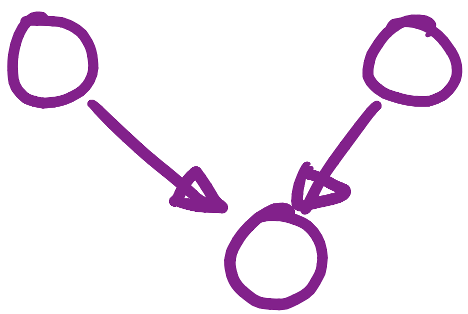
Admin

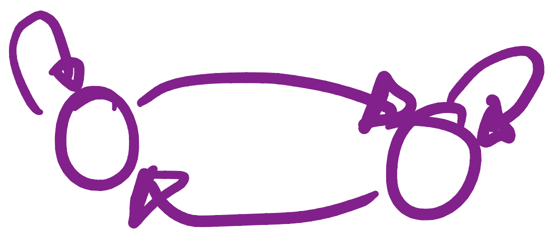
- ICA grades: all available, on Canvas -> ICA 22 (except for ICA 17)
- Test grades: all available, on Canvas
- HW grades: on Canvas through HW 6; HW 7 will get transferred early next week
- HW 8 grades: expect these early next week on Gradescope
- Canvas: Section 1 can see your current grade (w/o HW 7, ICA 17) now, including letter grade
    - Please double check these and reach out to me with any questions!
- Rounding – no rounding

Bayes Nets, part 2

With your neighbor, what are the difference(s) between a Bayes Net and a Markov Chain?

