SOLUTIONS

CS1800
Fall 2025
Recitation 2 - Practice Questions for Homework 1
September 17 & 18, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice Problems for Logical Equivalence (HW1, Question 1)

For each pair of logical expressions below, determine whether they are logically equivalent.

- If yes... Apply the laws of logical equivalence to prove that they are the same. Take one step at a time and label each step with one law.
- If no... give values for p and q that would yield a counterexample. Simplify both expressions to demonstrate that they are not the same.

For full credit, both *yes* and *no* answers should be clear, precise, and walk through your solution one small step at a time.

Part A $\neg(p \land (q \lor \neg p))$

$\neg p \lor \neg q$

Are the two expressions logically equivalent? _______

Demonstrate equivalence with a proof, or inequivalence with a thorough counterexample.

Solution: they are equivalent.

Proof:

$\neg(p \land (q \lor \neg p))$	
$\equiv \neg p \lor \neg(q \lor \neg p)$	DeMorgan
$\equiv \neg p \lor (\neg q \land \neg\neg p)$	DeMorgan
$\equiv \neg p \lor (\neg q \land p)$	Double Negation
$\equiv (\neg p \lor \neg q) \land (\neg p \lor p)$	Distributive
$\equiv (\neg p \lor \neg q) \land T$	Negation
$\equiv \neg p \lor \neg q$	Identity

Part B $p \lor \neg(p \lor q)$

$p \lor \neg q$

Are the two expressions logically equivalent? _______

Demonstrate equivalence with a proof, or inequivalence with a thorough counterexample.

Solution: they are equivalent.

Proof:

$(p \lor \neg(p \lor q))$

$\equiv (p \lor (\neg p \land \neg q))$ DeMorgan

$\equiv (p \lor \neg p) \land (p \lor \neg q)$ Distributive

$\equiv T \land (p \lor \neg q)$ Negation

$\equiv (p \lor \neg q)$ Identity

Part C $(p \land q) \lor (p \land \neg q)$

q

Are the two expressions logically equivalent? _______

If yes, prove using the laws of logical equivalence.
If no, give values for p and q that would yield a counterexample and simplify both expressions.

Solution: they are NOT equivalent.

Proof with counterexample
Let $p = F,\ q = T$

Simply both expressions with these values plugged in:

$(p \land q) \lor (p \land \neg q)$	q
$(F \land T) \lor (F \land \neg T)$ $\equiv F \lor (F \land F)$ $\equiv F$	T

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Practice Problems for English & Logic (HW1, Question 2)

Consider the statements S, R, and U, below, about Laney's dogs, Grizz and Carol.

S = Carol takes Grizz's ball
R = Carol barks at Grizz
U = Grizz is annoyed.

A Translate into logical statements, using only the symbols ¬, ∧, ∨, and/or ⇒ :
Carol takes Grizz's ball, or she barks at him, but never both.

Solution: $(\neg R \wedge S) \vee (R \wedge \neg S)$

Equivalent: $(R \vee S) \wedge \neg(R \wedge S)$

B Translate into logical statements, using only the symbols ¬, ∧, ∨, and/or ⇒ :
Grizz is annoyed but Carol isn't barking at him.

Solution: $(U \wedge \neg R)$

Evaluate each of the statements below as either True or False.

C $(26 \geq 26) \wedge (26 > 27)$

Solution: False
Steps:
$26 \geq 26 = True$
$26 > 27 = False$
$True \wedge False = False$

D $(26 \geq 26) \vee (26 > 27)$

Solution: True
Steps:
$26 \geq 26 = True$
$26 > 27 = False$
$True \vee False = True$

E $\neg(19 = 3 + 3 + 13) \wedge (5 \times 2 = 15)$

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Solution: False

Steps:

$19 = 3 + 3 + 13 = True$

$\neg(19 = 3 + 3 + 13) = False$

$5x2 = 15 = False$

$False \wedge False = False$

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Practice Problems for Truth Tables (HW1, Question 3)

Recall that the number of rows in a truth table must be 2^n, where n is the number of simple statements involved. Each column should add one simple step to a previous column(s). That's how we make the logical equivalence convincing -- by making it easy to show the reader how we get from one compound statement to the next.

