SOLUTIONS

CS1800
Fall 2025
Recitation 8 - Practice Questions for Homework 5
October 29 & 30, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice Problems for Pigeonhole (HW5, Q1)

A The 11 thieves from the movie *Ocean's 11* decide to get hotel rooms at the casino they're robbing. They book 6 rooms. At least one room is guaranteed to have at least how many thieves?

Solution -
The generalized pigeonhole principle gives us, if we have k boxes and $n > k$ objects, then at least one box has at least $\lceil n/k \rceil$ objects.

Here we have rooms are the boxes ($k = 6$) and thieves are the objects ($n = 11$) so we get $\lceil 11/6 \rceil = 2$.

B The 11 thieves from the movie *Ocean's 11* decide to get hotel rooms at the casino they're robbing. What is the minimum number of rooms they need to book to guarantee that no one shares a room with more than 3 people (i.e., no more than 4 total people per room)?

Solution -
The generalized pigeonhole principle gives us, if we have k boxes and $n > k$ objects, then at least one box has at least $\lceil n/k \rceil$ objects.

Here we have rooms are the boxes (k =?) and thieves are the objects ($n = 11$) and we want the minimum k such that $\lceil 11/k \rceil = 4$

We can achieve this with $k = 3$ rooms, because $\lceil 11/3 \rceil = 4$ while $\lceil 11/2 \rceil = 6$

C You can order a meal-deal at Veggie Galaxy with a vegan sandwich, fries, and a drink. There are three options for the sandwich, three for the fries, and three for the drink.

What is the minimum number of customers at Veggie Galaxy needed to guarantee that at least two orders will be identical?

Solution:

Number of total possible orders +1.

How many total possible orders? (Also in Part A!)
Product rule applies. There are three parts to the order, and three options for each part.

$3 \cdot 3 \cdot 3 + 1 = 28$

D There are 50 baskets of apples. Each basket contains at least one but no more than 24 apples. Show that there are at least 3 baskets containing the same number of apples.

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Solution

- Boxes: # apples in a given basket. There are 24 of these (i.e., if you've got a basket of apples, then there 24 options for the # things inside)
- Objects: baskets. There are 50 of these.

The ratio *n/k* of objects to boxes applies and we have *ceil*(50/24) = 3. Therefore, there must be at least 3 baskets with the same number of apples.

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Practice Problems for Conditional Probability (HW5, Question 2)

A Suppose we have a bit string of length 5, randomly generated such that all bit strings are equally likely. What is the probability that the string contains at least three consecutive ones, given that the first bit is a zero?

Solution:

This is a conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$, with E = the set of bit strings with at least three consecutive 1s and F = the set of bit strings where the first bit is a zero.

There are a total of $2^5 = 32$ bit strings of length 5.

$P(E \cap F)$ =the probability of three consecutive 1s AND the first bit is a zero. There are three of these (01111, 01110, 00111), giving us $3/32 = .09375$

$P(F) = .5$ = the probability that our 5-bit string starts with a zero, which is .5

Finally, we put everything together to get...

$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{.09375}{..5} \approx 18.8\%$

Coco Gauff has a bucket of tennis balls at the U.S Open; 25 of the tennis balls are pink and 50 of them are orange.

B Coco draw three tennis balls at random. What is the probability that none of them is pink?

Solution

No pink means we draw an orange one AND another orange one AND another orange one.

Order doesn't matter, so what we compute here is:

- Event space: Number of ways to choose 3 orange ones: $\binom{50}{3}$
- Event space: Number of ways to choose 3: $\binom{75}{3}$

Giving us the probability:

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$P(no\ pink)\ = P(all\ orange)\ = \frac{\binom{50}{3}}{\binom{75}{3}} =.2902$

(Another way to think about it is picking one of the 50 oranges out of the 75 total, then there are 74 left and we pick one of the 49 oranges, etc.)

$P(no\ pink)\ = P(all\ orange)\ = 50/75 \cdot 49/74 \cdot 48/73 \approx .2902$

C Coco draws 3 tennis balls at random from the bucket. She sees that one of them is pink. What is the probability that she got exactly three pink tennis balls, given that she knows she got at least one pink tennis ball?

Solution:

This is a conditional probability, $P(E|F)\ = \frac{P(E \cap F)}{P(F)}$, with E = the set of all 3-item draws that have exactly 3 pinks, and F = the set of all 3-item draws that have at least one pink.

$P(E\ \cap\ F)$ =the probability of getting exactly three pinks AND at least one pink, which simplifies to the probability of getting exactly three pinks. There are 25 pinks in the bucket,, and order doesn't matter.

