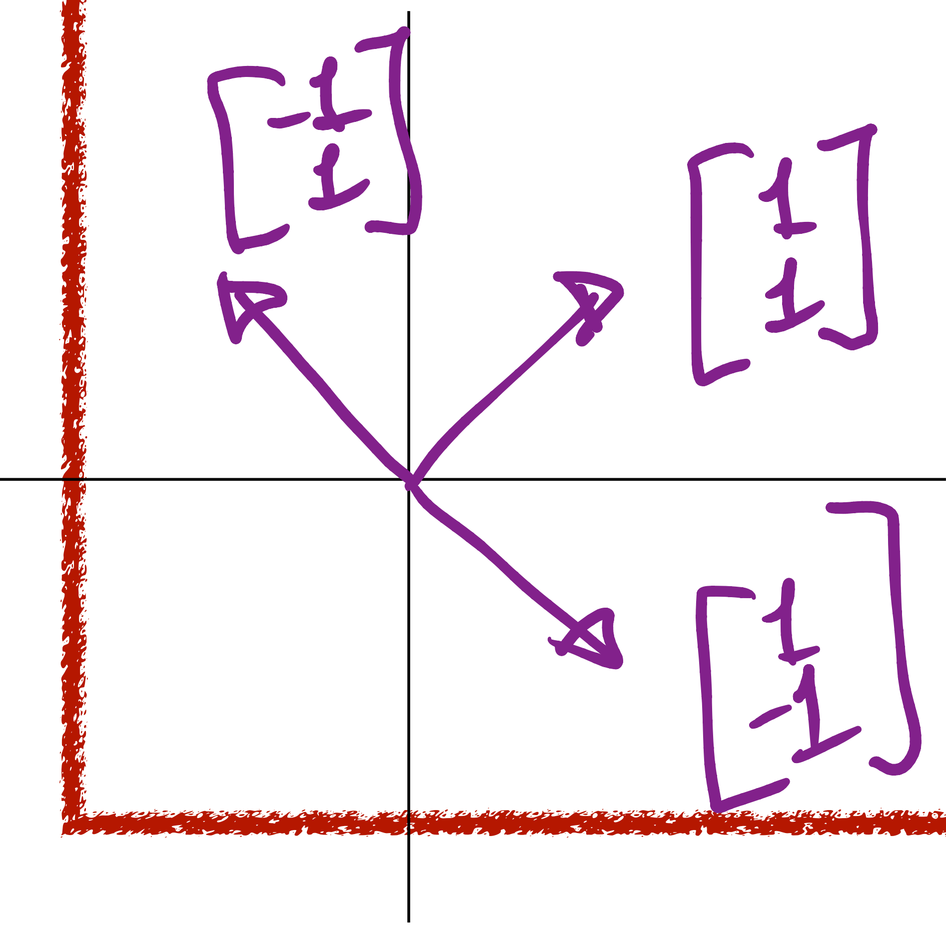
- Vectors are **orthogonal** if all angles between them are 90 degrees





