

SOLUTIONS

CS1800

Fall 2025

Recitation 6 - Practice Questions for Homework 4

October 15 & 16, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

SOLUTIONS

Practice Problems for Counting (Focus on Permutations, Combinations) (HW4, Q1)

- A** There are 100 people in your graduating class, and the Class Clown, Best Dressed, and Nicest Human are given out. How many ways are there to select these winners, assuming a person cannot win more than one honor?

Solution

Product rule: $100 \cdot 99 \cdot 98 = 970200$

- B** Now your graduation committee has decided it's OK for someone to win more than one honor. How many ways are there now?

Solution

Product rule: 100^3

- C** You can order a meal-deal at Veggie Galaxy with a vegan sandwich, fries, and a drink. There are three options for the sandwich, three for the fries, and three for the drink.

How many different orders can be created?

Solution

Product rule applies. There are three parts to the order, and three options for each part.

$$3 \cdot 3 \cdot 3 = 27$$

A *bit string* is a sequence of 0s and 1s that is treated as string/word rather than a numerical value (e.g., **001** is not the same as **01**).

- D** How many bit strings exist of length 8?

Solution: product rule! There are 2 choices for the first digit AND 2 for the second, etc.

This gives us $2^8 = 256$

- E** How many bit strings of length 10 start AND end with 0?

Solution: $2^8 = 256$

- F** How many bit strings of length 10 start OR end with 0, but not both?

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Solution: Sum rule! We could start with a 0 but end with 1 ($2^8 = 256$), or end with a 0 but start with a 1 ($2^8 = 256$), giving us $256 + 256 = 512$.

G How many bit strings of length 10 have exactly one 1?

Solution: 10. The 1 could go at position 1, 2, 3, ...10.

H How many bit strings of length 10 have at most one 1?

Solution: 11. Sum rule! The 10 from the previous problem PLUS the bit string containing only zeroes.

SOLUTIONS

Practice Problems for Counting (Focus on Stars and Bars) (HW4, Question 2)

- A** How many ways are there to select 5 bills from a cash box containing \$1 bills, \$2 bills, \$5 bills, \$10 bills, \$20 bills, \$50 bills, and \$100 bills? Assume that the order in which the bills are chosen does not matter, bills of the same denomination are indistinguishable, and there are at least 5 of each type.

Solution

Bills are indistinguishable, and order doesn't matter: this is stars and bars!

- 7 types of bills, so we need 6 dividers to separate the categories
- We're making 5 selections. For example: * | | ** | * | * | | \implies this would be one \$1, no \$2s, 2 \$5s, 1 \$10, 1 \$20, zero \$50s and zero \$100s

Stars: 5

Bars: 6

$$C_{\#stars}^{\#stars + \#bars} = C_5^{5+6} = C_5^{11} = 462$$

- B** Rush and Laney are flipping a fair coin, and Rush has called Heads -- each time it comes up Heads, Rush gets a point, and each time it comes up Tails Laney gets a point. They flip the coin 10 times.

We consider an "outcome" the number of points each person has at the end of 10 flips; order doesn't matter. How many possible outcomes are there?

Solution

Order doesn't matter, because we're computing the number of points and not the permutations of H/T outcomes. This is a stars and bars problem!

Laney | Rush

xxx | xxxxxxx \iff Laney wins 3, Rush wins 7

xxxxx | xxxxx \iff Laney wins 5, Rush wins 5

Stars: 10

Bars: 1

$$C_{\#stars}^{\#stars + \#bars} = C_{10}^{1+10} = C_{10}^{11} = 11$$

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- C** How many ways are there to distribute 7 identical Cooper photos among 5 students so that no student receives more than 5 photos?

Start with the original version of the problem, and then subtract away invalid options:

Stars: 7

Bars: 4

$$C_{\#stars}^{\#stars + \#bars} = C_7^{7+4} = C_7^{11} = 330$$

Now subtract out the invalid solutions:

- Someone gets exactly 6 photos: There are 5 ways to choose the lucky student who gets 6, AND 4 ways to distribute the remaining photo: $5 \cdot 4 = 20$

OR

- Someone gets exactly 7 photos: There are 5 ways to choose the lucky student who gets all 7

So in total we get $330 - 20 - 5 = 305$

- D** We have 15 snickers bars. How many ways can we distribute them to 4 kids, if we insist that every kid gets at least one snickers?

Solution

Start by giving out 4 snickers, one to each kid.

Now we have 11 candy bars left. # stars = 11, #bars = 3

- $C(14,11) = 364$

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Practice Problems for Counting (Focus on: Permutations, Combinations, Overcounting) (HW4, Question 3)

A How many ways to permute the letters in *LANEY*?

Solution - These are all distinct letters, so it's simply $P(5, 5) = 5! = 120$

B How many ways to permute the letters in *LENNY*?

Solution - Now we have two n's, making the question a little more interesting, because the n's are indistinguishable.

We start with the same answer, $P(5, 5) = 5!$

But this overcounts, assuming that $N_1 N_2 ELY$ is a different permutation than $N_2 N_1 ELY$ when in reality they are the same: *NNELY*.

