CS1800
Fall 2025
Recitation 7 - Practice Questions for Quiz 2
October 22 & 23, 2025

Quiz Preparation

Our second quiz is coming up on October 24th! There are two questions on the quiz, and there are practice problems for each topic below. It can also be a useful study practice to go back and revisit previous recitation practice problems on the same topics for extra practice.

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

Practice for Set Equality (Quiz 2 Question 1)

Let our universe be all people in Boston. Suppose we the following sets:

- A -- runners in Boston
- B -- cyclists in Boston
- C -- dog lovers in Boston

Express each partition below as a set operation on A and/or B. Your solution should use the set operation symbols for intersection, union, complement, and difference, and no other symbols.

A Boston runners who also cycle

$A \cap B$

B Boston runners who don't cycle

$A - B$

Equivalent $A \cap \overline{B}$

C Boston runners who don't cycle and don't love dogs

Do these people exist??

$A - (B \cup C)$

Equivalent $A \cap \overline{B} \cap \overline{C}$

D Boston dog lovers who run but don't cycle

$A \cap C \cap \overline{B}$

Equivalent $(A \cap C) - B$
Equivalent $A \cap (C - B)$

E Bostonians who love dogs and running, or Bostonians who don't run but do cycle.

$(A \cap C) \cup (\overline{A} \cap B)$

F Prove that your answers for Part A and Part B don't intersect by using set equality laws and/or definitions. Take one step at a time and label each step with one law.

We are showing that
$(A \cap B) \cap (A - B) = \{\}$

Start:
$(A \cap B) \cap (A - B)$

$(A \cap B) \cap (A \cap \overline{B})$	Definition of difference
$A \cap B \cap A \cap \overline{B}$	Associative
$A \cap A \cap B \cap \overline{B}$	Commutative
$A \cap B \cap \overline{B}$	Idempotent
$A \cap \{\}$	Complement
$\{\}$	Domination

For each pair of set expressions below, determine whether the resulting sets are equal.

- **If yes...** Apply the laws of set equality to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give example elements for *A, B,* and *C* that would yield a counterexample. Simplify both sets to demonstrate that they are not the same.

Both *yes* and *no* answers should be clear, precise, and walk through your solution one small step at a time.

G $(A \cap B) \cup (A \cap \overline{B})$
B

Solution they are not equal

Counterexample: let $A = \{1, 2\}$, $B = \{3, 4\}$ *and* $U = \{1, 2, 3, 4\}$

The set $A \cap B = \{\}$

The set $\overline{B} = \{1, 2\}$

The set $(A \cap B) \cup (A \cap \overline{B})$ with these values plugged in:

$= \{\} \cup \{\{1, 2\} \cap \{1, 2\}\}$

$= \overline{\{1, 2\}}$

Since $\{1,2\}$ is clearly not equal to B, the sets are not equal.

H $(A \cap B) \cup (A \cap \overline{B})$

$\overline{A}$

Solution they are not equal

Counterexample: Let $A = \{1, 2, 3\}$ $B = \{3, 4\}$, $U = \{1, 2, 3, 4\}$

LHS:

$(A \cap B) \cup (A \cap \overline{B}) = (\{1, 2, 3\} \cap \{3, 4\}) \cup (\{1, 2, 3\} \cap \{1, 2\})$

$= \{3\} \cup \{1, 2\}$

$= \{1, 2, 3\}$

RHS:

$\overline{A} = \{4\}$

I $(A \cap B) - (A \cup B)$

$\{\}$

Solution they are equal

$(A \cap B) - (A \cup B)$

$= (A \cap B) \cap \overline{(A \cup B)}$ definition of difference

$= (A \cap B) \cap (\overline{A} \cap \overline{B})$ deMorgan

$= A \cap B \cap \overline{A} \cap \overline{B}$ Associative

$= A \cap \overline{A} \cap B \cap \overline{B}$ Commutative

$= \{\} \cap B \cap \overline{B}$ Complement

$= \{\}$ Domination

J $\overline{(A \cap (A \cup \overline{B}))}$

$\overline{A}$

Solution they are equal

Proof:

$\overline{(A \cap (A \cup \overline{B}))}$

$= (\overline{A} \cup \overline{(A \cup \overline{B})})$ DeMorgan

$= (\overline{A} \cup (\overline{A} \cap \overline{\overline{B}}))$ DeMorgan

$= (\overline{A} \cup (\overline{A} \cap B))$ Double complement

$= \overline{A}$ Absorption

Practice for Counting (Quiz 2 Question 2)

A Suppose a password system has the following restrictions:

- Must be 6-8 characters long
- At least one digit required
- Every character must be an uppercase letter or a digit

How many possible passwords are there?

Solution: (note that we wouldn't require the whole number on a quiz, just the expression)

First, consider passwords of the three different possible lengths, call them P6, P7, and P8.

For each length *n,* the product rule applies: there are 36^n ways to choose *n* uppercase letters / digits. But this includes passwords consisting of only letters, of which there are 26^n, and we need to subtract them out.

