CS2810 Day 26

Admin:
TRACE @ ~50%, please do this!

Review:
Normal Distribution
- Central Limit Theorem
- CDF

Hypothesis Testing
- Whats a p-value?
- Experimental Bias
- T-Tests
    - one vs two sided
- Chi Square Test
- Multiple Comparison Correction

Covariance
- Covariance Matrix
- Correlation
- Independence & Correlation / Covariance

Bayes Rule Problems:
- Binary Variables
    - p(covid | test positive for covid)?
- Parameterized Likelihoods
    - poisson: traffic flow rate problem on HW
    - binomial: coconut special ICA (day 23)

Bayes Nets:
- Identifying (conditional) independence
- How to compute conditional prob
    - step 1: rewrite without conditional
    - step 2c: build joint distribution table
    - step 3c: marginalize joint probs from step 1 via joint distribution table

Problem 1: "Five Second Rule"
Normal Distribution / Central Limit Theorem / CDF

Assume the snacks my daughter drops on the floor which the dog then eats is a poisson distribution with lambda = 15 snacks / day. (This model has one big glaring assumption problem ... what is it?)

Estimate the probability that the dog eats more than 7 pounds of dropped food over a whole year.

Write out and evaluate any assumptions you deem necessary.

Problem 2: Hypothesis Testing

Two artists sell their work at auction.

Artist A's works go for (in thousands of dollars):

3, 4, 5 $\bar{A} = 4$

Artist B's works go for (in thousands of dollars):

5, 6, 7, 8 $\bar{B} = 6.5$

$$T = \frac{\bar{A} - \bar{B}}{S} = \frac{4 - 6.5}{\sqrt{.75}} = -2.88$$

$$S^2 = \frac{\hat{\sigma}_A^2}{N_A} + \frac{\hat{\sigma}_B^2}{N_B} = \frac{1}{3} + \frac{5/3}{4} = .75$$

- Identify 3 sources of experimental bias under which this data could've been collected
- Perform any hypothesis test you deem relevant to answer the question: does B's work sell for more than A's?
    - whats is a type 1/2 errors in this case? can we say anything about prob of type 1/2 errors?

$$\hat{\sigma}_A^2 = \frac{1}{N-1} \sum_i (A_i - \bar{A})^2 = \frac{1}{3-1}\left((3-4)^2 + (4-4)^2 + (5-4)^2\right)$$
$$= \frac{1}{2}(1 + 0 + 1) = 1$$

PVAL $<$ $\alpha$ $\Rightarrow$ REJECT $H_0$ GIVES $P(\text{TYPE 1 ERROR}) < \alpha$

GROUND TRUTH

| | | $H_0$ | $H_1$ |
|---|---|---|---|
| ESTIMATE | $H_0$ | ✓ | TYPE II ERROR |
| | $H_1$ | TYPE I ERROR | ✓ |

MISSED DETECTION

FALSE ALARM

$B = 5 \quad 6 \quad 7 \quad 8 \qquad \bar{B} = 6.5$

$$\hat{\sigma}_B^2 = \frac{1}{N-1} \sum_i \left(B_i - \bar{B}\right)^2$$

$$= \frac{1}{4-1} \Big( (5-6.5)^2 + (6-6.5)^2 + (7-6.5)^2 + (8-6.5)^2 \Big) = \frac{5}{3}$$