So we take our original answer and divide by the number of overcounted N arrangements, giving us $\frac{P(5,5)}{P(2,2)} = \frac{5!}{2!} = 60$

C Laney is waiting for the northbound Redline at JFK station. Both Ashmont and Braintree trains come through, at unpredictable intervals. If Laney takes 10 trips where 6 are Ashmont and 4 are Briantree, how many ways can we order the trips as a sequence of Ashmont (A) and Braintree (B) trains? For example, AABBAABAB is one possibility.

Solution: $\frac{10!}{6! \cdot 4!}$

We again start with $P(10, 10) = 10!$, and divide out the overcounted A's (6 of them) and B's (4 of them).

D How many ways are there to split 10 basketball players into two teams?

Solution

Choose 5 players for one team C_5^{10}

AND choose 5 players for the other team, which really doesn't do anything because once you've chosen 5 there are 5 leftover, and they're all needed for the second team: $C_5^5 = 1$

So far we have $C_5^{10} \cdot 1$

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But, this will overcount! Because the teams are unnamed. So having players 1-5 on the first team and 6-10 on the second is the same as having players 6-10 on the first and 1-5 on the second. So we divide out the overcounting.

$$\frac{C_5^{10} \cdot C_5^5}{2!}$$

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Practice Problems for Counting (General) (HW4, Question 4)

- A** Lucy, Sally, Charlie Brown, and Linus are standing in a line to take a picture. In how many ways can they be arranged?

Solution $P(4, 4) = 4! = 24$

- B** How many ways to arrange this same group if Linus always wants to be in the leftmost position?

Solution No choice for where to put Linus, so then we're just arranging everyone else. The lineup will look like: **Linus** _ _ _

$P(3, 3) = 3! = 6$

- C** How many ways to arrange this same group if the leftmost position must always be occupied by someone whose name starts with "L"?

Solution Now we have a choice, it could be Lucy or Linus in that leftmost position, and then we arrange everyone else.

Linus _ _ _

OR

Lucy _ _ _

$2 \cdot P(3, 3) = 2 \cdot 3! = 12$

- D** How many ways to arrange this same group if there always needs to be an "L" person on the leftmost side OR the rightmost side (or both)?

Solution

Choose the "L" person and put them on the left: $2 \cdot P(3, 3) = 2 \cdot 3! = 12$

OR

Choose the "L" person and put them on the right: $2 \cdot P(3, 3) = 2 \cdot 3! = 12$

But this also count the following two arrangements as different:

- Choose Linus to go on the left, then Lucy ends up on the right: Linus _ _ Lucy
- Choose Lucy to go on the right and then Linus ends up on the left: Linus _ _ Lucy
- (And vice versa)

There are $2 \cdot 2! = 4$ of these arrangements but we counted 8 of them.

Total $12 + 12 - 4 = 20$

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- E** How many ways to arrange this same group if there always needs to be an “L” person on the leftmost side OR the rightmost side (but not both)?

Solution

Start with the solution to Part D: $12 + 12 - 4 = 20$

BUT REMOVE

The invalid arrangements:

Linus ___ Lucy

Lucy ___ Linus.

There are $2 \cdot 2! = 4$ of these

Giving us a total of $20 - 4 = 16$

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Practice Problems for Counting (Focus on: Counting Cards) (HW4, Question 5)
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- A** Suppose I roll two fair six-sided dice. In how many ways can I roll a 4 and 5? For this question, it doesn't matter which die has the 4 and which has the 5.

Solution

(4, 5) or (5, 4)
2 ways

- B** Suppose I roll two fair six-sided dice, one after the other. In how many ways can I roll a 4 and then a 5?

Solution

Just one.
(4, 5)

- C** Suppose I roll two fair six-sided dice. In how many ways can I roll two numbers whose product is 6?

Solution

4 ways
(1, 6), (6, 1), (2, 3), (3, 2)

- D** Recall that there are 52 playing cards in a standard deck. Every card has one suit (clubs, diamonds, hearts, or spades) and one value (2, 3, ..., 10, Jack, Queen, King, Ace). When you are dealt a hand of cards, you can rearrange them to your liking; so *in general* for these questions order doesn't matter.

How many 5-card hands exist?

Solution

There are $C(52, 5) = 2598960$ 5-card hands

- E** How many 5-card hands exist that contain no Aces?

Solution

Remove the 4 aces from consideration and we have are $C(48, 5) = 1712304$

- F** In Poker, a *flush* is a hand consisting of five cards of the same suit, regardless of value (ex: $10\heartsuit, 4\heartsuit, 6\heartsuit, J\heartsuit, 7\heartsuit$). How many hands constitute a flush in a standard 5-card draw of poker?

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(For those familiar with poker, your answer here should include straight flushes and royal flushes. If those terms aren't familiar to you, just stick with the description above and count all possible flushes!)

Solution

There are $C(4, 1)$ suits and $C(13, 5)$ values. Therefore, we have
 $C(4, 1) \cdot C(13, 5) = 4 \cdot 1287 = 5148$.