CS 2810 Day 23 April 15

Admin:
10 mins for TRACE
Quiz3_02: problem 2.iv & 3.iv

Bayes Rule
- binary problems
- parametric likelihoods
- bayes rule & independence

TRACE

TRACE feedback helps me be a better teacher for you all.

TRACE feedback helps NU identify strong / weak teachers.

Please take a few minutes to give feedback about what worked and what didn't.

CONDITIONAL PROB

(INTUITION: "ZOOM INTO CONTEXT B")
SEE LAST ICA DAY 22

PROB A,B HAPPEN TOGETHER

$$P(A|B) = \frac{P(A B)}{P(B)}$$

PROB A HAPPENS GIVEN CONDITION B

PROB B HAPPENS

CONDITIONAL PROB (USEFUL ALGEBRAIC MANIPULATION)

PROB A HAPPENS GIVEN CONDITION B

PROB A,B HAPPEN TOGETHER

PROB B HAPPENS

$$P(A|B) = \frac{P(A\,B)}{P(B)}$$

$$P(A|B)\,P(B) = P(A\,B)$$

Takeaway: Multiplying a conditional probability by the probability of condition yields a "full joint" probability of all variables happening together.
(we'll see it works for more than just two vars A, B too!)

Bayes Rule (Glorified Conditional Probability)

See previous "takeaway"

$$P(A|B)P(B) = P(AB) = P(B|A)P(A)$$

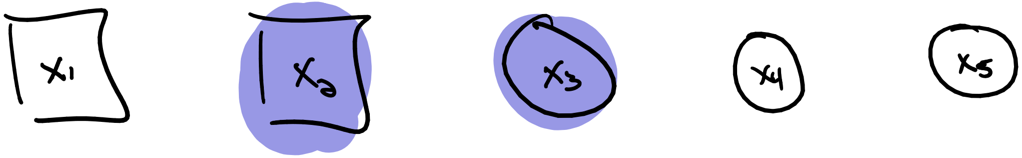
$$\Rightarrow \boxed{P(A|B) = \frac{P(B|A)P(A)}{P(B)}}$$

Notice: this formula "swaps" the order of the conditioning: P(A|B) on left P(B|A) on right
Its typical in a Bayes question to be given variables in one order while question asks for other.

MARGINALIZING (REMEMBER THIS?)

$B=1$ SHAPE IS BLUE

$C=1$ SHAPE IS CIRCLE



	$B=0$	$B=1$
$C=0$	x_1 1/5	x_3 1/5
$C=1$	x_4 x_5 2/5	x_2 1/5

$$P(B=1) = P(B=1, C=0) + P(B=1, C=1)$$

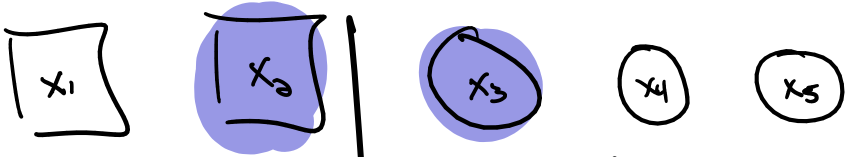
$$= 1/5 + 1/5$$

Remember: To compute P(B) we can sum P(B, A) for all outcomes in sample space of A

$$P(B=b) = \sum_a P(B=b, A=a)$$

Marginalizing (with a conditional)