A Let P = Biff sleeps in, and Q = it is Saturday. Write out a truth table for $Q \Rightarrow P$. How would express $Q \Rightarrow P$ in English?

Solution -- In English we might say "Biff sleeps in if it's Saturday", or "If it's Saturday, then Biff sleeps in" or "It is saturday only if Biff sleeps in"

p	q	$q \Rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

B Use a truth table to show that an implication and its contrapositive are equivalent, i.e.:
$p \Rightarrow q \;\equiv\; \neg q \Rightarrow \neg p$

Solution

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \Rightarrow \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

C Let P = the crowd at Barclay's does the wave, and Q = Ellie the Elephant dances. Write out a truth table for "The crowd at Barclays does the wave **if and only if** Ellie the Elephant dances"

Don't worry about the logic symbols here, we just want to identify truth values that respect the original statement. For each row, briefly explain your answer.

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P	*Q*	P if and only if Q	Explanation
T	T		
T	F		
F	T		
F	F		

Solution -- when we say "P if and only if Q", then Q has to be true in order for P to be True AND vice-versa. We've promised to wave if Ellie dances, and Ellie has promised to dance if we wave. Here's how it looks in a truth table:

P	*Q*	P only if Q	Explanation
T	T	T	Here Ellie has danced and we're doing the wave. Definitely respects the original statement.
T	F	F	Here Ellie didn't dance but we did the wave anyway. This does NOT respect the original statement, because we need her to dance in order to do the wave.
F	T	F	Ellie danced, but we didn't do the wave. This disrespects Ellie herself AND the original statement.
F	F	T	Ellie didn't dance and we didn't do the wave. A bummer, but still respects the original statement because we didn't fail on any promises.

D Let *P* = the Red Sox will win the world series this year, and *Q* = the Red Sox make another stupid trade. Write out a truth table for "The Red Sox will win the world series this year **unless** they make another stupid trade."

Don't worry about the logic symbols here, we just want to identify truth values that respect the original statement. For each row, briefly explain your answer.

P	*Q*	P unless Q	Explanation
T	T		
T	F		
F	T		
F	F		

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Solution -- another way to express this in English is "If the Sox don't make another stupid trade, then they'll win!" Stopping the bad trades is A WAY we can win, but we're not saying it's the only way.

P	*Q*	P unless Q	Explanation
T	T	T	Here we've both made the dumb trade and won the world series. This is OK to be true, because maybe we won the world series
T	F	T	Here we didn't make any stupid trades, and then we won! That's what we wanted and would respect the statement.
F	T	T	We made the trade, and then didn't win. As expected, and certainly respects the original.
F	F	F	We didn't make the trade, but then we didn't win either!! This does NOT respect the original statement.

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Practice Problems for Predicates (HW1, Question 4)

A Negate the predicate $\exists x(x^2 = 2)$, fully distributing the negation such that there is no $\neg$ at all in your final expression.

Solution

Negate the original statement: $\neg\exists x\,(x^2 = 2)$

Distribute the negation:

$\equiv \forall x\,\neg(x^2 = 2)$

$\equiv \forall x\ (x^2 \neq 2)$

B Name a domain for which your negated statement in Part A is True, and one domain for which it is False.

Solution

Domain where the negation is True: all integers

Domain where the negation is False: all real numbers

C Let our domain be the real numbers. Express the statement "Every real number except zero has a multiplicative inverse" in predicate logic.

Solution

$\forall x\,(x \neq 0) \Rightarrow \exists y(x \cdot y = 1)$

$\forall x\exists y\,(x \neq 0) \Rightarrow (x \cdot y = 1)$

D Let our domain be all students in CS1800 this semester. Let $F(x, y)$ be the statement "x and y are friends." Translate this predicate into natural English:

$\exists x\forall y\forall z\,(F(x, y)\ \wedge F(x, z)\ \wedge y \neq z)) \Rightarrow \neg F(y, z)$

Solution:

Break it down into pieces, left to right:

1. There exists a student, *x*
2. Such that, for all students y for all students z
3. X and y are friends
4. AND x and z are friends
5. AND y and z are not the same (i.e., x is friends with two different people, y and z)
6. Therefore, y and z are NOT friends

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Simplify: There is a student in cs1800 none of whose friends are also friends with each other.