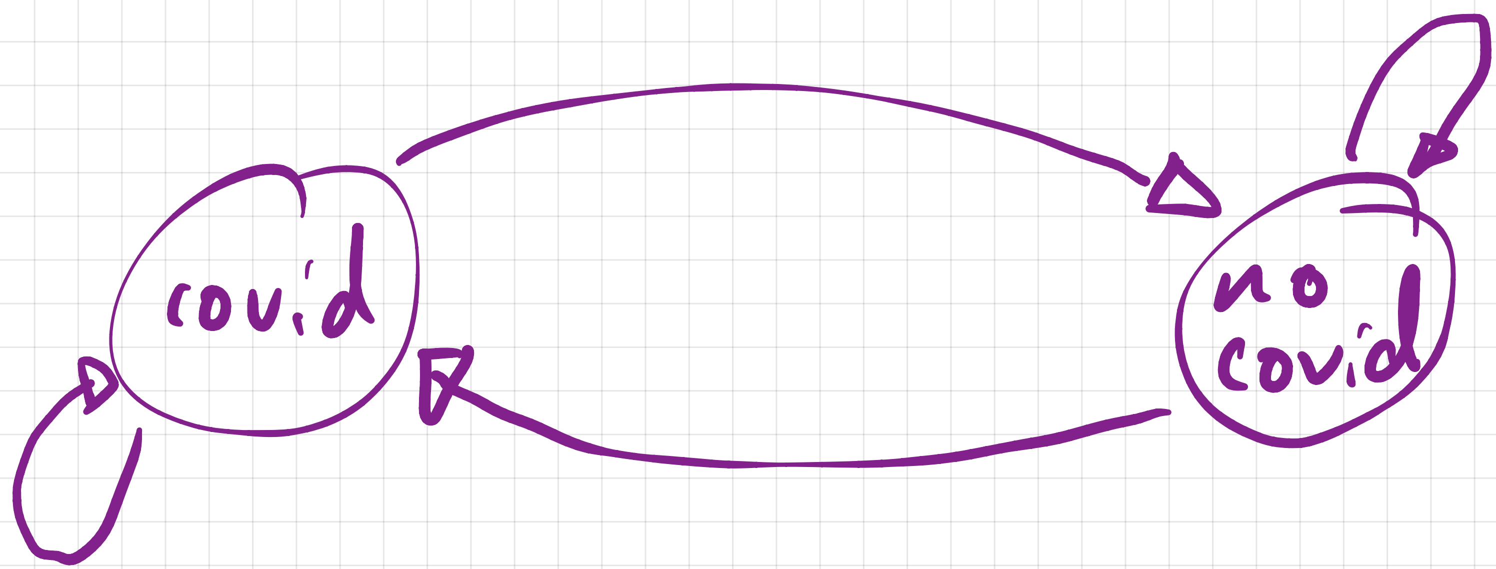


BN: representing conditional prob.
different relationships / dependencies

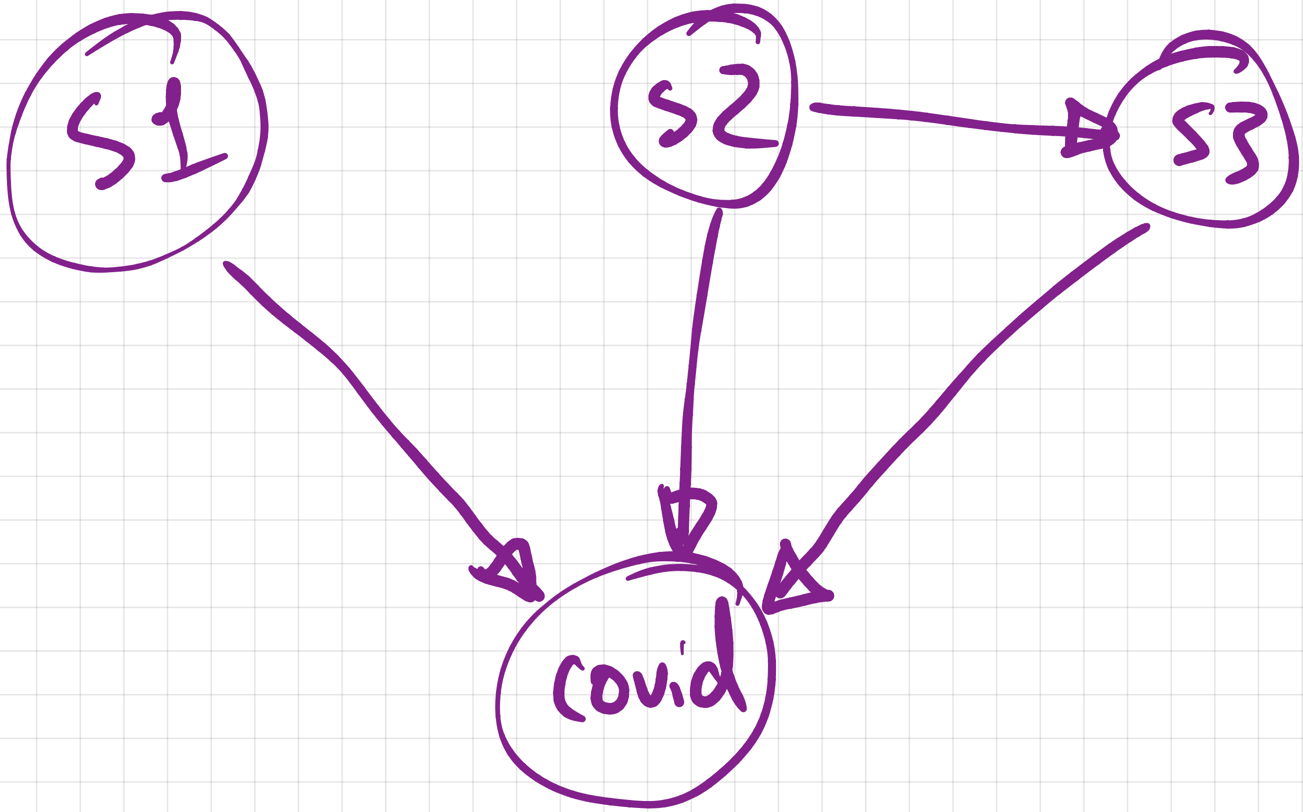


MC: showing how states change
(transition probabilities)

MC



BN



cycles? → no, we won't allow this

Taking a step back: Bayes Rule

- Another way to view conditional probabilities is as the probability of a hypothesis given the evidence:
  - $P(H|E) = \dfrac{P(H,E)}{P(E)}$
- Bayes' Rule, written this way is $P(H|E) = \dfrac{P(E|H)P(H)}{P(E)}$
- When to use Bayes' Rule?
  - When we know: $P(E|H)$
  - And can know or calculate: $P(H) + P(E)$
  - But don't know: $P(H,E)$

$P(H|E)$

$P(E|H)$

$P(H,E)$

$P(H)$

$P(E)$

Bayes nets

- So, in the real world, we often want to incorporate more information than one random variable.

