SOLUTIONS

CS1800
Fall 2025
Recitation 5 - Practice Questions for Homework 3
October 8 & 9, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice Problems for Set Equality (HW3, Question 1)

For each pair of set expressions below, determine whether the resulting sets are equal.

- **If yes...** Apply the laws of set equality to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give example elements for *A, B, C*, and the universal set *U* if necessary, that would yield a counterexample. Plug in your choices for *A, B, C* and *U* to show the sets are not the same.

For full credit, both *yes* and *no* answers should be clear, precise, and walk through your solution one small step at a time.

Part A $B \cup \overline{(A \cup B)}$

$B \cup \overline{A}$

Solution: the sets are equal

Proof:

$$
\begin{aligned}
& B \cup \overline{(A \cup B)} \\
= \; & B \cup (\overline{A} \cap \overline{B}) && \text{DeMorgan} \\
= \; & (B \cup \overline{A}) \cap (B \cup \overline{B}) && \text{Distributive} \\
= \; & (B \cup \overline{A}) \cap U && \text{Complement} \\
= \; & B \cup \overline{A} && \text{Identity}
\end{aligned}
$$

Part B $\overline{(A \cap (A \cup \overline{B}))}$

$\overline{B}$

Solution: they are not equal

Example: Let A = {1, 2, 3}, B = {3, 4}, and the universal set *U = {1, 2, 3, 4}*

This gives us the second expression directly, $\overline{B} = \{1, 2\}$

Simplify the first expression with these elements:

$$
\begin{aligned}
& \overline{(A \cap (A \cup \overline{B}))} \\
= \; & \overline{(\{1, 2, 3\} \cap (\{1, 2, 3\} \cup \{1, 2\})} \\
= \; & \overline{(\{1, 2, 3\} \cap (\{1, 2, 3\})} \\
= \; & \overline{(\{1, 2, 3\})}
\end{aligned}
$$

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$= \{4\}$

Part C $(A \cup \overline{(\overline{A} \cup \overline{B})}) \cap B$

$A \cap B$

Solution the sets are equal

Proof

$(A \cup \overline{(\overline{A} \cup \overline{B})}) \cap B$	
$= (A \cup (\overline{\overline{A}} \cap \overline{\overline{B}})) \cap B$	DeMorgan
$= (A \cup (A \cap B)) \cap B$	Double Complement
$= A \cap B$	Absorption

Part D $(A \cap B) \cup (C - A) \cup (C - B)$

$(A - \overline{B}) \cup C$

Solution: the sets are equal

Proof

$(A \cap B) \cup (C - A) \cup (C - B)$	
$= (A \cap B) \cup (C \cap \overline{A}) \cup (C \cap \overline{B})$	definition of difference
$= (A \cap B) \cup (C \cap (\overline{A} \cup \overline{B}))$	distributive
$= ((A \cap B) \cup C) \cap ((A \cap B) \cup (\overline{A} \cup \overline{B}))$	distributive
$= ((A \cap B) \cup C) \cap ((A \cap B) \cup \overline{(A \cap B)})$	deMorgan
$= ((A \cap B) \cup C) \cap U$	Complement
$= (A \cap B) \cup C$	Identity
$= (A - \overline{B}) \cup C$	Definition of difference

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Practice Problems for Set Builder Notation (HW3, Question 2)

Consider the subsets $A = \{6, 7, 9\}$ and $B = \{1, 3, 5, 7\}$ of the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Express each of the following sets in roster notation. This week's handout on sets defines all operators used below.

A $\{x \in A \mid x \text{ is prime}\}$

Solution
$\{7\}$

B $A - B$

Solution
$\{6, 9\}$

C $\{x \mid 3x \in A\}$ (*hint*:a universal set *U* has been declared above, ensure each *x* is in *U*)

Solution
$\{2, 3\}$

D $A \times B$

Solution
$\{(6, 1), (6, 3), (6, 5), (6, 7), (7, 1), (7, 3), (7, 5), (7, 7), (9, 1), (9, 3), (9, 5), (9, 7)\}$

E $A \Delta B$

Solution - elements that appear in A or B but not both
$\{1, 3, 5, 6, 9\}$

F $\overline{(A \cap B)}$

Solution
$\{1, 2, 3, 4, 5, 6, 8, 9\}$

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Practice Problems for Set Cardinality (HW3, Question 3)

Suppose you have sets A, B, C with the following cardinalities:

- $|A| = 12$
- $|B| = 18$
- $|C| = 7$
- $|A \cap B| = 8$
- $|A \cap C| = 3$
- $|B \cap C| = 2$
- $|A \cap B \cap C| = 1$

A What is $|A \cup B|$?