$P(E\ \cap\ F) = P(E) = \frac{\binom{25}{3}}{\binom{75}{3}} \approx .034$

(Again, we can also compute the probability of selecting the first pick one is 25/75, the next pink one is 24/74, etc., and we get the same answer)

$P(E\ \cap\ F)\ = 25/75 \cdot 24/74 \cdot 23/73 \approx .034$

$P(F)$ = the probability of getting at least one pink, for which we'll use the subtraction rule.

The probability of $P(\neg F)$, getting ZERO pinks, we already figured out! It's
$P(no\ pink)\ = P(all\ orange)\ \approx .2902$

Therefore$P(F) = 1 - .2902 \approx .7098$

Finally, we put everything together to get...
$P(E|F)\ = \frac{P(E \cap F)}{P(F)} = \frac{.034}{.7098} \approx 4.79\%$

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Practice Problems for Expected Value (HW5, Question 3)

Compute the expected value of each of the experiments in Parts A and B. Assume that the die faces are numbered $1, 2, ..., n$ for an n-sided die and that all outcomes are equally likely.

A A 3-sided die.

Solution
$EV(X) = 1/3 \cdot 1 + 1/3 \cdot 2 + 1/3 \cdot 3 = 2$

B A 4-sided die.

Solution
$EV(X) = 1/4 \cdot 1 + 1/4 \cdot 2 + 1/4 \cdot 3 + 1/4 \cdot 4 = 2.5$

C Is the expected value of a 6-sided die less than or greater than the value of an 8-sided die? Why?

Solution
Less than. Typically, the 8-sided die will show higher values than the 6-sided one.

D You are dealt 2 cards face down from a standard 52-card deck. The deck contains 26 red cards (all the hearts and all the diamonds). How many red cards would you expect to receive in your 2-card hand?

Solution:

Let $X = X_1 + X_2$ where X represents the number of red cards dealt, and X_i are indicator variables (0 or 1) denoting whether the ith card is a red card.

$E[X_1] = E[X_2] = 1 \cdot 26/52 + 0 \cdot 26/52 = .5$
So $E[X] = 2 \cdot E[X_1] = 2 \cdot .5 = 1$ red card

E Laney is playing video poker, always bets \$30 on a single hand. Then, one of two things happens:

- She loses the hand, and loses her \$30.
- She wins the hand, and profits \$60 (that's a total return of \$90).

No other outcomes are possible. What would the probability of winning need to be in order for Laney to expect to profit at least \$50?

Solution:

Let X be the random variable associated with the outcome of a hand of poker.

E[X] = $(-\ 30)(1\ -\ x)\ +\ (60)(x)$

Solve for x to find a percentage that would yield a profit ≥ \$50.
$(-\ 30)(1\ -\ x)\ +\ (60)(x)\ \geq\ 50$
$(-\ 30\ +\ 30x)\ +\ 60x\ \geq\ 50$
$90x\ \geq\ 80$
$x\ \geq\ 80/90$

The chance of winning would have to be at least $8/9$, or 88.89%.

Sanity-check, just to make sure it makes sense!

$$E[X]\ =\ (-\ \$30)(1/9)\ \ +\ \ (\$60)(8/9)$$

$$=\ \$50$$

And if we make the probability just a bit lower, we'd be under \$50:

$$E[X]\ =\ (-\ \$30)(1\ -.8888)\ \ +\ \ (\$60)(.8888)$$

$$=\ \$49.992$$

Practice Problems for Bayes Rule (HW5, Question 4)

A Kasing Lung uses three machines, X, Y, and Z to produce Labubus.

Suppose:
- Machine X produces 50% of the Labubus, of which 3% are defective
- Machine Y produces 30% of the Labubus, of which 4% are defective
- Machine Z produces 20% of the Labubus, of which 5% are defective.

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Find the probability that a randomly selected Labubu is defective.

Solution:
Let D denote the event that the Labubu is defective. Then by the law of probabilities
P(D) = P(X)*P(D|X) + P(Y)P(D|Y) + P(Z)P(D|Z)
= (0.5)*(0.03) + (0.3)*(0.04) + (0.2)*(0.05)
= .037

B Suppose a defective Labubu is found at the factory. Which machine is most likely to have built it?

Solution:
We need to find and compare P(X|D), P(Y|D), and P(Z|D)

From part A we have
P(D) = P(X)*P(D|X) + P(Y)P(D|Y) + P(Z)P(D|Z)
= 0.037

We can use this value with Bayes' formula
P(X|D) = P(X)*P(D|X) / P(D)
= ((.5)*(.03)) /0.037
= .405

P(Y|D) = P(Y)*P(D|Y) / P(D)
= ((0.3)*(0.04)) / 0.037
= .325

P(Z|D) = P(Z)*P(D|Z) / P(D)
= ((0.20)*(0.05)) / 0.037
= .270

Therefore, the likeliest scenario is that Machine X built the defective Labubu.

