SOLUTIONS

CS1800
Fall 2025
Recitation 9 - Practice Questions for Homework 6
November 5 & 6, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice Problems for Sequence Types & Formulas (HW6, Q1)

For each sequence below, do the following:

- Identify the sequence type (arithmetic, geometric, quadratic, or none of the above)
- If applicable, find the formula for a_k, the kth term (assume we count from one so the first term would be a_1)

A −28, −22, −16, −10, ...

Arithmetic. The common difference d is 6, and initial value a is -28, so our formula for a_k is...

$$a_k = (k - 1) \cdot d + a = (k - 1) \cdot 6 - 28$$

Sanity check:

$a_1 = 0 - 28 = -28$ (good!)

$a_2 = 6 - 28 = -22$ (good!)

$a_3 = 12 - 28 = -16$ (yep!)

$a_4 = 18 - 28 = -10$ (also yep!)

B 5, -7, -12, -19, ...

Fits no known pattern

C 5, -7, −19, −31, ...

Arithmetic. The common difference is -12 and initial value is 5, so our formula is...

$$a_k = (k - 1) \cdot d + a = (k - 1) \cdot (-12) + 5$$

Sanity check:

$a_1 = 0 + 5 = 5$ (good!)

$a_2 = -12 + 5 = -7$ (good!)

$a_3 = -24 + 5 = -19$ (yep!)

$a_4 = -36 + 5 = -31$ (also yep!)

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D 4, 20, 100, 500 ...

Geometric. The common ratio is 5 and initial value is 4, and so we have:

$$a_k = 4 \cdot 5^{k-1}$$

Sanity-check

$$a_1 = 4 \cdot 5^0 = 4$$

$$a_2 = 4 \cdot 5^1 = 20$$

$$a_3 = 4 \cdot 5^2 = 100$$

E 6, 11, 18, 27, 38, 51, ...

The differences between terms are: $5,\ 7,\ 9,\ 11,\ 13$...
The difference between differences is: 2

Therefore it must be quadratic!

So we need to solve for $a,\ b,\ c$in the equation $a_k = ak^2 + bk + c$.

We find $a_k$by solving a system of linear equations:

When k = 1.... $a + b + c = 6$
When k = 2.... $4a + 2b + c = 11$
When k = 3.... $9a + 3b + c = 18$

Let's subtract the first from the second:

$$\begin{array}{rl} & 4a + 2b + c = 11 \\ - & a + b + c = 6 \\ \hline & 3a + b = 5 \end{array}$$

So we can express b in terms of a: $b = 5 - 3a$

Now subtract the second from the third:

$$\begin{array}{rl} & 9a + 3b + c = 18 \\ - & 4a + 2b + c = 11 \\ \hline & 5a + b = 7 \end{array}$$

Plug in our new expression for b so we can solve for a:

$$5a + b = 7$$
$$5a + 5 - 3a = 7$$
$$2a = 2$$
$$a = 1$$

Now we can solve for b:

$$b = 5 - 3a$$
$$= 5 - 3$$
$$= 2$$

Finally, we can solve for c:

$$a + b + c = 6$$
$$1 + 2 + c = 6$$
$$c = 3$$

Let's sanity-check this thing: $a_k = ak^2 + bk + c = k^2 + 2k + 3$

$a_1 = 1 + 2 + 3 = 6$

$a_2 = 4 + 2 \cdot 2 + 3 = 11$

$a_3 = 9 + 2 \cdot 3 + 3 = 18$

$a_4 = 16 + 2 \cdot 4 + 3 = 27$

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Practice Problems for Term of the Sequence (HW6, Question 2)

What is the term a_3 of the sequence $\{a_n\}$, where a_k equals...

A $2^k + 1$

Solution: $a_3 = 2^3 + 1 = 9$

B $(k + 1)^{k+1}$

Solution: $a_3 = (3 + 1)^{(3+1)} = 4^4 = 256$

C $\lfloor k/2 \rfloor$

Solution: $a_3 = \lfloor 3/2 \rfloor = 1$

D $\lfloor k/2 \rfloor + \lceil k/2 \rceil$

Solution: $a_3 = \lfloor 3/2 \rfloor + \lceil 3/2 \rceil = 1 + 2 = 3$

Consider these initial terms of a sequence: $a_1 = 1,\ a_2 = 4,\ a_3 = 9,\ a_4 = 16$ ...