$H_0: N_B \le N_A$

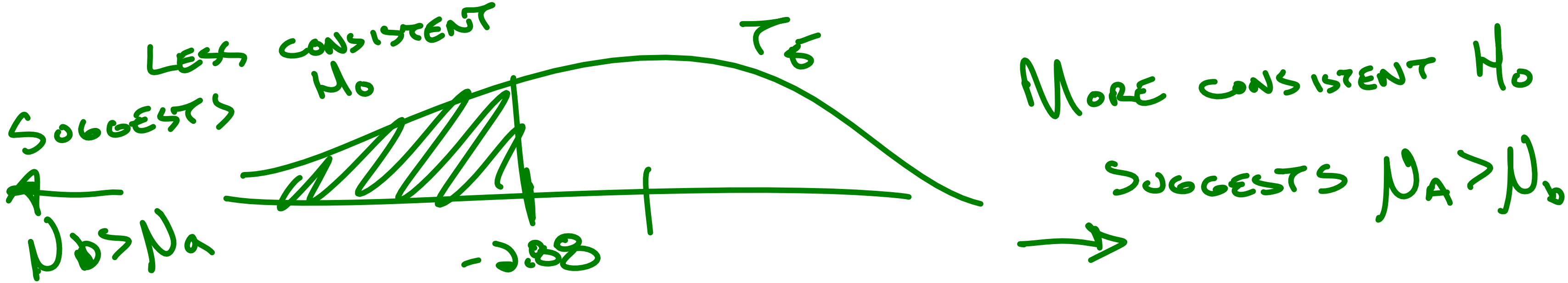
$H_1$: B's work sells more A's $N_B > N_A$

① Write $H_1$

② Compute T stat

$DF = N_A + N_B - 2$
$= 3 + 4 - 2 = 5$

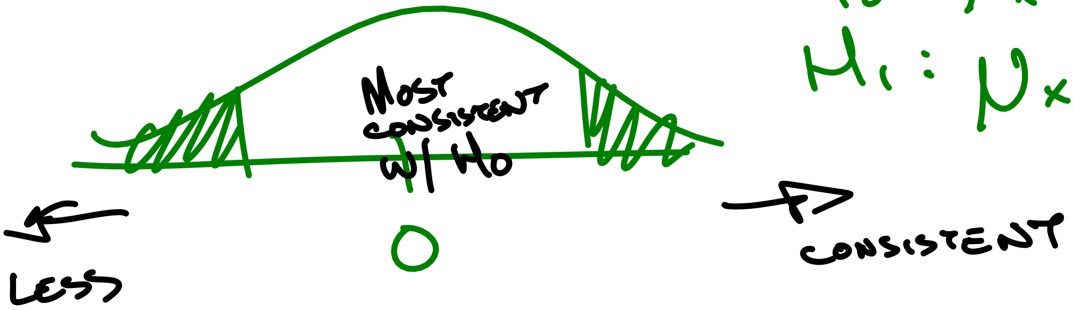
③ Build $P(T \mid H_0) \sim T_{DF}$



# TWO TAILS

$H_0: \mu_x = \mu_y$

$H_1: \mu_x \neq \mu_y$



P-value is the prob of an outcome less consistent with H0 is observed

$$P_{VAL} = T.CDF(-2.88, DF=5)$$
$$\hat{=} .017$$

$$\Rightarrow \quad P_{VAL} \hat{=} .017 < \alpha = .05$$

REJECT $H_0$ CLAIM $H_1$

$$\mu_b > \mu_a$$

$H_0$

$H_1$

P-VALUE $< \alpha = .05$

REJECT $H_0$ <u>CLAIM $H_1$</u>

ELSE

DO NOT REJECT $H_0$

Problem 3: Check, please!
The covariance (left) correlation (right) and mean (bottom left) of features describing diner bills is given below.
- Explain why tip (the total amount tipped) has a positive correlation with total_bill while tip_perc (tip as a percent of total) has a negative correlation.
- Describe the total_bill, smoker and size of the dining party which who gives the lowest tip (absolute, not perc)
- (+) The value 4 is most likely to belong to which of the five features below? Explain

| | total_bill | tip | smoker | size | tip_perc |
|---|---|---|---|---|---|
| **total_bill** | 79.252939 | 8.323502 | 0.371388 | 5.065983 | -0.184107 |
| **tip** | 8.323502 | 1.914455 | 0.003992 | 0.643906 | 0.028931 |
| **smoker** | 0.371388 | 0.003992 | 0.236845 | -0.061644 | 0.000916 |
| **size** | 5.065983 | 0.643906 | -0.061644 | 0.904591 | -0.008298 |
| **tip_perc** | -0.184107 | 0.028931 | 0.000916 | -0.008298 | 0.003730 |