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Practice Problems for (HW5, Question 5)

There are 6 different types of pies at Thanksgiving (pumpkin, pecan, apple, cherry, chocolate custard, and sweet potato). There is plenty to go around, and Laney picks 3 slices of pie at random.

A Assuming order doesn't matter and Laney doesn't mind repeating flavors, how many ways are there for her to have three slices of pie?

Solution

This is a stars and bars problem!

Laney selects 3 slices (stars), and there are 6 flavors giving us 5 dividers (bars).

$$C_{\#stars}^{\#stars\,+\,\#\,bars} = C_{3}^{3+5} = C_{3}^{8} = 56$$

B Assuming all outcomes are equally likely (e.g., having 3 slices of cherry is just as likely as two sweet potato and one pecan), what is the probability that Laney gets exactly two slices of pumpkin pie?

Solution

The sample space is all the 3-slice outcomes:

$$|S| = C_{\#stars}^{\#stars\,+\,\#\,bars} = C_{3}^{3+5} = C_{3}^{8} = 56$$

The event space is the number of pie-assortments that contain exactly two pumpkin. One way to pick the pumpkin and 5 ways to choose the remaining flavor

So altogether we get $|E| = 5$

Finally, $Pr(E) = |E|/|S| = 5/56 \approx .089$

C Assuming all outcomes are equally likely (e.g., having 3 slices of cherry is just as likely as two sweet potato and one pecan), what is the probability that Laney gets **at least** two slices of pumpkin pie?

Solution

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The sample space is all the 3-slice outcomes:

$$|S| = C^{\#stars + \# bars}_{\#stars} = C^{3+5}_{3} = C^{8}_{3} = 56$$

The event space is the number of pie-assortments that contain:

- Case 1 -- exactly three pumpkin. Only one outcome has this.
- Case 2 -- exactly two pumpkin. One way to pick the pumpkin and 5 ways to choose the remaining flavor

So altogether we get $|E| = 5 + 1 = 6$

Finally, $Pr(E) = |E|/|S| = 6/56 \approx .1071 = 10.71\%$

D Assuming all outcomes are equally likely (e.g., having 3 slices of cherry is just as likely as two sweet potato and one pecan), what is the probability that Laney gets three different flavors of pie?

Solution

The sample space is all the 3-slice outcomes:

$$|S| = C^{\#stars + \# bars}_{\#stars} = C^{3+5}_{3} = C^{8}_{3} = 56$$

The event space is the number of pie-assortments that contain three different flavors. Of the 6 possible flavors, we pick 3 of them.

So altogether we get $|E| = C^{6}_{3} = 20$

Finally, $Pr(E) = |E|/|S| = 20/56 \approx .357$

E Assuming all outcomes are equally likely (e.g., having 3 slices of cherry is just as likely as two sweet potato and one pecan), what is the probability that Laney gets only one flavor of pie?

Solution

The sample space is all the 3-slice outcomes:

$$|S| = C^{\#stars + \# bars}_{\#stars} = C^{3+5}_{3} = C^{8}_{3} = 56$$

The event space is the number of pie-assortments that contain three different flavors. Of the 6 possible flavors, we pick 1 of them.

So altogether we get $|E| \ = \ C^6_1 = 6$

Finally, $Pr(E) \ = |E|/|S| \ = \ 6/56 \approx .107$

F What is the expected number of distinct pie flavors on Laney's plate?

Solution

Let X be the random variable associated with the number of distinct flavors, and let $P(i)$ denote the probability of getting *i* flavors.. The expected value will be
$E[X] = 1 \cdot P(1) \ + 2 \cdot \ P(2) \ + 3 \cdot \ P(3)$

We've already computed $P(1)$ and $P(3)$ so the only thing we haven't covered is $P(2)$ which we can do with the subtraction rule.

Total outcomes 56
Outcomes with exactly 1 flavor or exactly three flavors: $20 + 6 = 26$
Remaining outcomes (i.e., exactly two flavors): $56 - 26 = 30$

Now we can plug into our formula
$E[X] = 1 \cdot P(1) \ + 2 \cdot \ P(2) \ + 3 \cdot \ P(3)$
$= 1 \cdot (6/56) \ + 2 \cdot \ (30/56) \ + 3 \cdot \ (20/56)$
=2.25 expected flavors!

(Want to compute P(2) directly? Look at like this;

$|S| \ = 56$

choose the two flavors = $C^6_2 = 15$

AND choose one of those two flavors for the remaining slice: $C^2_1 = 2$

$|E| \ = \ 15 \cdot 2 = 30$