E Each term is actually the value of a summation, because:

$a_1 = 1$

$a_2 = 1 + 3 = 4$

$a_3 = 1 + 3 + 5 = 9$

$a_4 = 1 + 3 + 5 + 7 = 16$

...

Let's generalize this idea. For a given a_k, fill in the summation formula:

$$a_k = \sum_{i=1}^{k} [???]$$

Solution

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Each term a_k is the sum of the first k odd integers. Now we can say:

$$a_k = \sum_{i=1}^{k} 2i - 1$$

F For the sequence in part E, we can express an arbitrary term a_k **recursively**, by referring to previous term(s). Come up with a recursive formula for a_k.

Solution

We can see that for each term, we take what we had previously and add on the next odd number. This gives us the recursive formula:

$$a_k = a_{k-1} + (2k - 1)$$

Base case: $a_1 = 1$

Sanity check (never hurts!), it should match the terms given in part E:

$$a_1 = 1$$
$$a_2 = a_1 + (4 - 1) = 1 + 3 = 4$$
$$a_3 = a_2 + (6 - 1) = 4 + 5 = 9$$
$$a_4 = a_3 + (8 - 1) = 9 + 7 = 16$$

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Practice Problems for Summations (HW6, Question 3)

A Give a formula, simplified as much as possible, for the sum of the first n terms of the sequence $5,\ 7,\ 9,\ 11,\ 13,\ \ldots$

Solution

This is an arithmetic sequence with $a = 5$ and $d = 2$

We know that the sum of the first n elements of an arithmetic sequence: $\frac{\#terms \cdot (first + last)}{2}$

- Number of terms: n
- First element: $a = 5$
- Last element: $a_n = 2(n - 1) + 5 = 2n + 3$

So we get: $= \frac{n \cdot (5+2n+3))}{2} = \frac{8n+2n^2}{2} = n^2 + 4n$

B What is the sum of the first 20 values of the sequence beginning with $-28,\ -22,\ -16,\ -10,\ \ldots$?

Solution

As we saw in Practice Problem #1, this is an arithmetic sequence with initial value -28 and common difference 6, giving us $a_k = (k - 1) \cdot 6 - 28$.

We can use the arithmetic-summation formula, or we can do fun summation math to end up with the formula for the sum of the first n positive integers. Let's try both ways:

Method #1: Sum of first n elements of any arithmetic sequence

Sum of the first n elements of any arithmetic sequence: $\frac{\#terms \cdot (first + last)}{2}$

- Number of terms: 20
- First element: a_1 =-28
- Last element: $a_{20} = (20 - 1) \cdot 6 - 28 = 86$

So we get: $= \frac{20 \cdot (-28 + 86)}{2} = 1160/2 = 580$

Method #2: Summation math until we get to the formula, where the sum of the first n *positive integers is* $\frac{(n)(n+1)}{2}$

We have $\sum_{k=1}^{20}[(k-1)\cdot 6-28]$

Start by pulling out the -28:

$$(-28\cdot 20)+\sum_{k=1}^{20}[(k-1)\cdot 6]$$

$$-560+\sum_{k=1}^{20}[(k-1)\cdot 6]$$

Pull out the 6:

$$-560+6\cdot\sum_{k=1}^{20}(k-1)$$

Now we can isolate the *k* inside the sigma:

$$-560+6\cdot\sum_{k=0}^{19}k$$

Now we have a Guassian term, the sum of the first 19 positive integers! Solve that:

$$-560+6\cdot\frac{(20)(19)}{2}$$

Finally, it's just arithmetic

$$-560+6\cdot 190$$
$$=-560+1140$$
$$=580$$(WOOHOO same as the other approach!!!!)

C What is the sum of the first 10 values of the sequence beginning with $4,\ 20,\ 100,\ 500,\ldots$?

Solution

As we saw in Practice Problem #1, this is a geometric sequence with initial value 4 and common ratio 5, giving us $a_k = 5^{k-1}\cdot 4$

.