  - **BUT** this often leads to *very* complex joint probability distributions

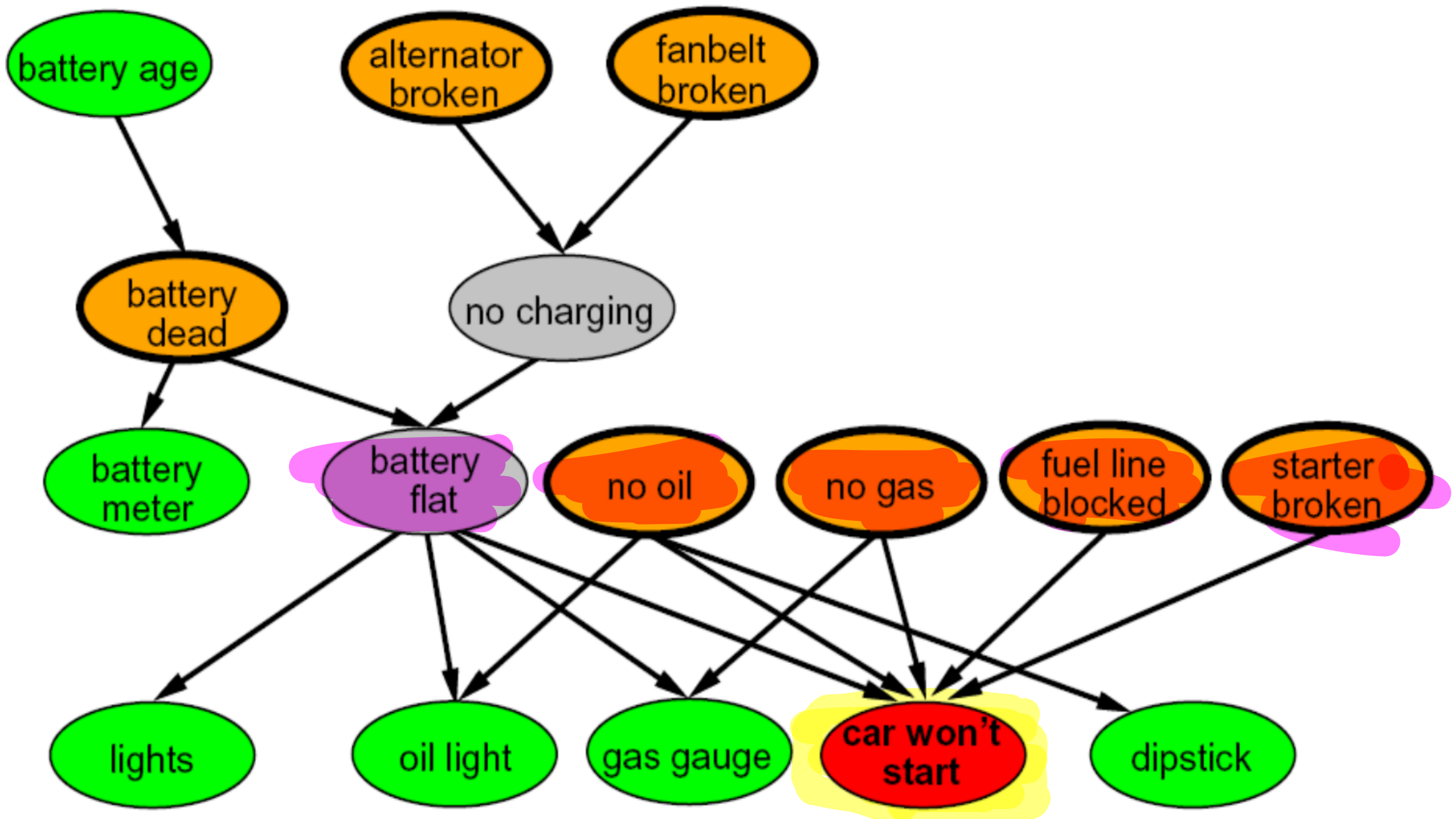
  - A **Bayes Net** (also known as a graphical model) is a way to encode conditional interdependencies and simplify the logic behind what's happening

- e.g. I want to compute P(illness | symptoms) or P(illness1,illness2,illness3| symptoms)

Bayes nets

UW CSE 473 notes (link at end)

- A more complex ("real world") example:



Taking a step back: independence

- So far, we've had two notions of independence for random variables:
- "regular" independence:
  - If A and B are independent, then $P(A, B) = P(A)P(B)$
  - example: A is the result of flipping a coin and B is a die roll
- conditional independence:
  - If A and B are conditionally independent, then $P(A \mid B, C) = P(A \mid C)$
  - example: A is the flu, B is a broken ankle, C is a fever

ICA 1: independence & conditional independence

Does independence imply conditional independence?