$B=1$	Shape is	Blue
$C=1$	Shape is	Circle



$$P(C=0)=2/5$$

$$P(B=1 \mid C=0)=1/2$$

$$P(B=0 \mid C=0)=1/2$$

$$P(C=1)=3/5$$

$$P(B=1 \mid C=1)=1/3$$

$$P(B=0 \mid C=1)=2/3$$

$$P(B=1)=\sum_c P(B=1\ C=c)$$

$$=P(B=1\ C=0)+P(B=1\ C=1)$$

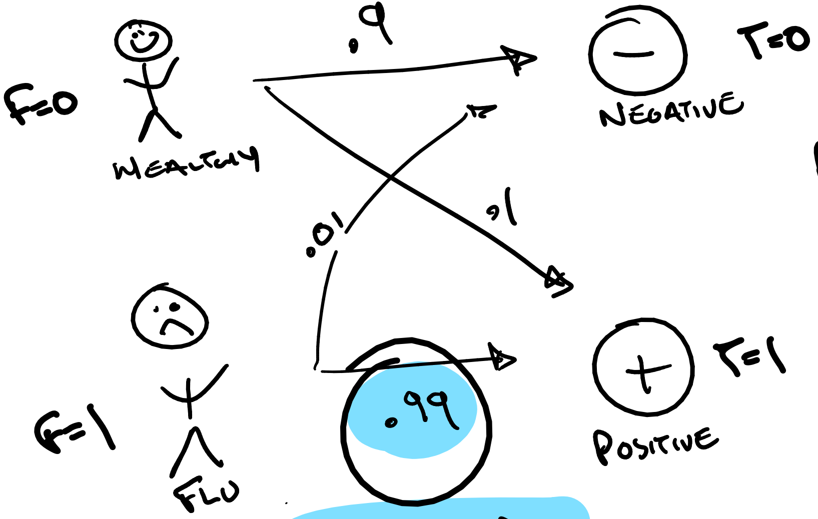
$$=P(B=1 \mid C=0)\,P(C=0)+P(B=1 \mid C=1)\,P(C=1)$$

$$=1/2\cdot 2/5+1/3\cdot 3/5$$

$$=1/5+1/5=2/5$$

Bayes Rule Ex

Given flu occurs in .04 of population, what is the ~~probability one has flu given they~~ test positive?



$$P(T=1|F=1) = .99$$

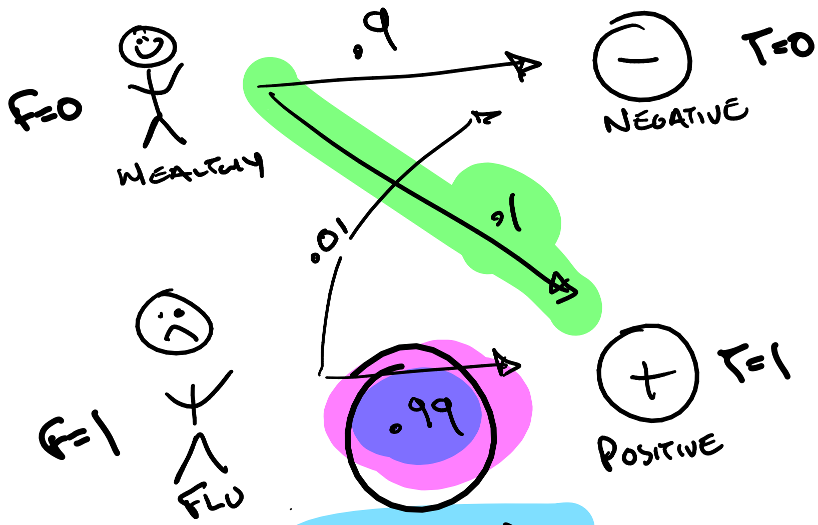
$$P(F=1) = .04$$

$$P(F=1|T=1) = \frac{P(T=1|F=1)P(F=1)}{P(T=1)}$$

$$= \frac{(.99)(.04)}{.1\cdot(.96)+(.99)(.04)}$$

Bayes Rule Ex

Given flu occurs in .04 of population, what is the ~~probability one has flu given they~~ test positive?