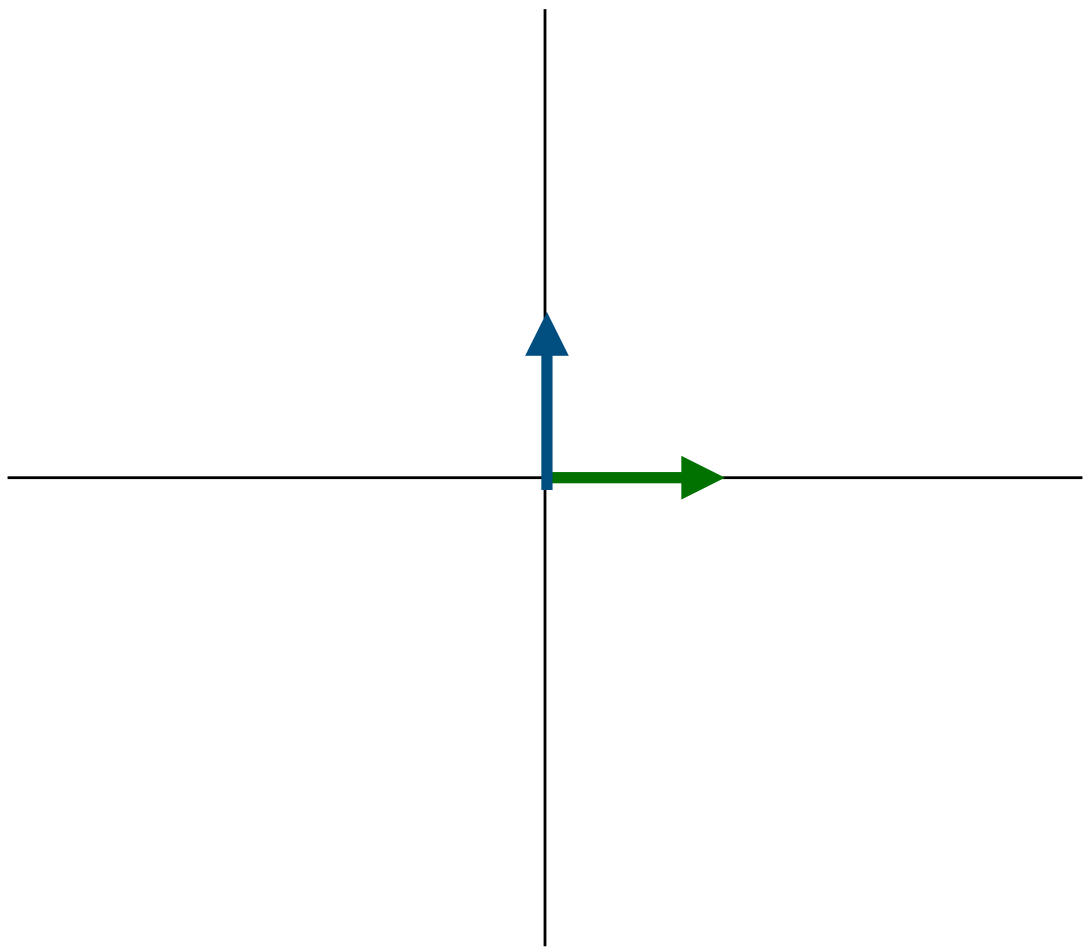
Orthogonality

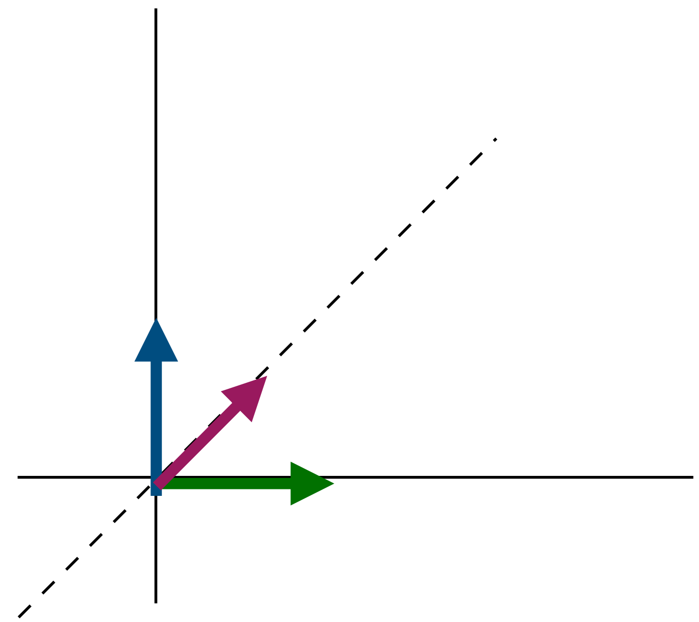
ICA Question 5: Give an example of a set of vectors in 2 or more dimensions that are orthogonal **and** have no coordinate values of 0.



Orthogonality

- Vectors are **orthogonal** if all angles between them are 90 degrees
- Vectors are **orthogonal** if their dot product is 0 zero





Schedule

Quiz 1 details on Thurs/Mon

Turn in ICA 7 on Gradescope

HW 2 is due next Sunday

- Quiz 1!

Felix's scheduled office hours will now be entirely on Calendly (currently T, R). Sign up with whatever quandaries you have at least an hour in advance!

They'll also appear on khouryofficehours from time to time.

Mon	Tue	Wed	Thu	Fri	Sat	Sun
February 7th Lecture 7 - Vector spaces **in Snell Engineering 108**	**Felix OH Calendly**		Lecture 8 - line of best fit **Felix OH Calendly**			**HW 2 due @ 11:59pm**
February 14th Lecture 9 - Polynomial best fit	**Felix OH Calendly**		Lecture 10 - **QUIZ 1 (HW 1 - 2), in class** **Felix OH Calendly**			~~[illegible]~~