Are W and T independent for this graph? $p(W,T) = P(W)P(T)$

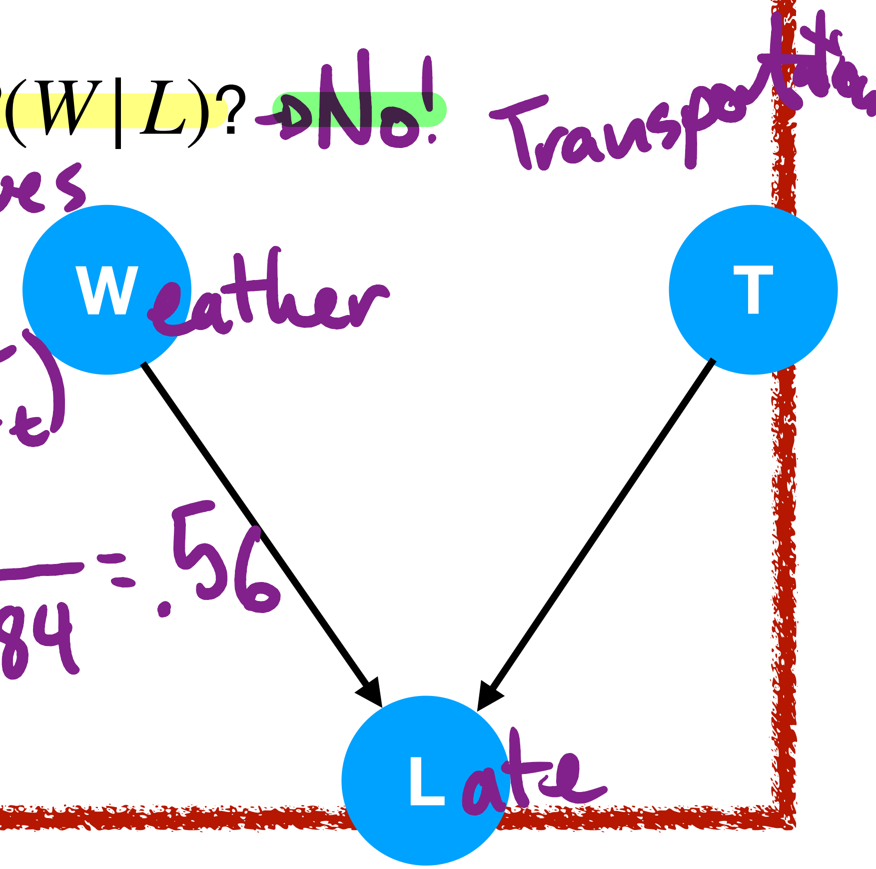
→ yes!

Are W and T conditionally independent? $P(W \mid L, T) = P(W \mid L)$? → No!

→ recommendation: pick some example values

$$P(W_r \mid L_y, T_t) = \frac{P(W_r, L_y, T_t)}{P(L_y, T_t)} = \frac{0.108}{\sum_w P(W, L_y, T_t)}$$