$$P(T=1|F=1) = .99$$

$$P(T=1) = P(T=1, F=0) + P(T=1, F=1)$$

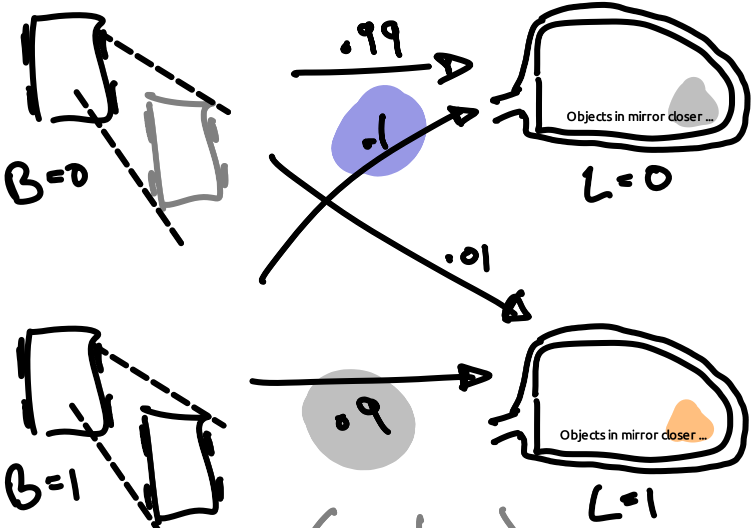
$$= P(T=1|F=0)P(F=0) + P(T=1|F=1)P(F=1)$$

$$= .1 \cdot (.96) + (.99)(.04)$$

In Class Assignment 1

A blind spot monitor produces a warning light (L=1) when it estimates that a car is in one's blind spot (B=1). Given that the light is off, whats the probability that a car is one's blind spot? (Assume that a car is in your blindspot .02 of the time while driving.)

$$P(B=1)=.02$$



$$P(L=1|B=1)=.9$$

$$P(B=1|L=0)=\frac{P(L=0|B=1)P(B=1)}{P(L=0)}$$

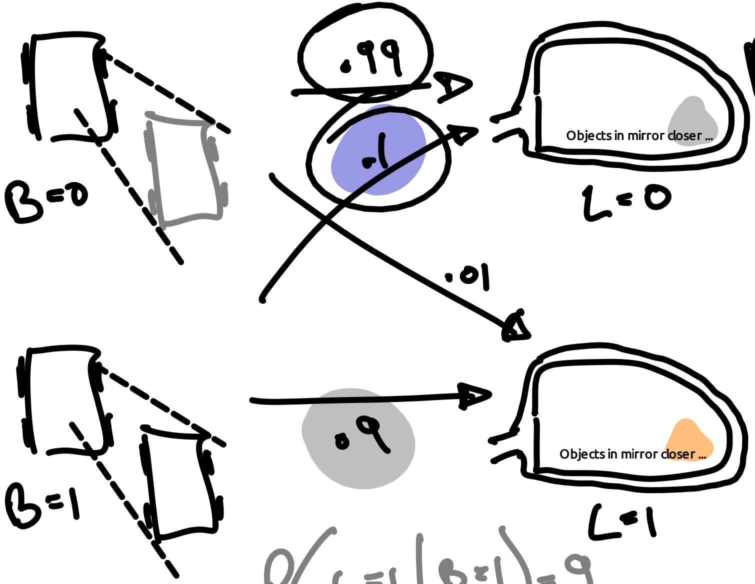
$$=\frac{.1\cdot(.02)}{(.99)(.98)+(.1)(.02)}$$

$$=0.002$$

In Class Assignment 1

A blind spot monitor produces a warning light (L=1) when it estimates that a car is in one's blind spot (B=1). Given that the light is off, whats the probability that a car is one's blind spot? (Assume that a car is in your blindspot .02 of the time while driving.)