E Let our domain be the integers. Translate this predicate into natural English:

$$\exists x \forall y\,(x \cdot y = 0)$$

Solution:

There exists an integer x such that, no matter what integer you multiply it by, you get zero.

F Negate the logic statement from part E, fully distributing the negation. Express the final result in logic and English.

Solution:

$\neg \exists x \forall y\,(x \cdot y = 0)$	
$\equiv \forall x \exists y \neg (x \cdot y = 0)$	Flip quantifiers
$\equiv \forall x \exists y (x \cdot y \neq 0)$	Apply negation

In English:

- For all integers x there exists an integer y such that $x \cdot y$ is not zero.
- Every integer can be multiplied by some integer and get a non-zero result.

G Let our domain be the real numbers. Translate this predicate into natural English:

$$\forall x \forall y\,((x \geq 0 \wedge y < 0) \Rightarrow (x - y > 0))$$

Solution:

A non-negative number minus a negative number is positive.

H Negate the logic statement from part G, fully distributing the negation. Express the final result in logic and English.

Solution:

$\neg \forall x \forall y\,((x \geq 0 \wedge y < 0) \Rightarrow (x - y > 0))$	
$\exists x \exists y\, \neg((x \geq 0 \wedge y < 0) \Rightarrow (x - y > 0))$	Flip quantifiers
$\exists x \exists y\, \neg(\neg(x \geq 0 \wedge y < 0) \vee (x - y > 0))$	Definition of implication
$\exists x \exists y\, \neg((x < 0 \vee y \geq 0) \vee (x - y > 0))$	Distribute negation
$\exists x \exists y (\neg(x < 0 \vee y \geq 0) \wedge \neg(x - y > 0))$	Distribute negation
$\exists x \exists y (\neg(x < 0) \wedge \neg(y \geq 0) \wedge \neg(x - y > 0))$	Distribute negation
$\exists x \exists y (x \geq 0 \wedge y < 0 \wedge (x - y \leq 0))$	Distribute negation

In English a few ways:

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- There exist real numbers *x* and *y* such that *x* is non-negative, y is negative, and *x*-*y* is zero or less.
- There exist a non-negative number *x* and a positive number *y* such that their difference is at most zero.

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Practice Problems for Implications and Proofs (HW1, Question 5)

A Suppose you know for sure that *If I study, then I will ace the first CS1800 quiz.* Which of the following must also be true?

(A) - If I don't study, I will not ace the first CS1800 quiz.
(B) - If I ace the first cs1800 quiz, then I studied.
(C) - I don't ace the cs1800 quiz, then I didn't study.

Solution -- C, it is the contrapositive.

B Let our domain be the positive integers. Prove the following statement by contrapositive: If $3n + 2$ is odd, then n is odd.

Begin by clearly stating what the contrapositive is. You can also use the definition that an odd number $n = 2k + 1$ for some integer *k,* and/or an even number $n = 2k$ for some integer *k.*

Solution -- proof by contrapositive:

- If n is even, then $3n + 2$ is even.

Define $n = 2k$ for some integer *k*.
Rewrite the expression: $3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1)$

Now we've expressed $3n + 2$ as a multiple of two, so it must be even.

C Let our domain be the positive integers. Prove the following statement by contrapositive: If n^2 is odd, then n is odd.

Begin by clearly stating what the contrapositive is. You can also use the definition that an odd number $n = 2k + 1$ for some integer *k,* and/or an even number $n = 2k$ for some integer *k.*

Solution -- proof by contrapositive:

- If n is even, then n^2 is even.

Define $n = 2k$ for some integer *k*.
Rewrite the expression: $n = 2k$, therefore $n^2 = n \cdot n = 2k \cdot 2k = 2(2k^2)$

Now we've expressed n^2 as a multiple of two, so it must be even.