$$= \frac{0.108}{0.108 + 0.084} = .56$$



(link: https://bit.ly/sec1bayes) "on time" sheet

$$P(W_r \mid L_y) = \frac{P(W_r, L_y)}{P(L_y)}$$

$$= \frac{\sum_T (W_r, L_y, T)}{\sum_{W,T} P(W, L_y, T)}$$

2 #s

4 #s

= not .56

$= .676$ _ _ _ - - -

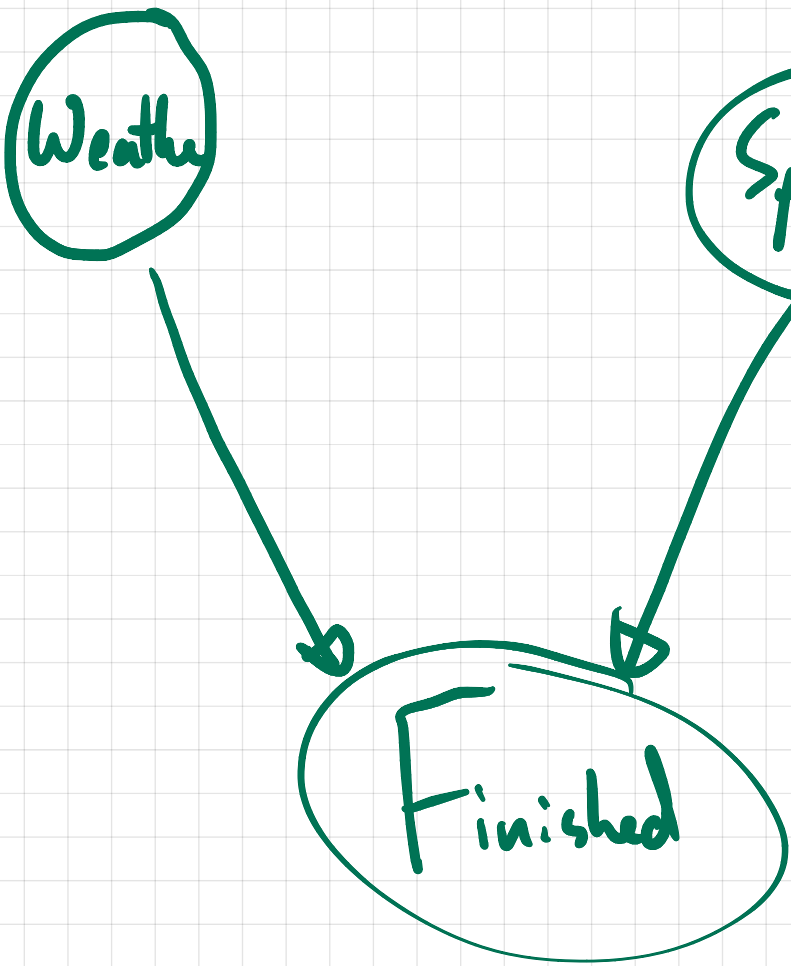
$P(L \mid W, T)$

$$P(L, W, T) \rightarrow P(L \mid W, T)\, P(W, T)$$

$$P(W, T) \rightarrow P(W)\, P(T)$$

To do: add slide clarifying/linking marathon example

marathon:



independences

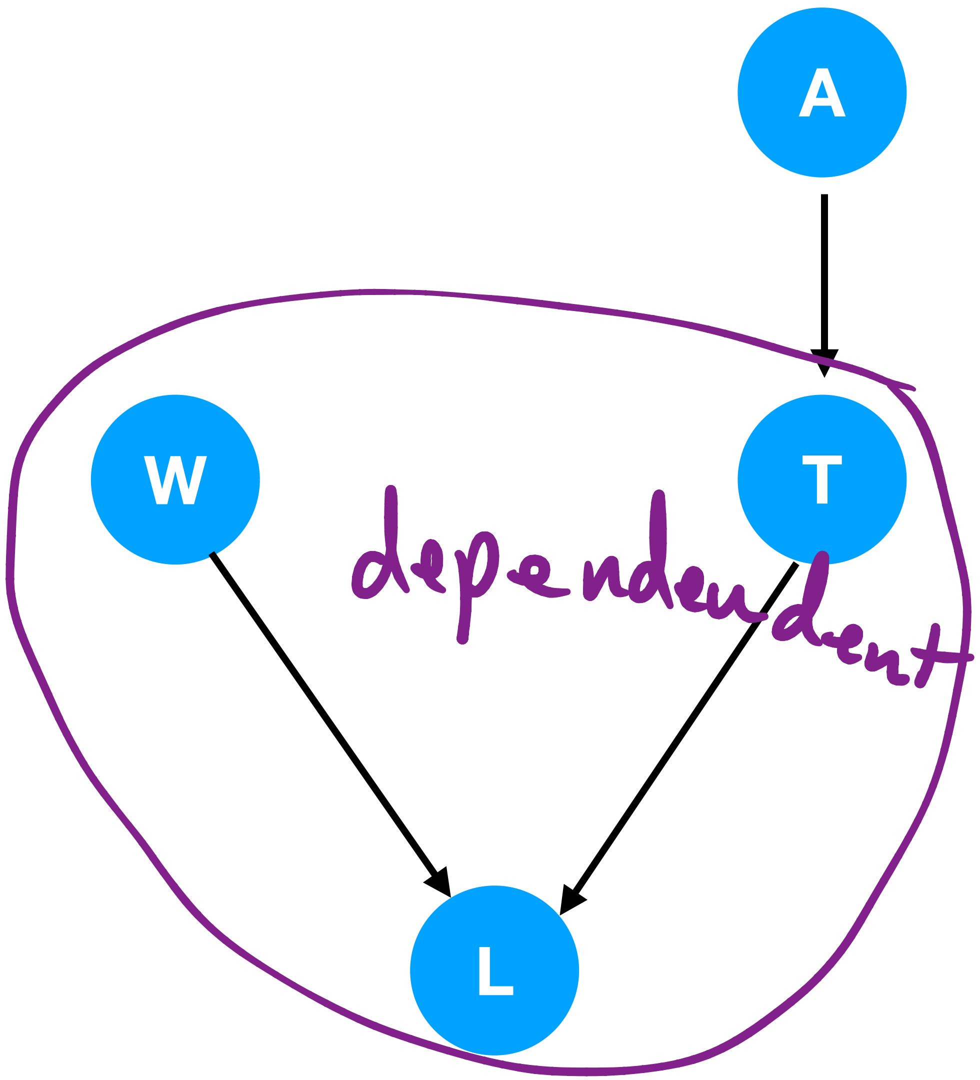
"regular"
↳ W, S

conditionally:
↳ none!
↳ *update* to lec 22

thanks!

