CS1800
Fall 2025
Recitation 12 - Practice Questions for Quiz 4 & Final Exam
December 3 & 4, 2025

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

For full credit, all four components (predicate -- provided, logic statement, base case, and inductive step) must be included and clearly labelled. Any application of the Inductive Hypothesis must be clearly noted. Take small steps in your proof, and remember that you are convincing the reader!

Practice Problems for Quiz 4 / Final Question #7 - Proof by Induction

A Exponential functions grow slower than factorial functions, e.g., $4^n < n!$. Prove by induction that the statement is true for all integers $n \geq 9$.

Predicate $P(x)$ states that $4^x < x!$

B Use mathematical induction to show the following inequality: for all integers $n \geq 3$, $2n+1 < 2^n$.

Predicate: $P(x)$states that $2x + 1 < 2^x$

C Use mathematical induction to show the following summation:

Predicate: $P(x)$states that $\sum_{i=1}^{x} i^3 = \frac{(x^2)(x+1)^2}{4}$

Practice Problems for Final Question #1 (Logical Equivalence)

See also: Recitation 4 Problem 1

Consider the logic statements below:

P = The Red Line is delayed
Q = Laney commutes by bike
R = Boston traffic is crazy

A Express the following English statement in logic, using only the statements defined above and logical operators ∨, ∧, ¬, and/or ⇒

If the Red Line is delayed or Boston traffic is crazy, then Laney commutes by bike.

B Express the following English statement in logic, using only the statements defined above and logical operators ∨, ∧, ¬, and/or ⇒

Boston traffic isn't crazy, but Laney commutes by bike if the red line is delayed.

C Are the expressions from Part A and Part B logically equivalent?

- **If yes...** Apply the laws of logical equivalence to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give values for *p, q* and *r* that would yield a counterexample. Simplify both expressions to demonstrate that they are not the same.

Practice Problems for Final Question #2 (Representation of Numbers and Predicate Logic)

See also: Recitation 4 Problem 2

Consider the following six-bit two's complement numbers:

- 111010_2
- 110011_2
- 010011_2
- 011011_2
- 100011_2

In the universe with only the five objects above, we define the following predicates:

- *pos*(*x*)... *x* is a positive number
- *neg*(*x*)... *x* is a negative number
- *val*(*x, y*)... *x* and *y* have the same absolute value
- *gt*(*x, y*)... *x* has is greater than *y*
- $x \neq y$... object *x* is different than object *y*

Using the predicates above, logical symbols $\vee$, $\wedge$, $\neg$, $\Rightarrow$ and quantifiers $\forall$, $\exists$ translate the following statements about our numbers into logic.

A There are at least two positive numbers with different absolute values.

B All positive numbers have the same absolute value.

C There is a negative number that is larger than all the other negative numbers.

D Negate the following statement, distributing the negation fully and labelling your steps along the way:

$\forall x \exists y\ (pos(x)\ \Rightarrow (neg(y)\ \wedge\ val(x,\ y))$

Practice Problems for Final Question #3 (Set Equality)

See also: Recitation 7 Problem 1

Consider the following pairs of shoes owned by one of your professors, this will be the Universal Set:
$\{superstar,\ endorphin\ pro,\ endorphin\ speed,\ novablast,\ samba, adizero\ adios,\ allbirds\}$
From those there are three sets of interest:

- $A = \{superstar,\ samba,\ allbirds\}$ casual shoes
- $B = \{superstar,\ samba,\ adizero\ adios\}$ Adidas shoes
- $C = \{adizero\ adios,\ endorphin\ pro,\ endorphin\ speed\}$ race shoes
- $D = \{adizero\ adios,\ endorphin\ pro,\ endorphin\ speed,\ novablast\}$ running shoes

Partition the shoes into the 4 subsets described below. Express each one as a set operation on *A, B, C and/or D*. Your solution should use the set operation symbols for intersection, union, complement, and difference, and no other symbols.

A Casual Adidas shoes

B The set containing Allbirds and Novablast but nothing else

C Running shoes that are race shoes or Adidas or both

D List the contents of both sets below in roster notation.
$A - (B \cup C)$
$A \cap \overline{B} \cap \overline{C}$

E Will the two sets from Part D always be the same? I.e., is $A - (B \cup C) = A \cap \overline{B} \cap \overline{C}$ true for any arbitrary sets *A, B, C?*

- **If yes...** Apply the laws of set equality to prove that they are the same. Take one step at a time and label each step with one law.
- **If no...** give example elements for *A, B,* and *C* that would yield a counterexample. Simplify both sets to demonstrate that they are not the same.

For full credit, take only step at a time, and clearly indicate which set equality law/definition is Applied.

Practice Problems for Final Question #4 (Counting)

See also: Recitation 7 Problem 2

Laney visits JP Licks and sees that there are 6 vegan flavors.

A Laney chooses 2 different vegan flavors for a double scoop. How many possible flavor combinations can she choose?

B If the order of the scoops matters (for example, chocolate on top of vanilla is different from vanilla on top of chocolate), how many possible double scoops are there?

C Laney chooses 2 vegan flavors for a double scoop, but now she might repeat a flavor. How many possible flavor combinations can she choose?

D Laney buys 3 cones, each with a double scoop chosen as in part C. If the cones are unlabeled, how many sets of 3 cones could she buy?

E How many ways are there to arrange the letters in the word *VAMPIRE*?

F How many ways are there to arrange the letters in the word *ROOKIE*?

G How many ways are there to arrange the letters in the word *LEVEL*?

H Laney is waiting for the eastbound Green Line at Hynes. Three of the four green line branches come through this station, at unpredictable intervals. If Laney takes 8 trips where 3 are on the B branch, 2 are on the C branch, and 3 are on the D branch, how many ways can we order a sequence of B, C, and D trains? (For example, CDCDBBBD is one possibility.)

I There are 12 dogs at the dog park. How many ways can you split them into three adorable teams of equal size, labelled Team A, Team B, and Team C?

J There are 12 dogs at the dog park. How many ways can you split them into three adorable teams of equal size without any labels?

Practice Problems for Final Question #5 (Probability)

See also: Recitation 10 Problem 1

A You flip three fair coins. What is the probability that all three are heads, given that at least two of them are heads?

B You flip 10 fair coins. What is the expected number of tails you'd see?

C You roll two fair six-sided dice. What is the probability that both dice show 6, given that at least one die shows 6?

Practice Problems for Final Question #6 (Sequences and Summations)

See also: Recitation 10 Problem 2

You deposit $100 into a savings account every month. The account earns 2% interest per month, and interest is applied before each new deposit. You make your first deposit right after opening the account, and you do this for 6 months total.

Let a_n be the amount in your account just after the *n*th deposit.

A What are the first four terms of the sequence representing the total amount of money in your account.

B Give a recursive formula for a_n including the base case.

Let's change it up and say you deposit $50 each month, earning 1% interest per month, for 4 months. Let b_n be the amount after the *n*th deposit.

C Give a recursive formula for a_n including the base case.

D What are the four terms of the sequence b_1, b_2, b_3, b_4?

At the Tesla factory, production of Cyber Trucks has been increasing ever since Prof. Derbinsky bought one. In the first month, Tesla made one Cyber Truck; in the second month they made two; in the third month they made three, and so on.

E Give a recursive formula for a_n including the base case.

F Give a closed-form formula for the *k*th term a_k .

G How many cars are produced in the first year?

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