| | total_bill | tip | smoker | size | tip_perc |
|---|---|---|---|---|---|
| **total_bill** | 1.000000 | 0.675734 | 0.085721 | 0.598315 | -0.338624 |
| **tip** | 0.675734 | 1.000000 | 0.005929 | 0.489299 | 0.342370 |
| **smoker** | 0.085721 | 0.005929 | 1.000000 | -0.133178 | 0.030820 |
| **size** | 0.598315 | 0.489299 | -0.133178 | 1.000000 | -0.142860 |
| **tip_perc** | -0.338624 | 0.342370 | 0.030820 | -0.142860 | 1.000000 |

mean of each feat:

| | |
|---|---|
| total_bill | 19.785943 |
| tip | 2.998279 |
| smoker | 0.381148 |
| size | 2.569672 |
| tip_perc | 0.160803 |

Problem 4: Sample mean / cov / corr compute

Compute the unbiased sample mean, covariance matrix and correlation matrices for the observations below

x = 4, 7, 9, 41
y = 3, 2, 1, 0

(each column above is a pair of observations (x0, y0) = (4,3), (x1, y1) = (7, 2), ...

Problem 5: Bayes

$P(X|A=1) \sim \text{Binom}(n=57, p=.001)$

Aliens, were they to exist on mars, would show up in .001 of photographs taken of the martian surface.

In the event Aliens don't exist, they'd never appear.

$P(X|A=0) \sim \text{Binom}(n=57, p=0)$

If we've taken 57 pictures of the surface of mars and an Alien hasn't shown up in any, whats the probability they exist?

Make any assumptions (i.e. a prior probability for aliens) you deem necessary. $P(A) = .5$
(Its a big drawback to bayesian analysis that we need to make a prior distribution ... feels rather subjective to estimate like this, right?)

A=1 EVENT ALIENS! ☺

X = # ALIEN PICS FROM 57

$$P(A=1 \mid X=0) = \frac{P(X=0 \mid A=1)\, P(A=1)}{P(X=0 \mid A=0)\, P(A=0) + P(X=0 \mid A=1)\, P(A=1)}$$

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

$P(X=0 \mid A=0) = 1$

$P(X=0 \mid A=1) = \text{Binom.PMF}(x=0, n=57, p=.001) = .945$

$$P(X=0) = P(X=0\ A=0) + P(X=0\ A=1)$$

$$= P(X=0 \mid A=0)\, P(A=0) + P(X=0 \mid A=1)\, P(A=1)$$

$$P(A=1 \mid X=0) = \frac{P(X=0 \mid A=1)\,P(A=1)}{P(X=0 \mid A=0)\,P(A=0) + P(X=0 \mid A=1)\,P(A=1)}$$

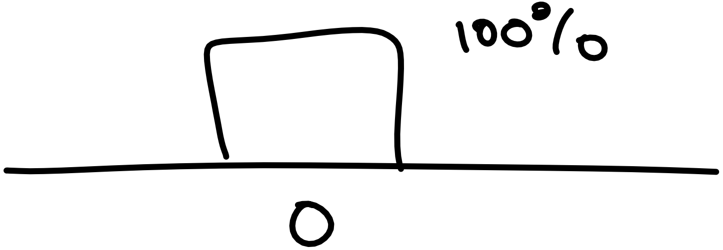
$$\boxed{P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)}}$$

$$P(X=0 \mid A=0) = 1$$

$$P(X=0 \mid A=1) = \text{Binom.PMF}(x=0,\ n=57,\ p=.001) = .945$$

$$P(A=1 \mid X=0) = \frac{.945 \cdot .5}{1 \cdot .5 + .945 \cdot .5} \simeq .4859$$