Taking a step back: independence

- So what independence **is** encoded in Bayes Nets?
  - Nodes are dependent only on their parents:
    - $P(L \,|\, T, A) = P(L \,|\, T)$
    - L is conditionally independent of A(larm)



Bayes nets

- Computing with Bayes Nets
  - Algebraically

    ↳ write expressions for sums needed $P(X_1, \ldots X_n)$

    $P(X_i \mid X_1 \ldots X_n)$
  - Spreadsheet (by "hand")

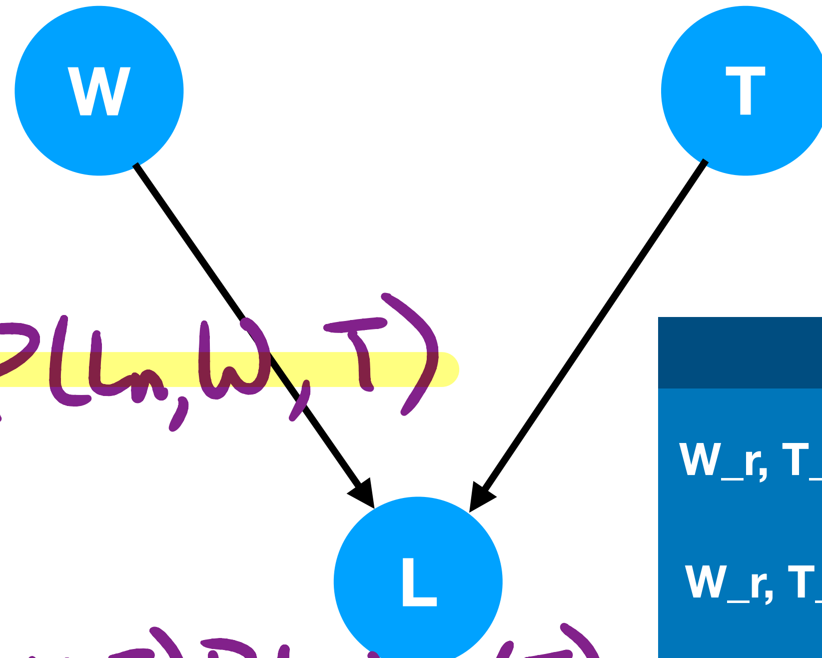
    ↳ enumerate all possibilities, let the spreadsheet do the math
  - Programming
    - Examples on piazza: https://piazza.com/class/ky1oss9wck43uh?cid=443

Last time: we were here....

Given the following Bayes Net, write **the algebraic expression** to calculate the probability Felix is not late (L_n).

W_rain	W_clear
0.6	0.4

T_bike	T_t
0.7	0.3



$$P(L_n, W, T) = \sum_{w,t} P(L_n, W, T)$$

$$P(L_n, W, T) = P(L_n | W, T) P(W) P(T)$$

	L_yes	L_no
W_r, T_b	0.3	0.7
W_r, T_t	0.6	0.4
W_c,T_b	0.1	0.9
W_c, T_t	0.7	0.3

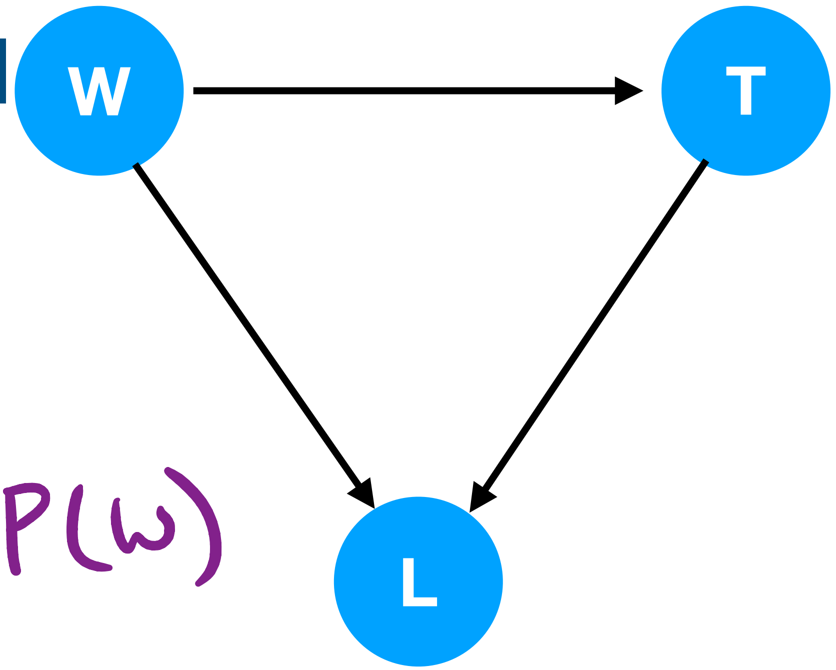
Last time: we were here... (spreadsheet-wise)