$|A \cup B| = |A| + |B| - |A \cap B| = 12 + 18 - 8 = 22$

B What is $|B \cup C|$?

$|B \cup C| = |B| + |C| - |B \cap C| = 18 + 7 - 2 = 23$

C What is $|A \cup B \cup C|$?

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
$= 12 + 18 + 7 - 8 - 3 - 2 + 1 = 25$

It's playoff season, and you're gathering your friends for some watch parties. You have asked a group of your friends and found that:

- 7 of your friends want to watch the WNBA playoffs (women's basketball).
- 5 of your friends want to watch the MLB playoffs (men's baseball).
- 3 of your friends want to watch both playoffs.
- 2 of your friends hate sports and don't want to watch either.

D How many friends did you ask?

This is the Principle of Inclusion/Exclusion. Let's define A = set of friends who like the WNBA, B = set of friends you like the MLB. We can compute the cardinality of their union:
$|A \cup B| = |A| + |B| - |A \cap B| = 7 + 5 - 3 = 9$

And then we add on the sports-haters and get 9+2=11 total people.

E How many friends like the WNBA and not the MLB?

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We have $|A| = 7$ for the total number of people who like the WNBA, but then $|A \cap B| = 3$ like both, giving us $7 - 3 = 4$ who like the WNBA and not the MLB.

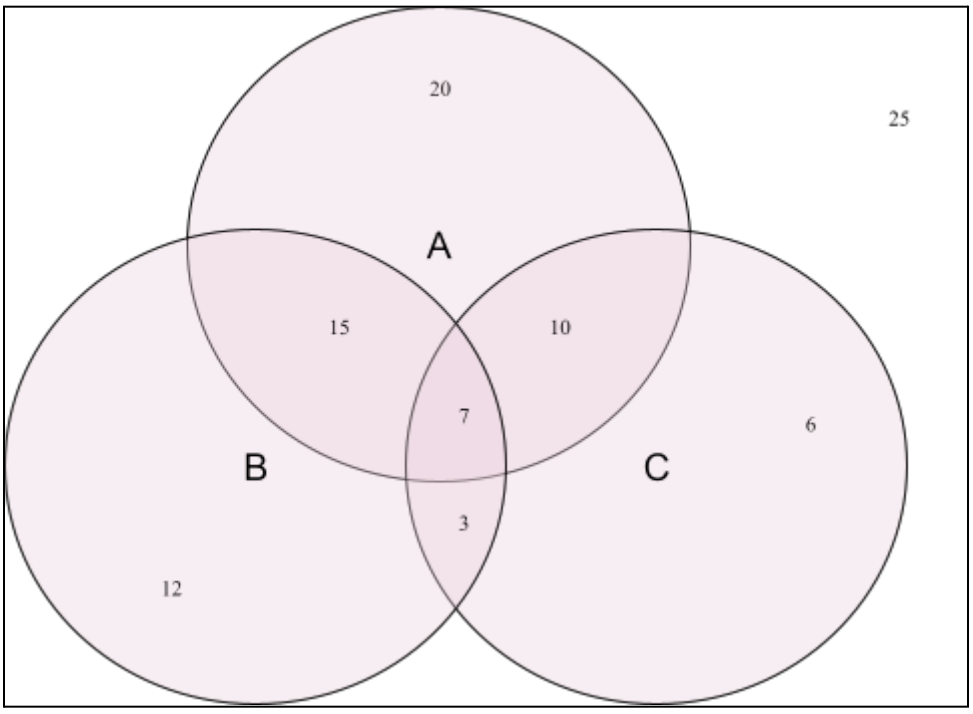
F How many friends want to watch exactly one sport?

$7 - 3 = 4$ who want to watch the WNBA only, and $5 - 3 = 2$ who want to watch the MLB only, giving us **6** friends who like exactly one sport.

G How many friends want to watch at least one sport?

This is back to PIE! $|A \cup B| = |A| + |B| - |A \cap B| = 7 + 5 - 3 = 9$

Answer the following questions based on the Venn Diagram below.



H What is the cardinality of the universal set, $|U|$?

Solution:
Venn Diagrams are disjoint, so we simply add all the components together

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$$20 + 12 + 6 + 15 + 10 + 7 + 3 + 25 = 98$$

I What is $|A|$?

Solution:

$$20 + 15 + 7 + 10 = 52$$

J What is $|A - B|$?

Solution:

$$20 + 10 = 30$$

K What is $|A \cup B|$?