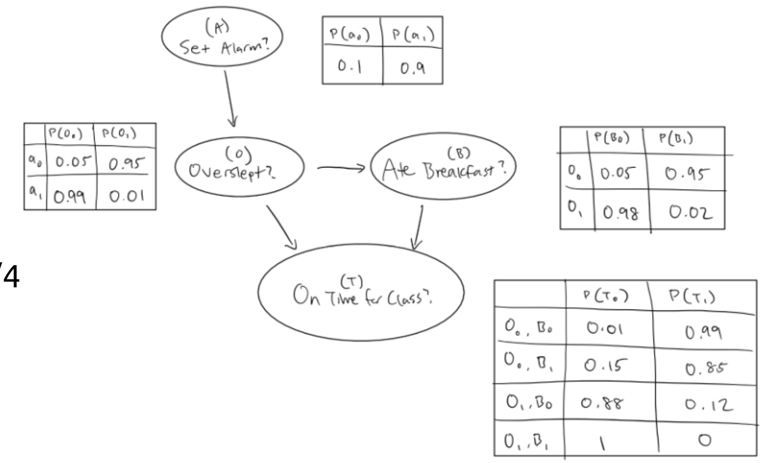
"PROBABILITY DISTRIBUTION"




$P(X|A=0)$

Problem 6: Bayes Net

1. Compute the joint distribution table for the Bayes Net
2. Compute prob one is on time for class.
3. Compute prob one is on time for class given they didnt set their alarm.
4. Compute prob one is on time for class given they didnt set their alarm and skipped breakfast.
5. Can you explain, via intuition informed by the network to the right, how prob from questions 2/3 and questions 3/4 compare? (e.g. why is 3 higher / lower than 2?)

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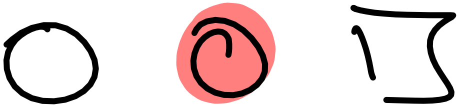
| (T) On Time For Class? | (B) Ate Breakfast? | (O) Overslept? | (A) Set Alarm? | P(TBOA) |
|---|---|---|---|---|
| t0 | b0 | o0 | a0 | 0.0000025 |
| t0 | b0 | o0 | a1 | 0.0004455 |
| t0 | b0 | o1 | a0 | 0.081928 |
| t0 | b0 | o1 | a1 | 0.0077616 |
| t0 | b1 | o0 | a0 | 0.0007125 |
| t0 | b1 | o0 | a1 | 0.1269675 |
| t0 | b1 | o1 | a0 | 0.0019 |
| t0 | b1 | o1 | a1 | 0.00018 |
| t1 | b0 | o0 | a0 | 0.0002475 |
| t1 | b0 | o0 | a1 | 0.0441045 |
| t1 | b0 | o1 | a0 | 0.011172 |
| t1 | b0 | o1 | a1 | 0.0010584 |
| t1 | b1 | o0 | a0 | 0.0040375 |
| t1 | b1 | o0 | a1 | 0.7194825 |
| t1 | b1 | o1 | a0 | 0 |
| t1 | b1 | o1 | a1 | 0 |