A	B	C	D	E	F	G	H	I	J	K	L	M
weather		transport		weather	transport	P(w, t)	late	weather	transport	P(l, w, t)		
rain	0.6	bike	0.7	rain	bike	0.42	yes	rain	bike	0.126	<- G2 * .3	<- P(L_y|w, t)P(w, t)
clear	0.4	T	0.3	rain	T	0.18	yes	rain	T	0.108	<- G3 * .6	
Sanity check	1	Sanity check	1	clear	bike	0.28	yes	clear	bike	0.028		
				clear	T	0.12	yes	clear	T	0.084		
					Sanity check	1	no	rain	bike	0.294		
							no	rain	T	0.072		
							no	clear	bike	0.252		
							no	clear	T	0.036		
									Sanity Check	1		
									Prob that I'll be late:	0.346		
									Prob that I won't be late	0.654		

ICA 2: T_bike

Given the following Bayes Net, use a spreadsheet to calculate the probability Felix is not late (L_n). Start by updating our calculations for T. What is the probability of T_{bike}?

W_rain	W_clear
0.6	0.4



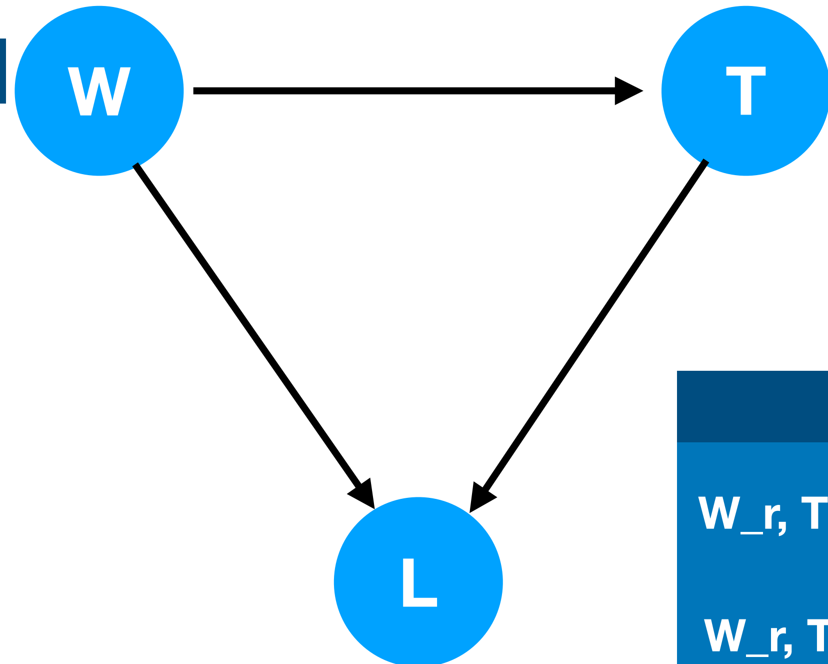
	T_bike	T_t
W_r	0.2	0.8
W_c	0.9	0.1

$P(W,T) = P(T|W)P(W)$

ICA 3: L_no

Given the following Bayes Net, use a spreadsheet to calculate the probability Felix is not late (L_n). Now that we have our calculations for T, is L_n lower or higher than it was before we added this dependency? (it was 0.654 before)