$P(B=1) = .02$



$P(L=1|B=1) = .9$

$$P(L=0) = P(L=0\ B=0) + P(L=0\ B=1)$$

$$= P(L=0|B=0)\,P(B=0) + P(L=0|B=1)\,P(B=1)$$

$$= (.99)(.98) + (.1)(.02)$$

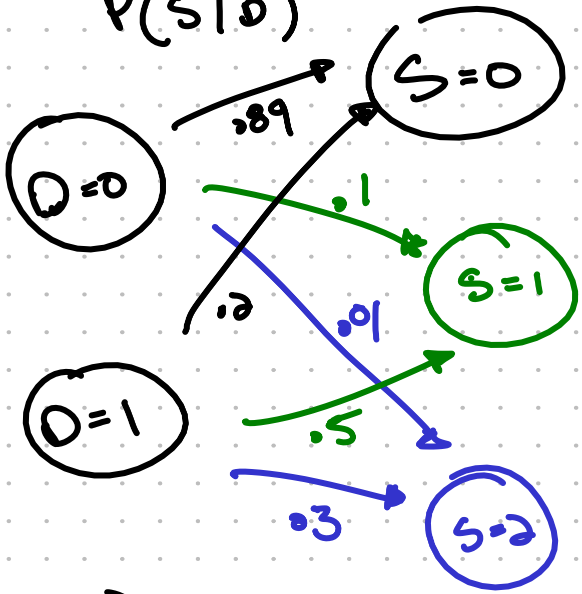
Making Bayes more useful (non-binary A, B variables):

Bayes is applicable in problems where each variable has more than 2 states too!

(see quick example on next page)

BAYES PRACTICE

$P(S|D)$



$P(D=1) = .09$

$D=0$ NO GOLD DEPOSIT

$D=1$ GOLD DEPOSIT

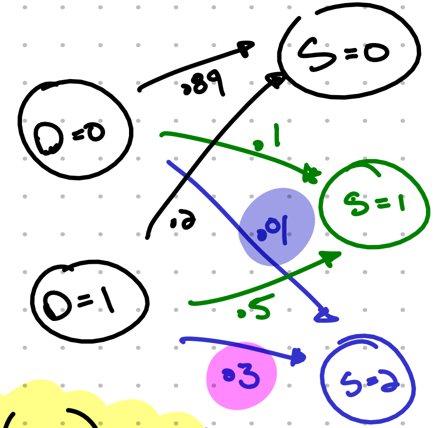
$S=0$ NO GOLD IN STREAM

$S=1$ LITTLE GOLD IN STREAM

$S=2$ MUCH GOLD IN STREAM

WHAT IS PROB OF DEPOSIT GIVEN MUCH GOLD IN STREAM?

$$P(D=1 \mid S=2) = \frac{P(S=2 \mid D=1)\, P(D=1)}{P(S=2)}$$



$P(D=1) = .09$

$$= \frac{P(S=2 \mid D=1)\, P(D=1)}{\sum_d P(S=2\ D=d)}$$

$$= \frac{P(S=2 \mid D=1)\, P(D=1)}{P(S=2 \mid D=0)\, P(D=0) + P(S=2 \mid D=1)\, P(D=1)}$$

$$= (.3 \cdot .09) / ((.01 \cdot .89 + .3 \cdot .09)) \approx .752$$

Making Bayes More Useful (parametric likelihoods):

Let's build models / problems where the conditional $P(B|A)$ is parametric
- binomial distribution
- poisson

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

$P(X \mid D=1) = \text{Poisson}(\lambda = 7)$

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

$P(X \mid D=0) = \text{Pois}(\lambda = .7)$

1% of streams are near gold deposits. $P(D=1) = .01$

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

$P(D=1 \mid X=3) = ?$

X = # NUGGETS FOUND AFTER 7 HOURS OF SIFTING

D = 1 IF DEPOSIT NEARBY

0 OTHERWISE

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

$$P(x \mid D=1) = \text{Poisson}(\lambda=7)$$

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

$$P(x \mid D=0) = \text{Pois}(\lambda=.7)$$

1% of streams are near gold deposits.

$$P(D=1) = .01$$

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

$$P(D=1 \mid x=3) = \frac{P(x=3 \mid D=1)\,P(D=1)}{P(x=3)} = \frac{\text{PMF}(x=3\ \lambda=7)\,(.01)}{\text{PMF}(x=3\ \lambda=.7)(.99) + \text{PMF}(x=3\ \lambda=7)(.01)}$$

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

$P(x \mid D=1) = \text{Poisson}(\lambda=7)$

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

$P(x \mid D=0) = \text{Pois}(\lambda=.7)$

1% of streams are near gold deposits.

