SOLUTIONS

CS1800
Fall 2025
Recitation 4 - Practice Questions for Quiz 1
October 1 & 2, 2025

Quiz Preparation

Our first quiz is coming up on October 3rd! There are two questions on the quiz, and there are practice problems for each topic below. It can also be a useful study practice to go back and revisit previous recitation practice problems on the same topics for extra practice.

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice for Logical Equivalence (Quiz 1 Question 1)

Consider the statements p and q, below

p = You drive over 65 miles per hour
q = You get a speeding ticket

Translate the following into logic statements, using only the symbols $\neg$, $\wedge$, $\vee$, and/or $\Rightarrow$:

A You do not drive over 65 miles per hour.

Solution $\neg p$

B You drive over 65 miles per hour, but you do not get a speeding ticket.

Solution $p \wedge \neg q$

C You will get a speeding ticket if you drive over 65 miles per hour.

Solution $p \Rightarrow q$

E You will get a speeding ticket only if you drive over 65 miles per hour.

Solution $q \Rightarrow p$

F If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

Solution $\neg p \Rightarrow \neg q$

Consider the statements p and q, below

p = It is below freezing
q = It is snowing

Express each of the propositions below as an English sentence.

A $p \wedge q$

Solution: It is below freezing and snowing

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B $p \wedge \neg q$

Solution: It is below freezing but it's not snowing

C $p \Rightarrow q$

Solution: If it's below freezing, then it is snowing

D $q \Rightarrow \neg p$

Solution: If it's snowing, then it is not below freezing

Are the expressions below logically equivalent?

- **If yes...** Apply the laws of logical equivalence to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give values for p and q that would yield a counterexample. Simplify both expressions to demonstrate that they are not the same.

Both *yes* and *no* answers should be clear, precise, and walk through your solution one small step at a time.

A $(\neg p \wedge (p \vee q)) \Rightarrow q$
T

Are the two expressions logically equivalent? _______

Demonstrate equivalence with a proof, or inequivalence with a thorough counterexample.

Solution: they are equivalent.

Proof:

$(\neg p \wedge (p \vee q)) \Rightarrow q$	
$((\neg p \wedge p) \vee (\neg p \wedge q)) \Rightarrow q$	Distributive
$(F \vee (\neg p \wedge q)) \Rightarrow q$	Complement
$(\neg p \wedge q) \Rightarrow q$	Identity
$\neg(\neg p \wedge q) \vee q$	Definition of implication
$\neg\neg p \vee \neg q \vee q$	DeMorgan

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$\neg\neg p \lor T$ T	Complement Domination

B $(p \land q) \Rightarrow r$

$(p \Rightarrow r) \land (q \Rightarrow r)$

Are the two expressions logically equivalent? _______

Demonstrate equivalence with a proof, or inequivalence with a thorough counterexample.

Solution: they are not equivalent

Proof by counterexample: let p = T, q = F, r = F. Plug these values in and simplify

$(p \land q) \Rightarrow r$ $(T \land F) \Rightarrow F$ $F \Rightarrow F$ T	$(p \Rightarrow r) \land (q \Rightarrow r)$ $(T \Rightarrow F) \land (F \Rightarrow F)$ $F \land T$ F

Use truth tables to prove the following Logical Equivalence laws.

Recall that the number of rows in a truth table must be 2^n, where n is the number of simple statements involved. Note that each column should ONE step to a previous column(s). That's how we make the logical equivalence convincing -- by making it easy to show the reader how we get from one compound statement to the next.

A DeMorgan's Law $\neg(p \lor q) \equiv \neg p \land \neg q$

Solution

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$
T	T	T	F	F	F	F

T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

B Associative Law $(p \vee q) \vee r \equiv p \vee (q \vee r)$

Solution

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

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Practice for Number Representation and Predicate Logic (Quiz 1 Question 2)

A Convert the unsigned number 1011011001_2 to decimal.

Solution:

$2^0 + 2^3 + 2^4 + 2^6 + 2^7 + 2^9$
$= 1 + 8 + 16 + 64 + 128 + 512$
$= 729_{10}$

B Convert the unsigned number 1011011001_2 to hexadecimal.

Solution:

Using the lookup table:

- $1001_2 = 9_{16}$
- $1101_2 = D_{16}$
- $10_2 = 2_{16}$

Putting them together: $2D9_{16}$

C Convert the unsigned number 289_{10} to binary.

Solution:

$289 \div 2 = 144\,R\,1$
$144 \div 2 = 72\,R\,0$
$72 \div 2 = 36\,R\,0$
$36 \div 2 = 18\,R\,0$
$18 \div 2 = 9\,R\,0$
$9 \div 2 = 4\,R\,1$
$4 \div 2 = 2\,R\,0$
$2 \div 2 = 1\,R\,0$
$1 \div 2 = 0R\,1$

Flip the column to get: 100100001_2

D Convert the unsigned number 289_{10} to hexadecimal.

Solution:

$289 \div 16 = 18\, R\, 1$
$18 \div 16 = 1\, R\, 2$
$1 \div 16 = 0\, R\, 1$

Flip the column to get: 121_{16}

The grid below shows Laney's (she/her) commuting experiment during a random 30 days when sometimes the orange line or red line was shut down, and sometimes not.

	B	B		
T	T	B	T	B
	B	T	T	T
	T	B	T	T
B	B	B	B	
	T			

Every object on the grid is a T (for the MBTA) or a B (for bicycle). Each object can be colored green or purple, with no exceptions. Any object can be above another object. We define these properties with predicates as follows:

- purple(x)... x is colored purple
- $B(x)$... x is the letter B
- $T(x)$... x is the letter T
- above(x, y)... object x is in a row that's above object y
- $x \neq y$... object x is different than object y

Mark each of the statements below as True or False for the grid above.

A $\exists x\, T(x)$

Solution:

True.There is an object that is a T.

B $\exists x\, T(x) \land B(x)$

Solution:

False.There is an object that is both a T and a B.

C $\forall x\, purple(x) \wedge T(x)$

Solution:

False. This says, Every object in our universe is both purple and a T, which is not true.

Using the predicates above, logical symbols $\vee, \wedge, \neg, \Rightarrow$ *and quantifiers* $\forall, \exists$ *translate the following statements about our grid into logic.*

D All T's are colored purple.

Solution:

$\forall x\, T(x) \Rightarrow purple(x)$

E There are two different green B's

Solution:

$\exists x, y \;\; B(x) \;\wedge B(y) \;\wedge\; x \neq y \;\wedge\; \neg purple(x) \wedge \neg purple(y)$

F There is a B that is colored green and is above every T.

Solution:

$\exists x \forall y \;\; B(x) \;\wedge\; \neg purple(x) \;\wedge\; (T(y) \;\Rightarrow above(x,\, y))$