Solution:

Directly from Venn Diagram

$$20 + 15 + 7 + 3 + 12 + 10 = 67$$

Applying PIE

$$|A \cup B| = |A| + |B| - |A \cap B| = 52 + 37 - 22 = 67$$

L What is $|A \cup B \cup C|$?

Solution:

Directly from Venn Diagram

$$20 + 15 + 7 + 3 + 12 + 10 + 6 = 73$$

Applying PIE

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$= 52 + 37 + 26 - 22 - 17 - 10 + 7 = 73$$

M What is $|(A \Delta B) \cup C|$?

Solution:

$$20 + 12 + 6 + 10 + 7 + 3 = 58$$

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Practice Problems for Set Functions (HW3, Question 4)

For any two sets *A* and *B*, define *foo(A, B)* as the set $\{b \mid \exists a \in A \ \exists b \in B \ (b = 3a)\}$. In the subparts below, you are given *A* and *B*, and you are asked for the set *foo(A,B)*. Give your answer in roster notation.

A If $A = \{2\}$ and $B = \{6\}$, what is *foo(A,B)*?

Solution: The set builder notation for foo(A,B), $\{b \mid \exists a \in A \ \exists b \in B \ (b = 3a)\}$, gives us a description of an arbitrary element of the set foo. This arbitrary element comes from set B, and is equal to 3a for some element in A.

{6}

B If $A = \{2\}$ and $B = \{5, 6, 7, 8, 9\}$, what is *foo(A,B)*?

Solution:
{6}

C If $A = \{2, 3\}$ and $B = \{5, 6, 7, 8, 9\}$, what is *foo(A,B)*?

Solution:
{6, 9}

D If $A = \{2, 3\}$ and $B = \{5, 7, 10, 11, 12\}$, what is *foo(A,B)*?

Solution:
{}

For any two sets *A* and *B*, define *bar(A, B)* as the set $\{c \mid \forall a \in A \ \exists b \in B \ (c = a + b)\}$. In the subparts below, you are given *A* and *B*, and you are asked for the set *bar(A,B)*. Give your answer in roster notation.

E If $A = \{2, 3\}$ and $B = \{1, 2\}$, what is *bar(A,B)*?

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Solution: We're building a set with values *c* that can be constructed by taking every element in A and finding a value in B that we can add to it and get *c*. So we need to take the 2 from A, and the 3 from A, and find two different values in B to add on and get to the same answer:

- 4 = 2+2, 3 + 1

{4}

F If $A = \{2, 3\}$ and $B = \{0, 1, 2, 4, 5, 6\}$, what is *bar(A,B)*?

Solution:

- 3 = 2+1, 3 + 0
- 4 = 2+2, 3 + 1
- 7 = 2+5, 3 + 4
- 8 = 2+6, 3 + 5

{3, 4, 7, 8}

Each of the collections listed below is either (1) a powerset of a set *A,* or (2) could not possibly be the powerset of anything. Your solution should: define *A* in roster notation, or declare that the collection could not be the powerset of anything.

I {{},{*a*},{*b*},{*c*}}

Solution: *cannot be the powerset of anything*

J {{},{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

Solution: powerset of {1, 2, 3}

K {{}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

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Solution: cannot be the powerset of anything

Practice Problems for Counting (HW3, Question 5)

A Your running wardrobe consists of 5 singlets, 3 pairs of shorts, and 17 sports bras. How many different outfits can you make which have one singlet, one pair of shorts and one sports bra?

Solution: product rule! There are 5 choices for the singlet AND 3 for the shorts AND 17 for the sports bra.

This gives us $5 \cdot 3 \cdot 17 = 255$

B Suppose you have two sets, A and B with $|A| = 10$, $|B| = 5$. What is: the largest possible value for $|A \cap B|$, the smallest possible value for $|A \cap B|$, and the possible values for $|A \cup B|$?

Solution: largest possible value for $|A \cap B| = 5$
smallest possible value for $|A \cap B| = 0$
Values for $10 \leq |A \cup B| \leq 15$

C If there are n Khoury faculty members in Boston, how many ways are there to form a faculty committee of any size?

Solution: product rule!
Every prof can be on the committee, or not.
This means for every prof there are two choices (on/off the committee), giving us 2^n

(and then we could modify this to be $2^n - 1$ for practical reasons, so we don't get a committee of size 0)

D How many three-letter initials (such as RBG) contain exactly two As? (Assume all letters are capital.)

Solution:
Sum rule!. The initials can have the non-A as the first letter, OR the second letter, OR the third letter.

For each of those cases, there are 25 possibilities for the non-A letter.

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$25 + 25 + 25 = 75$