Sum of the first *n* elements of a geometric sequence: $\frac{ar^n-a}{(r-1)}$

- $n = 10$
- $a = 4$
- $r = 5$

So we get: $= \frac{4 \cdot 5^{10} - 4}{4} = 9765624$

Method #2: Fun sneaky way

Another approach is to solve this problem in a similar way that Gauss summed up the first 100 positive integers.

We just need to know the value of the 10th term, which is $5^9 \cdot 4 = 7812500$

Let $S = 4 + 20 + 100 + 500 + \ldots + 7812500$

Multiply every number by 5 to get $5S = 20 + 100 + 500 + \ldots + 39062500$. Now we have pairs of identical numbers.

$5S - S = S$so we can subtract one formula from the other:

$$20 + 100 + 500 + \ldots + 781250 + 39062500$$
$$- (4 + 20 + 100 + 500 + \ldots + 781250)$$

$$4S = 39062500 - 4$$
$$S = 9765624$$

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Practice Problems for Combining Summations (HW6, Question 4)

A Write the expression $3 + 6 + 9 + 12 + \cdots + 60$ in sigma notation

Solution: This is an arithmetic sequence, so we're adding multiples of 3:. $\sum_{i=1}^{20} 3i$

Recall that the sum of the first n positive integers is $\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$

We recommend applying it to solve the problems below!

B Find $\sum_{i=2}^{n} i$

Solution: This is pretty close to the original, we're just subtracting off the first value $i = 1$.

$\sum_{i=2}^{n} i = \frac{(n)(n+1)}{2} - 1$

C Find $\sum_{i=1}^{n-1} i$

Solution: This is pretty close to the original, we're just subtracting off the last value $i = n$.

$\sum_{i=1}^{n-1} i = \frac{(n)(n+1)}{2} - n$

Now we just need to simplify, and we get $\frac{n^2+n}{2} - \frac{2n}{2} = \frac{n^2-n}{2} = \frac{(n)(n-1)}{2}$

D Find $\sum_{i=1}^{n} (i - 1)$

Solution: Same as part C!

$\frac{(n)(n-1)}{2}$

E Find $\sum_{i=50}^{n} i$. Simplify your expression as much as possible.

Solution:

We can apply the summation formula above, but not directly for the whole summation. But, we can break the summation into parts.

$$\sum_{i=50}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{49} i$$

Now we can apply the arithmetic summation formulation to simplify the second term $\sum_{i=1}^{49} i = \frac{49 \cdot 50}{2} = 1225$, giving us:

$$\sum_{i=50}^{n} i = \frac{(n)(n+1)}{2} - 1225$$

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Practice Problems for Sequences in Real Life (HW6, Question 5)

Daniel loses a bet to Laney and owes her $50,000. She sets terms on the money, though, just in case Daniel is a deadbeat who refuses to pay up. He owes $50,000 in month one. But in month two, she demands to be paid $56,000. Month three it's $62,720 and month four it's $70,246.40.

A Identify the type of sequence Laney is using (arithmetic, geometric, or quadratic). Use the formula for that type to determine a closed-form formula for the amount Daniel will owe Laney in month k.

Solution

$a_1 = 50000$
$a_2 = 56000$
$a_3 = 62720$
$a_4 = 70246.40$

Is this arithmetic? Let's check the differences between values.

a_1 to a_2: 6000
a_2 to a_3: 6720
a_3 to a_4: 7526.4

Doesn't look arithmetic.

Is this geometric? Let's check the ratios between values.

a_1 to a_2: 1.12
a_2 to a_3: 1.12
a_3 to a_4: 1.12

It's geometric!

Now we have initial value $a = 50000$ and common ratio $r = 1.12$

It is geometric.
This gives us closed-form formula: $a_k = (50000)(1.12)^{k-1}$

Sanity-check! Not required as part of your answer, but it's good to make sure the formula

re-derives the values we know about.

$$a_1 = (50000)(1.12)^0 = 50000$$
$$a_2 = (50000)(1.12)^1 = 56000$$
$$a_3 = (50000)(1.12)^2 = 62720$$
$$a_4 = (50000)(1.12)^3 = 70246.4$$ boom!!!!!

B Use your formula to determine how much Daniel will owe Laney in month 12.

$$a_{12} = (50000)(1.12)^{11} = \$173,927.50$$