SOLUTIONS

CS1800
Fall 2025
Recitation 10 - Practice Questions for Quiz 3
November 12 & 13, 2025

Quiz Preparation

Our third quiz is coming up on November 14th! There are two questions on the quiz, and there are practice problems for each topic below. It can also be a useful study practice to go back and revisit previous recitation practice problems on the same topics for extra practice.

Recitations

CS1802 Recitations are dedicated time set aside to work on practice problems that specifically prepare you for the current homework or upcoming quiz.

Recitations are in-person and attendance is expected.

The solutions are published at the same time as the problems, so you can check your work. There is no need to submit anything.

Approaching the Problems

These practice problems are labelled according to which Homework or Quiz topic they will help you prepare for. You do not need to complete every practice question; we encourage you to do at least one per topic, and to prioritize the topics you would like to practice.

Instructors & Teaching Assistants

Your recitation is led by a Khoury College professor, assisted by a knowledgeable and wonderful Teaching Assistant. Professors and TAs are fantastic resources, and you have the opportunity in recitation to work with them in a smaller group -- I strongly recommend you take advantage of the time to review your solutions to these practice problems, ask for help on the homework, or review material from lecture.

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Practice for Probability (Quiz 3 Question 1)

A A roulette wheel has 38 slots: 18 red, 18 black, and 2 green. Suppose you spin the wheel once.

What is the probability that the spin lands on red given that it is not green?

Solution:
This is a conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$, with E = the set of red spins, and F = the set of non-green spins.

$P(E \cap F)$ =the probability of red AND non-green.
$P(E \cap F) = 18/38$

$P(F)$ = probability of non-green= $36/38$

Finally, we put them together to get
$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{18/38}{36/38} = 18/38 \cdot 38/36 = 18/36 = 1/2$

B We spin the wheel 5 times. We care about the numbers only. What is the probability of getting the outcome *13-15-6-15-28?*

Solution:
This is one specific outcome out of all possibilities, so our final answer will be 1/something.

We now just need the sample space, because we are equally likely to land on any single number

Here order matters, and repetition is OK, so we have $|S| = 38^5$
And finally our probability $1/38^5$

C We spin the wheel 5 times. We care about the color **only** and not the numbers. What is the probability of getting the outcome *RRBGG?*

Solution:
This is different than Part B, because we are not equally likely to land on any color. The solution here is: $18/38 \cdot 18/38 \cdot 18/38 \cdot 2/38 \cdot 2/38$

D We spin the wheel 5 times. We care about the color **only** and not the numbers. What is the

expected number of Reds in our outcome?

Solution:
Let $X = X_1 + X_2 + X_3 + X_4 + X_5$ where X represents the number of red outcomes and X_i are indicator variables (0 or 1) denoting whether the ith spin comes up red.

$$E[X_1] = E[X_2] = E[X_3] = E[X_4] = E[X_5] = 1 \cdot 18/38 + 0 \cdot 18/38 + 0 \cdot 2/38 = 18/38$$

So $E[X] = 5E[X_1] = 5 \cdot 18/38 \approx 2.37$

E Let's modify the roulette game with a payout: You win $5 if the ball lands on red, lose $3 if it lands on black, and lose $10 if it lands on green.. How much do you expect to win on one spin?

Solution:
Win $5 with probability 18/38 (red)
Lose $3 with probability 18/38 (black)
Lose $10 with probability 2/38 (green)

We apply the expected value formula directly Pr(si) * Xi

- $E[X] = 5 \cdot 18/38 - 3 \cdot 18/38 - 10 \cdot 2/38$
  $= \$.421$

F You draw 2 cards from a standard deck without replacement.What is the probability that they are both hearts?

Solution:
Order matters: $13/52 \cdot 12/51 = .0588$

Order doesn't matter: $\frac{C(13,2)}{C(52,2)} = \frac{78}{1326} = .0588$

G You draw 2 cards from a standard deck without replacement.What is the probability that they are both hearts, given that at least one is a heart?

Solution:
This is a conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$, with E = the set of all 2-card hands that are both hearts, and F = the set of all 2-c-ard hands with at least one heart.

The event of both, $E \cap F$, is the same as just E which we already computed.

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$$Pr(E \cap F) = 78/1326 = .0588$$

The event space F is the events where we have at least one heart, i.e.,. the complement of zero hearts. There are 39 non-hearts, therefore:

- There are $C(39, 2) = 741$ ways to have zero hearts.
- $Pr(F) = 1 - 741/1326 = .4411$

Giving us: $P(E|F) = \frac{P(E \cap F)}{P(F)} = .0588/.4411 = .1333$

H You draw 2 cards from a standard deck without replacement. Given that both cards in your two-card hand are red (hearts or diamonds), what is the probability they're both hearts?

Solution:

This is a conditional probability, $P(E|F) = \frac{P(E \cap F)}{P(F)}$, with E = the set of all 2-card hands that are both hearts, and F = the set of all 2-card hands that are both red.

The event of both, $E \cap F$, is the same as just E which we already computed.

$$Pr(E \cap F) = 78/1326 = .0588$$

The event space F is the event where we have two red cards:

- There are $C(26, 2) = 325$ ways to have two red cards
- $Pr(F) = 325/1326 = .2451$

Giving us: $P(E|F) = \frac{P(E \cap F)}{P(F)} = .0588/.2451 = .24$

I You draw 2 cards from a standard deck without replacement. How many hearts do you expect to have?