- P6: $36^6 - 26^6$ = 1,867,866,560
- P7: $36^7 - 26^7$ = 70,332,353,920
- P8: $36^8 - 26^8$ = 2,612,282,842,880

Finally, apply the sum rule, because we can choose a P6 or a P7 or a P8 password: $\mathbf{36^6 - 26^6 + 36^7 - 26^7 + 36^8 - 26^8}$

(This is 2,684,483,063,360 total possible passwords.)

B How many **bit** strings of length 9 contain exactly 4 zeroes?

Solution: Choose the positions of the four zeroes, everything else is a one

$C(9,4) = 9!\ / 4!\,5! = 126$

C How many **decimal** strings of length 4 contain exactly 3 zeroes?

Solution: Choose the positions of the three zeroes, but now other value could be 1-9.

$C(4,3) = 4!\ / 3!\,1! = 4$

The remaining value: 9 choices

In total: $4 \cdot 9 = 36$

D How many integer solutions are there for the equation

$x_1 + x_2 + x_3 + x_4 = 50$

where x_1, x_2, x_3, x_4 are non-negative integers?

Solution

We're selecting 50 indistinguishable things and assigning them to 4 buckets. For example:

$x_1 = 10,\ x_2 = 3,\ x_3 = 20,\ x_4 = 17$

We have #stars = 50, # bars = 3

- $C(53, 50) = 23426$

E How many integer solutions are there for the equation

$x_1 + x_2 + x_3 + x_4 = 50$

where x_1, x_2, x_3, x_4 are **positive** integers?

Solution

Adding constraints makes this stars and bars problem a bit more complicated. Let's turn this into a problem we know we can solve. Rewrite each $x_i = y_i + 1$ with $y_i \geq 0$, so that we guarantee the x's are greater than zero:

- $x_1 = y_1 + 1$
- $x_2 = y_2 + 1$
- $x_3 = y_3 + 1$
- $x_4 = y_4 + 1$

Plug them back in:

$(y_1 + 1) + (y_2 + 1) + (y_3 + 1) + (y_4 + 1) = 50$

Subtract to isolate the y's:

$y_1 + y_2 + y_3 + y_4 = 46$

Now we can solve it straightforwardly! We have 46 stars and three bars, giving us

$C(49, 46) = 18424$

F How many integer solutions are there for the equation

$$2x_1 + 2x_2 + 2x_3 + 2x_4 = 50$$

where x_1, x_2, x_3 are non-negative integers?

Divide both sides by two:

$$x_1 + x_2 + x_3 + x_4 = 25$$

Now we're solving an easier version of the problem and we get 25 stars and 3 bars so that's $C(28, 3) = 3276$

G University House of Pizza on Huntington Ave. offers 8 different toppings. You want to order a pizza with 3 different toppings. How many different pizzas can you create?

Solution : order doesn't matter, no repetition. This is a combination!

$C(8, 3) = 56$

H In a race with 7 runners, how many different ways can the gold, silver, and bronze medals be awarded?

Solution : order matters, no repetition. This is a permutation!

$P(7, 3) = 210$

I How many ways are there to roll three six-sided dice so that the sum of their values is 4? Order matters here, such that the roll (1, 1, 2) is different than (1, 2, 1).

Now we essentially have $d_1 + d_2 + d_3 = 4$ but with the constraint that no value can be zero.

Because this problem is so small, we could just do it by hand:

- (1, 1, 2)
- (1, 2, 1)
- (2, 1, 1)

There are three ways!

Let's also do this the "real" way: we'll count all the solutions and subtract out the invalid ones.

Without constraints, $d_1 + d_2 + d_3 = 4$ has $C(6, 4) = 15$ solutions.

Invalid options to subtract:

- d_1 is zero. Then we're distributing the 4 stars between the remaining two dice, giving us $C(5, 4) = 5$
- d_2 is zero but d_1 is not (because we already counted it). Then have $C(5, 4) - 1 = 4$
- d_3 is zero but d_1 and d_2 are not. Then have $C(5, 4) - 2 = 3$

So in total we have $C(6, 4) - (5 + 4 + 3) = 15 - 12 = 3$

J How many ways are there to roll three six-sided dice so that the sum of their values is 7? Order matters here, such that the roll (1, 1, 5) is different than (1, 5, 1).

Solution

Now we essentially have $d_1 + d_2 + d_3 = 7$ but with two constraints:

- No value can be 0
- No value can be 7

So let's start with solving the easier version of the problem, and then subtract out the invalid possibilities. How many solutions are there for

$$d_1 + d_2 + d_3 = 7$$

Without any constraints, and any d can be 0-7, his gives us 7 stars and 2 bars, so a total of $C(9, 7)$ ways.

Invalid option: one of the dice is zero:

- Invalid: d_1 is zero. This means the seven stars are distributed between the remaining two dice, giving us $C(8, 7) = 8$ ways to do this.
- Invalid: d_2 is zero but d_1 is not zero (because we already counted it). We have $C(8, 7) - 1 = 7$ ways to do this.
- Invalid: d_3 is zero but d_1 is not zero and d_2 is not zero We have $C(8, 7) - 2 = 6$ ways to do this.

(Do we also need to worry about the possibility with a 7 in the mix? Nope, we already took care of it! Because 7 would only appear in one die when we have zeroes in the other two.)

So in total we have $C(9, 7) - (8 + 7 + 6) = 36 - 21 = 15$