W_rain	W_clear
0.6	0.4



	T_bike	T_t
W_r	0.2	0.8
W_c	0.9	0.1

	L_yes	L_no
W_r, T_b	0.3	0.7
W_r, T_t	0.6	0.4
W_c,T_b	0.1	0.9
W_c, T_t	0.7	0.3

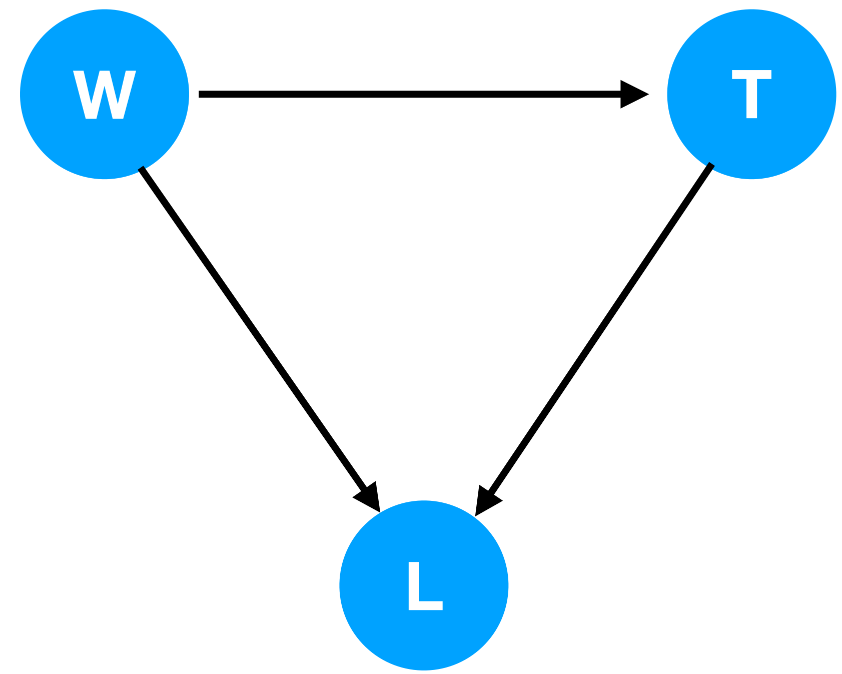
ICA 4: given L_no

What is $P(W, T \mid L_n)$ algebraically? What is $P(W_r, T_b \mid L_n)$?

$$P(W,T \mid L_n) = \frac{P(W,T,L_n)}{P(L_n)}$$

$$= \frac{P(W,T,L_n)}{\sum_{w,t} P(W,T,L_n)}$$

$$P(W_r,T_b \mid L_n) = \frac{P(W_r,T_b,L_n)}{\sum_{wt} P(W,T,L_n)}$$



Link to spreadsheet computations for these Bayes Nets!

- https://bit.ly/sec1bayes
    - (we added the previous example to this sheet in real time :D)

Summary: Bayes Rule

- Bayes Rule:
  - Bayes' rule denotes the relationship between $P(A \mid B)$ and $P(B \mid A)$
  - $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$
  - When calculating $P(B)$ for the denominator, it's often useful to calculate this as the sum of $\sum_i P(B \mid A_i)P(A_i)$
  - On HW 9/Test 4: yes

Summary: Naïve Bayes Classifiers

- Naïve Bayes Classifiers:
  - Why: grounding Bayes' Rule in a real-world example
  - Main idea: leverage Bayes' Rule to decide on the class of an unknown data point
  - On HW 9/Test 4: not explicitly

Summary: Bayes Nets

- Bayes Nets
    - Why: models much more complex relationships/dependencies
    - Main idea: use conditional probabilities and conditional independences to make computations tractable when many factors are given
    - On HW 9/Test 4: yes

Break time!

- Go do ICA 23. Passcode is "secret"
- While you are waiting —> give Felix your mini-project questions

mini-projects @ 1:07

Mini-projects Questions

format:

↳ can I include pictures? yes!

prob/stats:

↳ ok if some recommendations pertain to different components of the system? yes!

HW 9 questions

P1: ticket class - lower is better

[1, ... 5]

Schedule

4/25: Review (yes, there will be an ICA on the 25th)

Mini-project: must email Felix to request an extension (by default no late passes).

Test 4: May 4th, 1 - 3pm, Snell Engineering 108, you'll have 90 minutes for this test (expect it to be about the same length as Test 3 though).

Test 4 review: April 29th (Friday) @ 10am w/ Prof. Higger (will be recorded)

Mon	Tue	Wed	Thu	Fri	Sat	Sun
April 18th **No lecture - Patriot's Day**	**Felix OH Calendly**	**Felix OH Calendly**	**Felix OH Calendly** Lecture 23 - Bayes Nets, part 2, mini-project work day			
April 25th Lecture 24 - review **Mini-project due @ 11:45am**		**Felix OH Calendly** **HW 9 due @ 11:59pm**		**Review @ 10am (zoom)**		
May 2nd	**Felix OH**	**Test 4, 1 - 3pm, Snell Eng 108**				

More recommended resources on these topics

- YouTube: Berkeley AI, Section 5: Probability, Bayes Nets
- UW CSE 473, Bayes' Nets: https://courses.cs.washington.edu/courses/cse473/19sp/slides/cse473sp19-BayesNets.pdf
- Does independence imply conditional independence? https://stats.stackexchange.com/questions/51322/does-independence-imply-conditional-independence