Solution:

Let $X = X_1 + X_2$ where X represents the number of hearts and X_i are indicator variables (0 or 1) denoting whether card i is a heart.

$$E[X_1] = E[X_2] = 1 \cdot 13/52 + 0 \cdot 39/52 = 13/52$$

So $E[X] = 2E[X_1] = 2 \cdot 13/52 = .5$

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Practice for Sequences & Summations (Quiz 3 Question 2)

Consider the sequence that starts with the terms $3,\ 5,\ 7,\ 9,\ \ldots$

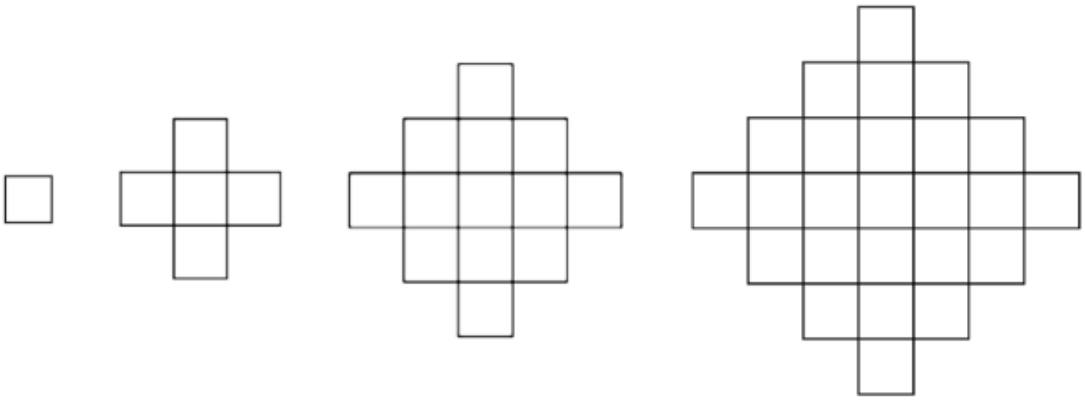
A What type of sequence is this?

Solution: arithmetic, with a common difference of 2 and initial value 3.

B What is the value of the 12th term, a_{12}?

Solution: $a_{12} = 3 + 11 \cdot 2 = 25$

For this problem, a_k denotes the number of squares in each diagram, so as depicted below,
$a_1 = 1,\ a_2 = 5,\ a_3 = 13,\ a_4 = 25$,etc.



C What type of sequence do the squares represent -- arithmetic, geometric, quadratic, or none?

Solution
Our sequence goes $1,\ 5,\ 13, 25$

The differences are: $4,\ 8,\ 12$
The second-level differences are $4,\ 4,\ 4$

So it's quadradtic!

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D How many squares would be in the 10th figure?

For this we need to solve the quadratic formula.

So we need to solve for $a,\ b,\ c$ in the equation $a_k = ak^2 + bk + c$.

We find a_k by solving a system of linear equations:

When $k = 1$.... $a + b + c = 1$
When $k = 2$.... $4a + 2b + c = 5$
When $k = 3$.... $9a + 3b + c = 13$

Let's subtract the first from the second:

$$\begin{array}{rl} & 4a + 2b + c = 5 \\ - & a + b + c = 1 \\ \hline & \end{array}$$

$3a + b = 4.$
$b = 4 - 3a$

Let's subtract the second from the third:

$$\begin{array}{rl} & 9a + 3b + c = 13 \\ - & 4a + 2b + c = 5 \\ \hline & \end{array}$$

$5a + b = 8$

Plug in b in terms of a, and solve for a.
$5a + (4 - 3a) = 8$
$2a = 4$
$a = 2$

Solve for b:
$b = 4 - 3a = -2$

Solve for c:
$a + b + c = 1$
$2 - 2 + c = 1$
$c = 1$

Now we can plug these values in for $a_{10} = 2 \cdot 100 - 2 \cdot 10 + 1 = 181$

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A staircase is built using blocks. Each step is one block tall and one block deep.

- A 1-step staircase uses 1 block
- A 2-step staircase uses 3 blocks
- A 3-step staircase uses 6 blocks
- A 4-step staircase uses 10 blocks
- A 5-step staircase uses 15 blocks

The volume of each block is 27 cubic inches.

E How many blocks would be needed for a 7-step staircase?

Solution

Our sequence goes $1,\ 3,\ 6,\ 10,\ 15,\ \ldots$

The differences are: $2,\ 3,\ 4,\ 5,\ \ldots$
The second-level differences are $1, 1, 1, 1\ldots$

So it's quadratic!

For this we need to solve the quadratic formula.

So we need to solve for $a,\ b,\ c$ in the equation $a_k = ak^2 + bk + c.$

We find a_k by solving a system of linear equations:

When $k = 1$.... $a + b + c = 1$
When $k = 2$.... $4a + 2b + c = 3$
When $k = 3$.... $9a + 3b + c = 6$

Let's subtract the first from the second:

$$\begin{array}{rl} & 4a + 2b + c = 3 \\ - & a + b + c = 1 \\ \hline & 3a + b = 2. \end{array}$$

$b = 2 - 3a$

Let's subtract the second from the third:

$$\begin{array}{rl} & 9a + 3b + c = 6 \\ - & 4a + 2b + c = 3 \\ \hline & 5a + b = 3 \end{array}$$

Plug in b in terms of a, and solve for a.

$5a + (2 - 3a) = 3$

$2a = 1$

$a = 1/2$

Solve for b:

$b = 2 - 3a = 2 - 3/2 = 1/2$

Solve for c:

$a + b + c = 1$

$1/2 + 1/2 + c = 1$

$c = 0$

Now we can plug these values in for $a_7 = 1/2 \cdot 49 + 1/2 \cdot 7 + 0 = 28$

F What is the total volume of a 7-step staircase?

From Part E, we know that there are 28 blocks, and this gives us a total of 28 * 27 = 756 cubic inches