DON'T CARE ABOUT O

$$P(t_1 \mid a_0 b_0) = \frac{P(t_1 a_0 b_0)}{P(a_0 b_0)}$$

All Rows w/ $a_0$ $b_0$



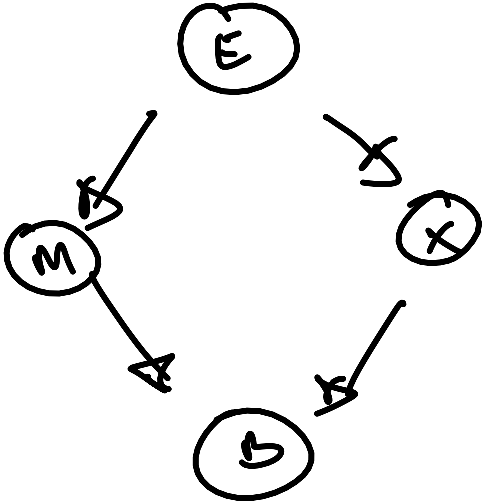
$C=1$ CIRCLE

$R=1$ RED

| | $C=0$ | $C=1$ |
|---|---|---|
| $R=0$ | $1/3$ | $1/3$ |
| $R=1$ | $0$ | $1/3$ |

$$P(R=0) = P(R=0\ C=0) + P(R=0\ C=1)$$

DON'T CARE ABOUT C




$$P(E)$$

$$P(EX) = P(X|E)\,P(E)$$

$$P(MEX) = P(M|EX)\,P(EX)$$

$$\downarrow$$

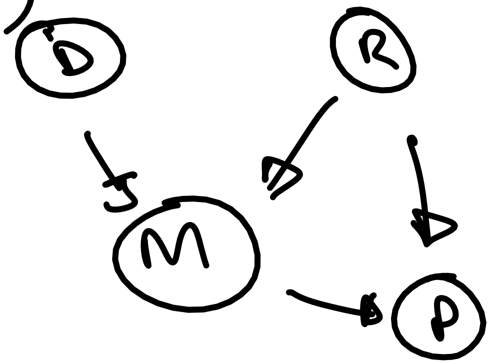
$$P(M|E) \text{ SINCE}$$

INDEP

$$P(RM) = P(M|R)\,P(R)$$

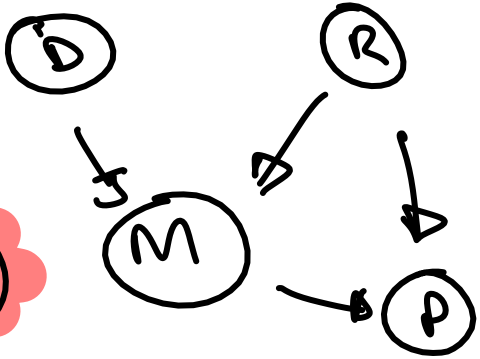
R, D INDEP

$$P(RM|D) = P(M|RD)\,P(R|D)$$
$$= P(M|RD)\,P(R)$$

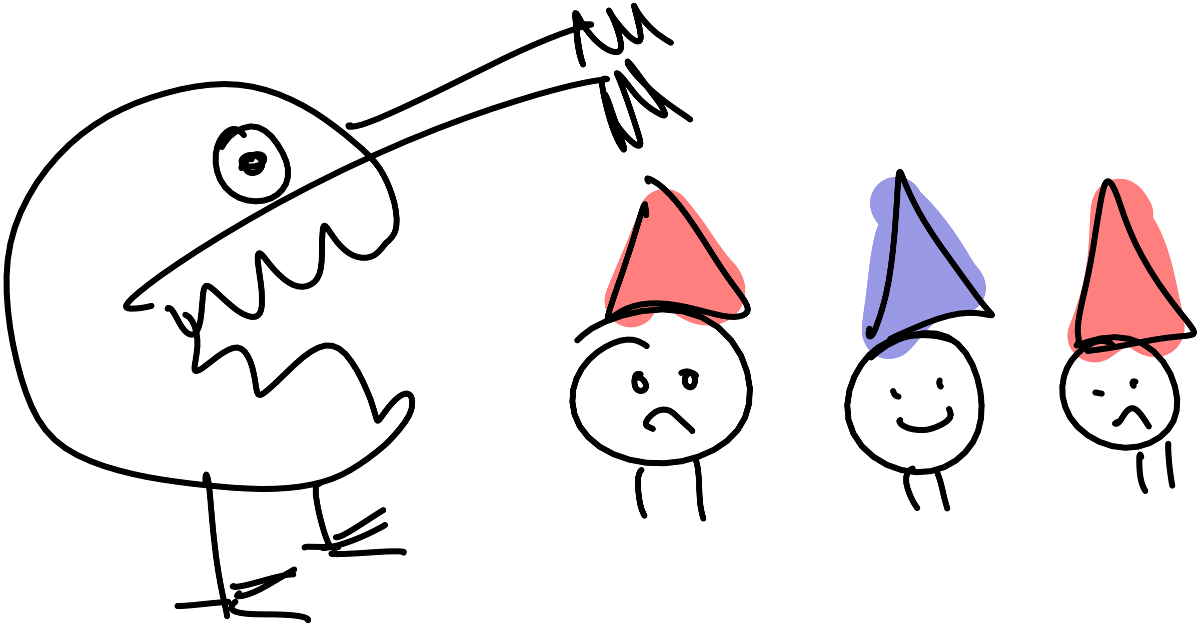


$$P(RMD) = P(M|DR)\,P(DR)$$

$$= P(M|DR)\,P(D)\,P(R)$$




$$\rightarrow P(RMD) = P(RM|D)\,P(D)$$



$$P(A|B) = \sum_{CD} P(ACD|B)$$