$P(D=1) = .01$

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

$$P(x=3) = P(x=3, D=0) + P(x=3, D=1)$$
$$= P(x=3 \mid D=0)P(D=0) + P(x=3 \mid D=1)P(D=1)$$
$$= \text{PMF}(x=3, \lambda=.7)(.99) + \text{PMF}(x=3, \lambda=7)(.01)$$

BAYES RULE TERMS HAVE NAMES

T TARGET VARIABLE OF INTEREST (GOLD DEPOSIT NEAR)

E EVIDENCE (# NUGGETS)

likelihood: probability of evidence
under each possible target outcome

"a priori" / prior
distribution of target variable
before observing any evidence

$$P(T|E) = \frac{P(E|T)\,P(T)}{P(E)}$$

"a posteriori" / posterior
the probability of target variable
after observing the evidence

If a stream is near a gold deposit, one typically finds a gold nugget after an hour of sifting.

If a stream is not near a gold deposit, one typically finds a gold nuggest after a full day of sifting work (10 hours).

1% of streams are near gold deposits.

PRIOR

LIKELIHOOD

If we find 3 nuggets after 7 hours of sifting a particular stream, whats the probability that this stream is near a gold deposit?

POSTERIOR

D=1 DEPOSIT NEARBY ← TARGET

X=# NUGGETS/7 HR ← EVIDENCE

In Class Assignment

$P(X|B=0) = \text{BINOM}(n=5, p=.05)$

In a typical box of chocolates, only 5% of chocolates are coconut flavored.

In a "coconut special", 50% of the chocolates are coconut flavored.

$P(X|B=1) = \text{BINOM}(n=5, p=.5)$

If one selects 5 chocolates out of a box and observes that 3 are coconut flavored, whats the probability that the box is a "coconut special" box?

$P(B=1|X=3)$

Assume that coconut special boxes are as common as typical chocolate boxes.

$.5 = P(B=1) = P(B=0)$

$X = \#$ COCONUT CHOC IN 5 SAMPLES

$B = 1$ IF COCONUT SPECIAL

0 NORMAL BOX CHOC

In Class Assignment

$P(X|B=0) = \text{Binom}(n=5, p=.05)$

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If one selects 5 chocolates out of a box and observes that 3 are coconut flavored, whats the probability that the box is a "coconut special" box?

Assume that coconut special boxes are as common as typical chocolate boxes.

$.5 = P(B=1) = P(B=0)$

$\text{PMF}(x=3, n=5, p=.5)$

$$P(B=1|X=3) = \frac{P(X=3|B=1)P(B=1)}{P(X=3)} \simeq \frac{.156}{.157} = .99$$

$$P(x=3) = P(X=3 \;\; B=0) + P(X=3 \;\; B=1)$$

$$= P(X=3 \mid B=0)\, P(B=0) + P(X=3 \mid B=1)\, P(B=1)$$

$$= \mathrm{PMF}(X=3, n=5, p=.05)\left(1/2\right) +$$

$$\mathrm{PMF}(X=3, n=5, p=.5)\left(1/2\right)$$

$$\simeq .157$$

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent
if observing any outcome of one doesn't
impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x, Y=y) = P(X=x)P(Y=y)$$

Bayes Rule shows the equivilence of the algebraic and intuitive definitions above!

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x\ Y=y) = P(X=x)\,P(Y=y)$$

$$P(X|Y) = \frac{P(X\,Y)}{P(Y)} = \frac{P(X)\,P(Y)}{P(Y)} = P(X)$$

Notice that P(X|Y) = P(X). Observing Y has no impact on the prob of X!

$P(E_0) < \alpha$

E_i = i-TH HYPOTHESIS WAS TYPE I ERROR

$$P(FWE) = 1 - P(\text{NO TYPE I ERRORS})$$
$$= 1 - (1-\alpha)^N$$

BONFERRONI MOTIVATION

(AFTER CLASS QUESTION)